Performance of Diversity Schemes & Spread Spectrum Systems*

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Abstract: Fading is one of the inherent effects experienced in any wireless channel; Diversity schemes help to mitigate the effect of fading. This paper deals with performance of diversity systems and Spread Spectrum technologies. Section 1 briefly touches upon diversity systems and their performance in the presence of correlated noise. Performance of modulation schemes like PSK, DPSK and FSK for a MRC and selection diversity system is discussed in sections 2 and 3 respectively. Section 4 considers Macro diversity and briefly discusses its performance. An introduction to spread spectrum, its operation, the various techniques and its performance under noisy conditions is presented in section 5. PN sequences that are so vital for spread spectrum are studied in detail with attention being given to their properties in section 6. Section 7 introduces Multiple Access Interference.

1. Diversity: Ever since H.O. Peterson and H.H. Beverage developed the first diversity receiver at Long Island, New York in the 1920's diversity schemes have been used to improve the performance when there is fading. In diversity, the receiver is provided with multiple copies of the information that is transmitted over independent channels (Real or Virtual). There are various diversity schemes like frequency, time, space, angle and polarization diversity. Despites their differences all of them provide multiple copies of the message at the receiver. Based on our observations so far it appears that as we increase the number of branches in the diversity scheme, the performance in terms of the signal strength increased either linearly or exponentially. The probability of the received signal strength being below a threshold reduces (these observations are based on our derivation of the *cdf*). However when the noise is correlated increasing the number of branches causes detioration in performance.

1.1 Effects of Correlated Noise: In all our derivations so far we had assumed the noise in the branches to be uncorrelated and independent. However in reality as this does not hold in most of the cases, we analyze diversity systems when there is correlation between the branches. Selection and switched diversity are not affected by correlation between the branches as at any time only one of the branches is connected to the output. MRC and equal gain combining are on the other hand affected by correlated noise.

Reasons for Correlated noise: Equipment such as amplifiers, frequency synthesizers may be shared between the branches depending on the implementation in which case they may introduce noise that is correlated. Before the development of MIMO, MRC was extensively used, as it was strong.

Denote the noise signal in the i^{th} branch as n_i, assume without loss of generality that the noise is zero mean and unit variance. i.e.

$$E(n_i) = 0 \quad , \; E(n_i^2) = 1 \qquad (1)$$

Let the correlation between the noise components be represented as

$$\rho_{ij} = E(n_i n_j) = \rho \qquad \forall \; i \neq j \qquad (2)$$

* Based on the lecture notes of Dr. Narayan Mandayam in Wireless Communication Technologies, Lectures 15 and 16.

The resulting noise power in a system with equal gain combining with M branches is

$$E(n^2) = E[(\sum_{i=1}^{M} n_i^2)] \qquad (3)$$

Splitting this into two summations with the same index terms in one and others in another, we have

$$\rightarrow \sum_{i=1}^{M} E(n_i^2) + \sum_{i=1}^{M} \sum_{j=1, j\neq i}^{M} E(n_i n_j) \quad (4)$$

Making use of the fact that the noise is unit variance and has a cross correlation of ρ, we have the noise power as

$$E(n^2) = M[1+(M-1)\rho] \qquad (5)$$

This is because there are M^2 terms totally out of which there are M terms with the same index and hence (M^2-M) terns with correspond to the cross correlation terms.

Recall that when we had uncorrelated noise, the noise power was M. Thus when there is correlation the noise power increases by a factor 1+ (M-1)ρ. Thus even though the signal power increases as we increase M, due to the new form of the noise power, it is possible that for some values of M and ρ, the performance detiorates as M. This results in a decrease in SNR as M increases. Hence when the noise is correlated it is possible that for some values of M and ρ the SNR decreases as M increases and hence one cannot unanimously state that as M increases the performance improves.

2 BER performance of MRC Diversity with M branches:

We obtain the BER by evaluating the conditional probability of SNR and averaging it out with respect to the *pdf* of the SNR.

2.1 BPSK with MRC:

Assume that the detector makes decision on the output signal from the MRC. The resulting SNR is the sum of the SNR in each of the branches. Recalling that the gains were g_i=k.a_i, we have

$$\gamma = \sum_{i=1}^{M} \gamma_i = \frac{E_b}{N_0} \sum_{i=1}^{M} a_i^2 \qquad (6)$$

Conditioned on γ, the probability of error is

$$P_e(\gamma) = Q(\sqrt{2\gamma}) \qquad (7)$$

The probability of error due to the MRC is

$$P_d = \int_0^\infty Q(\sqrt{2\gamma}).P_{MRC}(\gamma)\, d\gamma \qquad (8)$$

Recalling P_{MRC} as

$$P_{MRC}(\gamma) = \frac{1}{(M-1)!} \frac{\gamma^{M-1}}{\gamma_0^M} \exp(\frac{-\gamma}{\gamma_0}) \;\; \gamma \geq 0 \qquad (9)$$

The probability of error is given as

$$P_d = \int_0^\infty Q(\sqrt{2\gamma}).\frac{1}{(M-1)!} \frac{\gamma^{M-1}}{\gamma_0^M} \exp(\frac{-\gamma}{\gamma_0}) d\gamma \quad (10)$$

Integrating, we have

$$\rightarrow (\frac{1-\mu}{2})^M \sum_{i=0}^{M-1} {}^{M+1-i}C_i \, (\frac{1+\mu}{2})^i \;\; , \mu = \sqrt{\frac{\gamma}{1+\gamma_0}} \qquad (11)$$

we need to be careful here as γ_b is the SNR per bit and differs for the various diversity schemes. For time and frequency diversity γ_b=M.γ_0 as we transmit the bit M times whereas for space diversity $\gamma_b=\gamma_0$.

Time and Frequency Diversity: γ_b=M. γ_0
At BER= 10^{-5}
M=2 gains 20dB over M=1
M=4 gains 29dB over M=1

Space Diversity: $\gamma_b=\gamma_0$
At BER=10^{-5}
M=2 gains 23dB over M=1
M=4 gains 35 dB over M=1

In case of space diversity as we don't waste energy retransmitting the same bit, the gain is greater than in the previous case (time or frequency diversity).

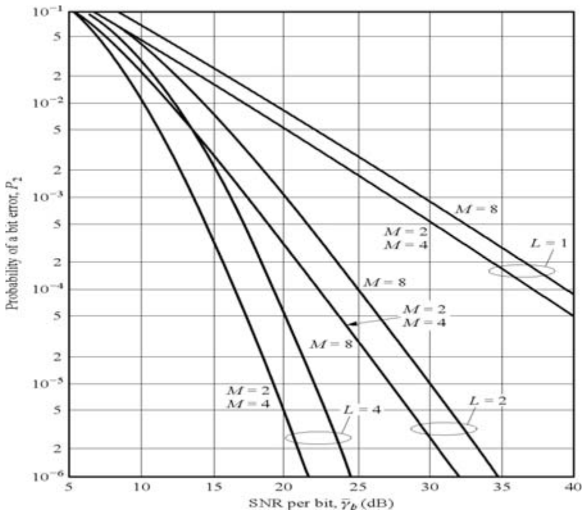

Figure 1. Probability of error with L branches & order of modulation M.

Diversity offers immense gains (20-30dB) as we see here and hence practically we stick to M=2 as the gain is high and after that the gains are affected by the law of diminishing returns.

For large SNR $\gamma_0 >> 1$
It is found that

$$\frac{1+\mu}{2}=1 \ \& \ \frac{1-\mu}{2}=\frac{1}{4\gamma_0} \tag{12}$$

Note that

$$\sum_{i=0}^{M+1-i} {}^{M+1-i}C_i = {}^{2M}C_M \tag{13}$$

for large SNR (i.e. when γ_0 >10dB)

$$P_d = {}^{2M-1}C_M . (\frac{1}{4\gamma_0})^M \tag{14}$$

When we did only BPSK we found that the bit error rate was α $1/4\gamma_0$. Here it goes down as $1/(4\gamma_0)^M$. Thus as M increases, the decay of P_d is much faster. With an Additive White Gaussian noise Channel the decay was exponential; fading made it linear. By introducing diversity we have been able to improve the decay.

2.2 FSK with MRC: In case of FSK with MRC the error is derived similarly as

$$P_d = \binom{2M-1}{M} (\frac{1}{2\gamma_0})^M \tag{15}$$

2.3 DPSK: Since coherent detection is difficult we often go for DPSK. This is used in many systems.

$$P_d = \binom{2M-1}{M} (\frac{1}{2\gamma_0})^M \tag{16}$$

2.4 Non-Coherent Orthogonal FSK:

$$P_d = \binom{2M-1}{M} (\frac{1}{\gamma_0})^M \tag{17}$$

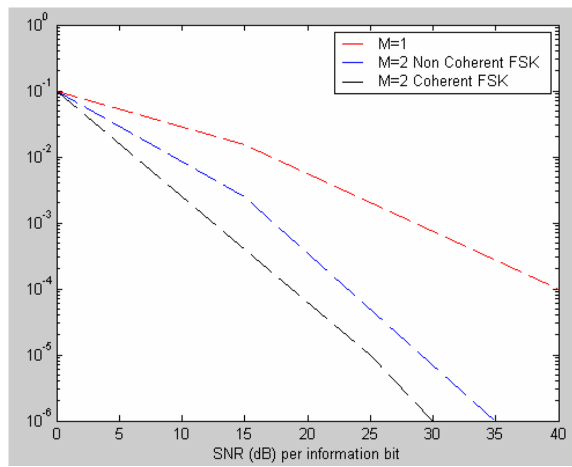

Figure.2 Error performance comparison of FSK schemes

3. Performance of Selection Diversity

$$P_d = \int_0^\infty P_e(\gamma).P_{SELECTION}(\gamma)\, d\gamma \tag{18}$$

Where we know that for selection diversity

$$P_{SELECTION}(\gamma) = \frac{M}{\gamma_0}.\exp(\frac{-\gamma}{\gamma_0}).(1-\exp(\frac{-\gamma}{\gamma_0}))^{M-1}$$

for non coherent FSK, $P_e(\gamma) = \frac{1}{2}.\exp(\frac{-\gamma}{2})$

(20)

hence we have

$$P_d = \frac{M}{2\gamma_0}\int_0^\infty \exp(-\gamma(\frac{1}{2}+\frac{1}{\gamma_0}).(1-\exp(\frac{-\gamma}{\gamma_0}))^{M-1}d\gamma \tag{21}$$

Where we have

$$(1-\exp(\frac{-\gamma}{\gamma_0}))^{M-1} = \sum_{k=0}^{M-1}(-1)^k \binom{M-1}{k}\exp(\frac{-k.\gamma}{\gamma_0}) \tag{22}$$

$$\rightarrow P_d = \frac{M}{2\gamma_0}\sum_{k=0}^{M-1}(-1)^k \binom{M-1}{k}\int_0^\infty \exp(-\gamma(\frac{1}{2}+\frac{1}{\gamma_0}+\frac{1}{k})d\gamma$$

simplifying further we have

$$P_d = M\sum_{k=0}^{M-1}(-1)^k \binom{M-1}{k}.\frac{1}{\gamma_0+2+k} \tag{23}$$

for selection diversity this is difficult to evaluate. As expected this is much poorer in performance when compared to MRC.

4. Macro Base Station Diversity:

This is primarily used to deal with shadowing on the downlink. Let s_k be the average received signal strength from the k^{th} base station. We assume that for the average received signal, Rayleigh (Small Scale Fading) is averaged out. Thus s_k are lognormal random variables.

Let there be M base stations of which only one transmits to the mobile at any time. The base station of choice is determined by looking at the strongest received signal from the mobile on the reverse link. All base stations talk with each other and decide which as to which base station is going to serve the mobile. The channel is not really symmetric as the uplink and downlink differ from each other but we assume that the received signal strength is a good measure. The base station that receives the strongest signal from the mobile is chosen.

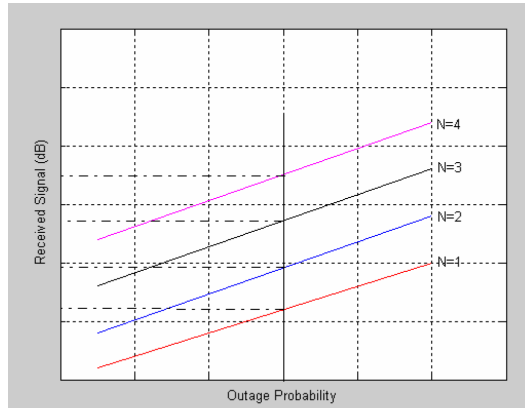

Figure.3 Performance of Macro Diversity in terms of Outage Probability vs. the received signal value (dB) with regard to the mean value.

5 Spread Spectrum Modulation:

Although the first intentional use of Spread Spectrum, was probably by Armstrong in the late '20's or early '30's with wideband FM, early spark gap "Wireless era" transmitters actually used Spread Spectrum, since their RF bandwidths were much wider than their information bandwidth. The real impetus for Spread Spectrum came with World War II. Spread spectrum systems are characterized by the use of transmission bandwidths in excess of the required bandwidth. They offer immunity to jamming and covertness to communication due to the low probability of interception.

Definition: “Spreading” refers to expansion of bandwidth well beyond what is required to transmit the data.

Principle: Due to the spreading (in the frequency domain) the signal gets buried in noise. As a consequence its exact location in the spectrum is not known and hence a jammer would have to jam the entire spectrum of white noise. This would require infinite power.

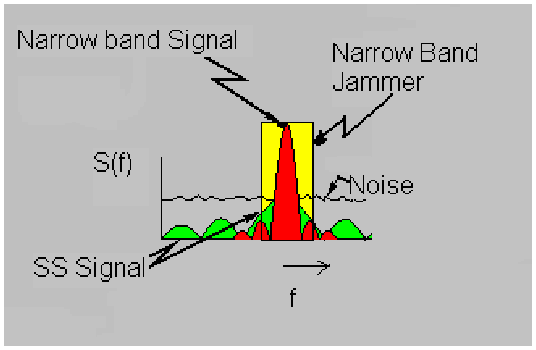

Figure.4 Principle of Spread Spectrum Systems

As a result a narrow band jammer has little impact on spread spectrum.

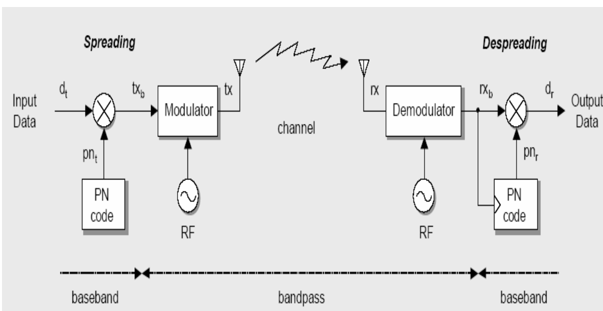

Figure.5 Spread Spectrum System

5.1 Spreading Techniques:

Direct Sequence Spread Spectrum: The spreading sequence along with the basic information (bit sequence) is used to modulate their RF carrier. Direct sequence is used in a variety of systems such as 802.11b, UMTS.

$$s(t) = \sqrt{2\rho}\, b(t)\cos(\omega_c t + \phi).c(t) \qquad (24)$$

b(t) is the bit sequence and c(t) is the spreading sequence.

Frequency Hopping: The system "hops" from frequency to frequency over a wide band. The specific order in which frequencies are occupied is a function of the code sequence. This is made use of in Blue tooth systems.

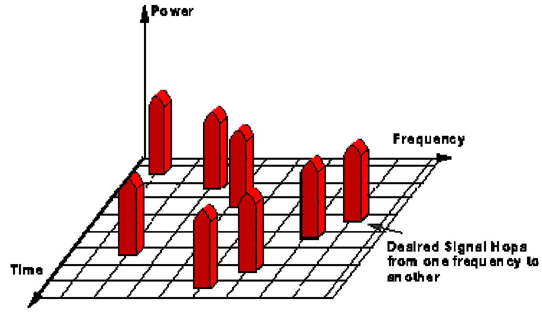

Figure.6 Hopping of frequency with time in a frequency hopping system.

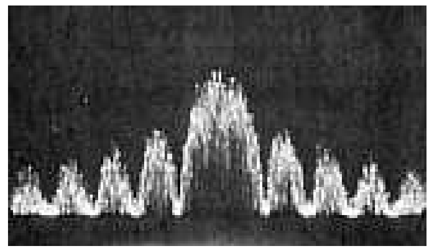
Figure.7 Spectrum of a Direct Sequence Spectrum. Courtesy Spread spectrum magazine.

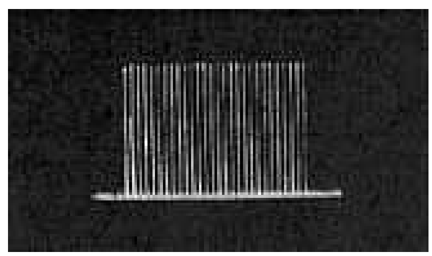
Figure.8 Spectrum for Frequency hopping system. Courtesy Spread Spectrum magazine.

DS-CDMA:

We make use of BPSK for transmission

$$b(t) = \sum_{i=-\infty}^{\infty} b_i P_T(t - iT) \quad , b_i\, \varepsilon \{-1,1\} \qquad (25)$$

$$P_T(x = 1 \;\; , 0 \le x \le) T$$

$$= 0 \quad else$$

$$c(t) = \sum_{n=-\infty}^{\infty} c_n\, \varphi(t - nT) \qquad (26)$$

$c_n\, \varepsilon \{-1,1\}$. φ(t) is the chip waveform that is time limited to $[0,T_c]$. For spreading to occur the period the period of the chip sequence should be lesser than the bit sequence i.e. $T_c<T$.

$$\frac{1}{T}\int_{0}^{T_C}\varphi^2(t)\,dt=1 \qquad (27)$$

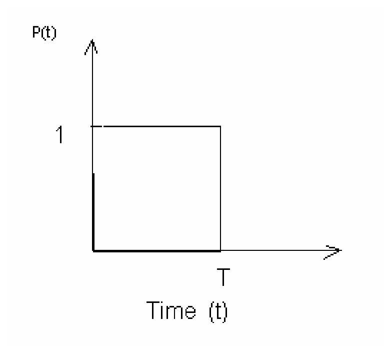

Figure.9 $P_T(t)$

Popular choices:

$$\varphi(t)=P_{TC}(t)\ \ \&\ \varphi(t)=\sqrt{2}\sin c\left(\frac{\Pi t}{T_C}\right)P_{TC}(t) \qquad (28)$$

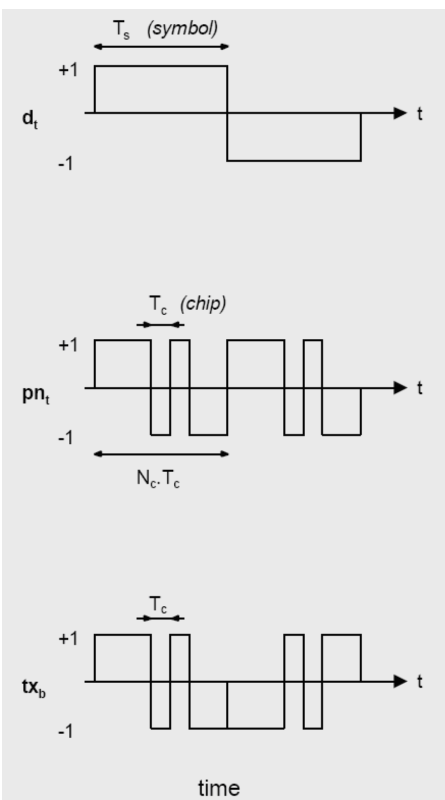

Figure.10 Bit sequence, Spreading sequence and the spread sequence

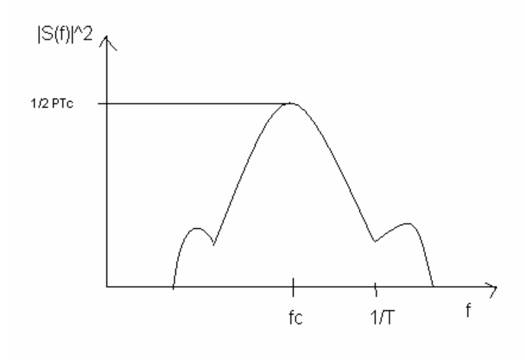

Figure.11 Square of the spectrum of the un-spread signal

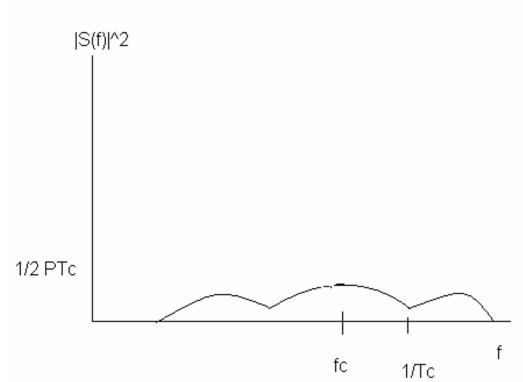

Figure.12 Square of the spectrum for the spread signal

Processing or Spreading Gain:

$$\Delta N=\frac{T}{T_C} \qquad (29)$$

It is the extent to which the spreading of the bandwidth occurs.

5.2 Bit Error Performance of Spreading a signal on AWGN channel:

Receiver: for BPSK, the received signal is

$$r(t)=\sqrt{2\rho}\,b(t)\cos(\omega_c t+\phi).c(t)+\eta(t) \quad (30)$$

where $\eta(t)$ is Gaussian noise with zero mean and PSD $N_0/2$. We assume that the carrier phase is known and hence there are no phase offsets.

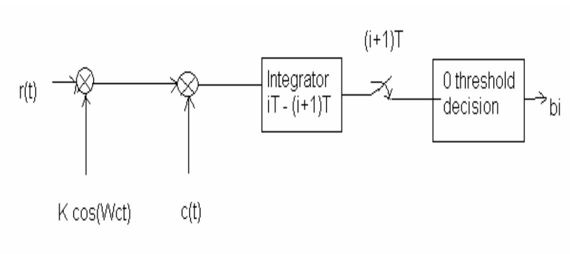

Figure.13 Correlation receiver with k $= \sqrt{2/T}$.

Matched Filter Receiver: A matched filter receiver eliminates the need for an analog multiplier. Regardless of whether it is a matched filter or correlator receiver, both produce the same output.
The output of the matched filter is given as

$$z = \sqrt{E_b} + \eta^1 \qquad (31)$$

where we have

$$\eta^1 = \int \eta\ (t) \sqrt{\frac{2}{T}}\ \cos(\omega_c t)\, dt \qquad (32)$$

i.e. the projection of the noise onto $\sqrt{\frac{2}{T}}\ \cos(\omega_c t)$

η^1 is normal with zero mean i.e. $N(0,N_0/2)$. Hence

$$z = N(\pm\sqrt{E_b}\,, \frac{N_0}{2}) \qquad (33)$$

$$P_b = P[z > o\,|\,b_i = -1] = Q(\sqrt{\frac{2E_b}{N_0}}) \qquad (34)$$

By symmetry the total probability of error is given as

$$P_e = Q(\sqrt{\frac{2E_b}{N_0}}) \qquad (35)$$

Thus in AWGN channel spread spectrum systems have no performance gain as BPSK too yields the same probability of error. However we still use Spread Spectrum as it offers resistance to jamming and has low probability of interception.

Advantages of CDMA:

There is graceful degradation in performance with an increase in the number of users unlike in the case of TDMA or FDMA where the number of slots is fixed and an increase causes much degradation in performance.

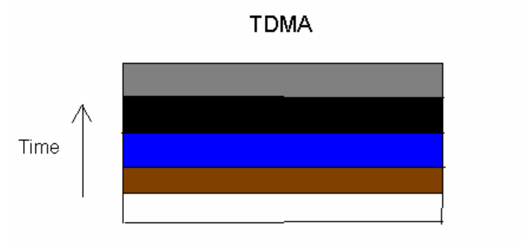

Figure.14 Principle behind TDMA. Division of time slots for different users.

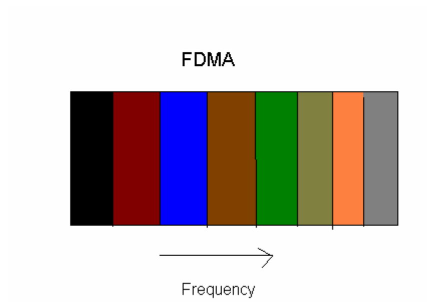

Figure.15 Division of frequency bands for individual users.

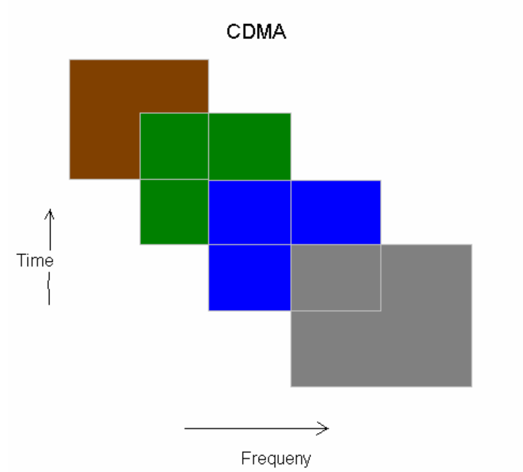

Figure.16 Contemporary approach of CDMA involving use of unique codes.

6. PN Sequences:

These are commonly used to generate the spreading waveform. A PN

(Pseudo Noise) sequence us a periodic binary sequence with a noise like waveform that's generated by means of a feedback register that uses m stages (each of which is a flip flop) and a logic circuit.

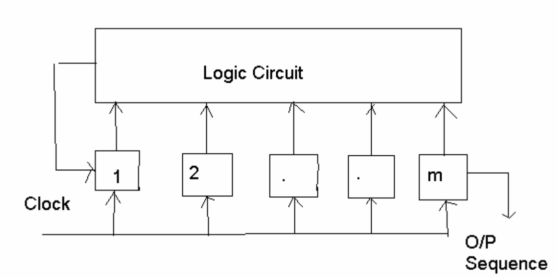

Figure.17 PN sequence Generator

The flip-flops are regulated by a single timing clock and at each tick of the clock the following things happen

1) The state of each flip flop is shifted to the next one down the line
2) The logic circuit computes some Boolean function of the m states.
3) The result of this operation is fed to flip flop 1.
4) The output of the m^{th} flip flop is the output sequence.

Let $S_j(k)$ denote the state of the j^{th} flip flop after the k^{th} clock pulse. The state of the shift register after the clock pulse is $\{S_1(k), S_2(k), \ldots\ldots\ldots\ldots S_m(k)\}$

$$S_j(k+1) = S_{j-1}(k) \quad for\, k \le 0, 1 \le j \le m \qquad (36)$$

$S_0(k)$ is the the input applied to the first flip flop after the k^{th} clock pulse.

$$S_0(k) = F(S_1(k), S_2(k) \ldots\ldots\ldots S_m(k)) \qquad (37)$$

where F is some Boolean function.

6.1 Observations: For some fixed m, the Boolean function uniquely determines the subsequent sequence of states i.e. uniquely determines the PN sequence. If the Boolean function is modulo-2 addition then the feed back shift register is called "linear". When we have a linear register, we exclude the "all zeros" state as the circuit gets stuck in this state and would not move out. Thus a PN sequence produced by a LFSR with m flip-flops can't exceed 2^m-1 in period. When the period is exactly 2^m-1, the PN sequence is called **M sequence** (Maximum length).

Example

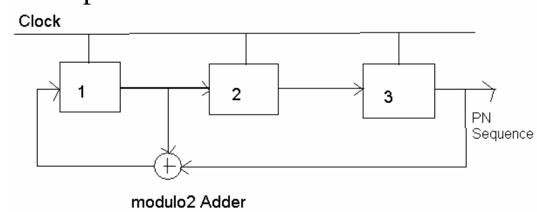

Figure.18 PN sequence generator with m=3 and period 7.

Example: Consider a PN sequence generator with m=3 i.e. 3 flip flops where the modulo2 of the first and third stages are fed back to the first stage. Table 1 shows the logical operation of the generator. This has a period of 2^3-1 =7. The choice of the initial state was arbitrary. Any other choice produces a cyclically shifted sequence. If we know the logic, it is hence possible for us to recreate the sequence at the receiver. Synchronization between the PN sequences at the transmitter and receiver is important.

Clock Count	Element 1	Element 2	Element 3	Output PN
1	1	1	0	0
2	1	1	1	1
3	0	1	1	1
4	1	0	1	1
5	0	1	0	0
6	0	0	1	1
7	1	0	0	0
8	1	1	1	0

Table.1 Logic table with the state of the logic elements and the output PN sequence. The states for count 1 & 8 are the same indicating a period of 7 (2^3-1).

The PN sequence is a binary sequence, however in order to do spreading we need +1 & –1 and thus we map o to –1 and 1 to +1.

6.2 Properties of M sequences:
M sequences have many properties of truly random binary sequences.

$$c(t) = \sum_{n=-\infty}^{\infty} c_n P_{TC}(t - nT) \qquad (38)$$

where

$$c_n = +1 \, with\ probability\ p = \frac{1}{2}$$
$$= -1 \qquad p = \frac{1}{2}$$

Let the period of the original waveform be

$$T_b = N T_c ,\, N = 2^{M-1} \qquad (39)$$

N is the number of chips per bit. T_c is the chip duration of the M sequence. The auto correlation of the periodic signal c(t) of period T_b is given as

$$R_c(\tau) = \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} c(t)\, c(t-\tau)\, d\tau \qquad (40)$$

$$\rightarrow = 1 - \frac{N+1}{N T_c}|\tau|, \; |\tau| \le T_c$$
$$= \frac{-1}{N} \qquad for\ rest\ of\ period \qquad (41)$$

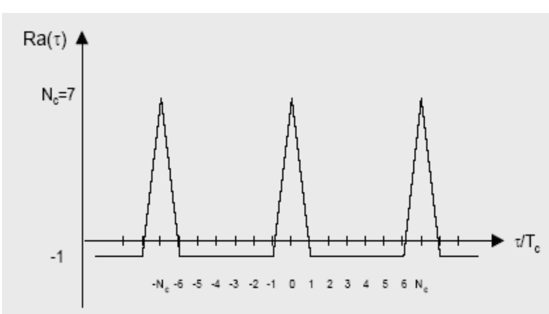

Figure.19 Autocorrelation for PN sequence with m=3 and N (period) =7.

For a truly random binary sequence cross correlation=1/N and auto correlation=1 which resemble the correlation function of the PN sequence. Hence for analysis a random binary sequence is a close enough modeling approximation.

Note that the correlation property of such spreading sequences allows DS-SS communications to have the following benefits:
1) Combat multiple access interference by assigning PN sequences to different users.
2) Combat multi path fading (CDMA offers multipath fading resistance)

7. Simplified model of MAI:
For MAI (Multiple Access Interference), we consider the received signal as the sum of the intended signal with the interference signal (signal meant for the other user) and noise.

$$r(t) = \sqrt{\frac{2E_1}{T_b}}\, b_1(t).c_1(t).\cos(\omega_c t) + \sqrt{\frac{2E_2}{T_b}}\, b_2(t).c_2(t).\cos(\omega_c t) + \eta(t) \qquad (42)$$

where η(t) is AWGN.

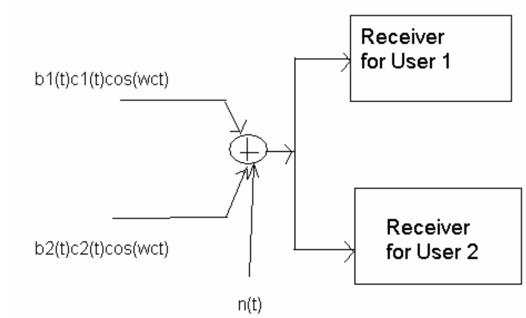

Figure.20 Signals for receivers 1 & 2 being added up with noise in the channel & being received at either of the receivers.

For user 1 we correlate with $\sqrt{\frac{2}{T_b}} c_1(t)\cos(\omega_c t)$. Hence we get

$$r_1 = \sqrt{E_1} + \sqrt{E_2} \int_0^{T_b} c_1(t)\, c_2(t)\, dt + \eta_1 \qquad (43)$$

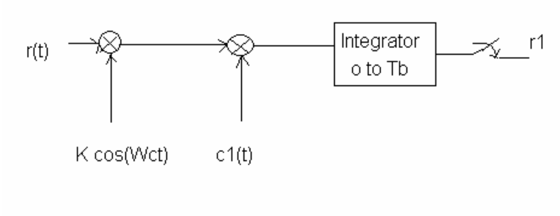

Figure.21 Receiver structure for User 1

Instead of making the second integral zero by making $c_1(t)$ and $c_2(t)$ orthogonal, we make use of the fact that the cross correlation is 1/N. thus when N is large even if both of them use the same sequence, due to delay (due to the physical separation) the cross term is less i.e. if $c_1(t)$ and $c_2(t)$ are assigned random spreading codes, then

$$\int_{o}^{T_b} c_1(t).c_1(t-\tau)dt = 0 \qquad (44)$$

for any large N. In case of the IS 95 system (CDMA-2G) synchronization between the base stations is necessary. This is to ensure that the PN sequences from two adjacent stations remains orthogonal even after transmission.

7.1 Multipath or ISI:

$$r_1 = \sqrt{E} + \sqrt{E_1}\int_{0}^{T_b} c_1(t)\,c_1(t-\tau)\,dt + \eta$$

The second term becomes zero for large N. Thus CDMA allows multiple users to access the channel and combats multipath. This provides an inbuilt provision for diversity.

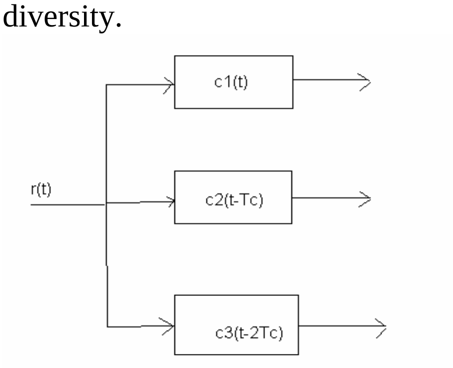

Figure.22 Rake Receiver

Advantages of CDMA:

1) Multiple access (Graceful degradation as the number of users increases)
2) Provides robustness to fading/ Multipath.
3) Universal frequency re use (All base stations use the same frequency)
4) Soft hand offs.
5) Voice activity detection (much more difficult to do in case of TDMA/ FDMA).

8.Conclusion: Regardless of whether the noise being correlated diversity systems offer better performance when compared to systems without diversity. MRC and selection diversity based systems differ in their susceptibility and performance in the presence of correlated noise. Spread Spectrum systems use bandwidths in excess of the bandwidths required and provide immunity to jamming. PN sequences are used in both direct and frequency hopping systems. Their attractive properties and their resemblance to a truly random sequence have ensured their wide spread use. Finally the significance of these spreading codes is realized in the presence of interference wherein they help to reduce the interference.

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