Wireless Communications Technologies

Course No: 16:332:546

Solution to Homework 3

1. Attenuation due to large-scale shadow fading is modelled as a lognormal random variable. Specifically, the lognormal random variable Ω_v when measured on a dB scale results in a Gaussian random variable. The transformation is $\Omega_{v(dB)} = 10\log_{10}(\Omega_v)$, where $\Omega_{v(dB)}$ is a Gaussian random variable with pdf given as

   $$p(\Omega_{v(dB)}) = \frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp(-\frac{(\Omega_{v(dB)} - \mu_\Omega)^2}{2\sigma_\Omega^2})$$

   where $\mu_\Omega = E[\Omega_{v(dB)}]$ and $\sigma_\Omega^2 = Var[\Omega_{v(dB)}]$

   The pdf of the lognormal random variable Ω_v is derived as follows:

   Let $\Omega_{v(dB)}$ be the Gaussian random variable with pdf given as

   $$p(\Omega_{v(dB)}) = \frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp(-\frac{(\Omega_{v(dB)} - \mu_\Omega)^2}{2\sigma_\Omega^2})$$

   where $\mu_\Omega = E[\Omega_{v(dB)}]$ and $\sigma_\Omega^2 = Var[\Omega_{v(dB)}]$

   If $\Omega_{v(dB)} = 10\log_{10}(\Omega_v)$, then the pdf of Ω_v, $p(\Omega_v)$ is given by solving

   $$p(\Omega_v)d\Omega_v = p(\Omega_{v(dB)})d\Omega_{v(dB)}$$

   $\Rightarrow$

   $$p(\Omega_v) = p(\Omega_{v(dB)})\frac{d\Omega_{v(dB)}}{d\Omega_v}$$

   which yields

   $$p(\Omega_v) = \frac{10/ln(10)}{\sqrt{2\pi}\Omega_v\sigma_\Omega}\exp(-\frac{(10\log_{10}(\Omega_v) - \mu_\Omega)^2}{2\sigma_\Omega^2})$$

2. The mean excess delay and the RMS delay spread for the multipath profile given in Figure 1 are:

   Mean excess delay: $\bar{\tau} = 4.38\,\mu s$

   Second moment: $\bar{\tau^2} = 21.07\mu s^2$

   The RMS delay spread is $\sigma_\tau = \sqrt{\bar{\tau^2} - \bar{\tau}^2} = 1.37\mu s$

   The coherence bandwidth is $B_c \approx 1/\sigma_\tau = 730\,kHz$.

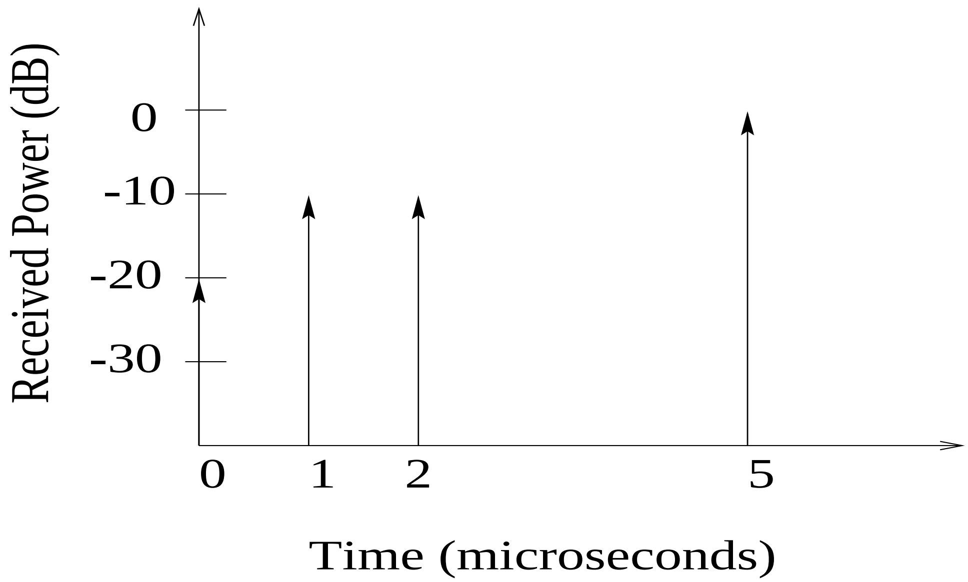

Figure 1: Multipath Profile $P(t)$ *vs.* t

3. Consider M-ary phase shift keying signaling. The psd for such a signal is given as

$$S_{vv}(f) = E_s[\frac{\sin(\pi f T)}{\pi f T}]^2$$

where T is the symbol period and E_s is the energy per symbol.

To derive a general expression (as a function of M) for the bandwidth efficiency $\eta_B = \frac{R_b}{B}$, we see that the null-to-null bandwidth B is $2/T$ where $T = \log_2(M)/R_b$. Therefore

$$\eta_B = \frac{R_b}{B} = \frac{\log_2(M)}{2}$$