Wireless Communications Technologies

Course No: 16:332:546

Homework 3 Solutions

1. The derivation of the PSD for M-ary PSK is text book stuff. You can find a detailed derivation in any of the graduate level communications text books. For example, you can find it in Stuber's book (one of the reference books for the class) on pages 197-200. The PSD is of the form:

$$S_{vv}(f) = \frac{A^2 T_b \log_2 M}{2} [\frac{\sin(\pi f T_b \log_2 M)}{\pi f T_b \log_2 M}]^2$$

2. The BER vs. SNR plots for M-ary FSK were done in class :-)

3. For $\sqrt{M} - PAM$, $s_m(t) = A_m s(t) = A_m \Re[u(t) e^{j2\pi f_c t}]$ with $A_m = 2m - 1 - \sqrt{M}$, $m = 1, 2, \cdots, \sqrt{M},\ 0 \le t \le T$

   The energy is given as $\mathcal{E}_m = \int_0^T s_m^2(t) dt = A_m^2 \mathcal{E}$,
   where the pulse energy is $\mathcal{E} = \frac{1}{2} \int_0^T \mid u(t) \mid^2 dt$

   Since the $\sqrt{M} - PAM$ system has only one-half the power of $M - QAM$ system (denoted as P_{av},

$$P_{\sqrt{M}-PAM} = \frac{1}{2} P_{av} = \frac{1}{T} \int_0^T s_m^2(t) dt = \frac{\mathcal{E}}{T} \frac{M-1}{3}$$

   Therefore, $\mathcal{E} = \frac{3 P_{av} T}{2(M-1)}$

   The output of a matched filter (matched to the pulse $u(t)$) is given as

$$U = \Re[e^{j\phi} \int_0^T r(t) u^*(t) dt] = \Re[e^{j\phi} \int_0^T (\alpha e^{-j\phi} A_m u(t) + z(t)) u^*(t) dt] = \mu_m + \nu,$$

   where $\mu_m = 2\alpha \mathcal{E} A_m$ and $\nu = \Re[\int_0^T z(t) u^*(t) dt]$ is a Gaussian random variable with $\mu_\nu = 0$ and $\sigma_\nu^2 = 2\mathcal{E} N_0$

   As a result, U is Gaussian with mean μ_m and variance $2\mathcal{E} N_0$

   In a QAM constellation, in any particular row, the end points are such that errors occur only in one direction. In this case, we have $\sqrt{M}$ points in constellation. Therefore,

$$P_{\sqrt{M}} = \frac{\sqrt{M} - 1}{\sqrt{M}} P(\mid U - \mu_m \mid > 2\alpha \mathcal{E}),$$

   which can be shown to be

$$P_{\sqrt{M}} = 2(1 - \frac{1}{\sqrt{M}}) Q(\sqrt{\frac{6}{M-1} \frac{\gamma_s}{2}}),$$

   where $\gamma_s = \alpha^2 P_{av} T / N_0$

4.

$$D^2 = \lim_{N\to\infty} \int_0^{NT} [s(t;\mathbf{x}^{(i)}) - s(t;\mathbf{x}^{(j)})]^2 dt$$

$$= \lim_{N\to\infty} [2N\mathcal{E} - 2\int_0^{NT} s(t;\mathbf{x}^{(i)}) s(t;\mathbf{x}^{(j)}) dt]$$

$$= \lim_{N\to\infty} [2N\mathcal{E} - 2\int_0^{NT} \frac{2\mathcal{E}}{T} \cos(2\pi f_c t + \phi_i(t)) \cos(2\pi f_c t + \phi_j(t)) dt]$$

Assuming $f_c T \gg 1$,

$$= \lim_{N\to\infty} [2N\mathcal{E} - \frac{2\mathcal{E}}{T} \int_0^{NT} \cos(\Delta_\phi(t)) dt]$$

$$= \frac{2\mathcal{E}}{T} \int_0^{\infty} (1 - \cos(\Delta_\phi(t))) dt = 2log_2 M E_b \frac{1}{T} \int_0^{\infty} (1 - \cos(\Delta_\phi(t))) dt$$