1. The series expansion for the Bessel function of order \( m \) of the first kind, \( J_m(z) \) is

\[
J_m(z) = \left(\frac{z}{2}\right)^m \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \Gamma(m + k + 1)} \left(\frac{z}{2}\right)^{2k}
\]

2. In general,

\[
J_m(z) = \frac{1}{\pi} \int_0^\pi \cos(mt - z \sin(t)) \, dt, \quad m = 0, 1, 2, \ldots
\]

3. The modified Bessel function of order \( m \), \( I_m(z) \) is given as the solution to the following differential equation:

\[
\frac{d^2 w}{dz^2} + \frac{1}{z} \frac{dw}{dz} - \left(1 + \frac{m^2}{z^2}\right) w = 0
\]

The relation to the Bessel function of the first kind is given as

\[
I_m(z) = i^{-m} J_m(iz)
\]

4. (a)

Let \( r(t) = r_I(t) + r_Q \). Then it follows by definition that

\[
E[|r(t)|^2 | r(t+\tau)|^2] = E[r_I^2(t)|r_I^2(t+\tau)|] + E[r_Q^2(t)|r_Q^2(t+\tau)|] + E[r_I^2(t)|r_Q^2(t+\tau)|] + E[r_Q^2(t)|r_I^2(t+\tau)|]
\]

Note the following two facts:
• If $X$ and $Y$ are Gaussian random variables with zero mean (see Papoulis or any basic probability book for details):


• Since $r(t)$ is WSS, we have $\phi_{rrrr}(t) = \phi_{rr}(0)$ and $\phi_{rqq}(t) = -\phi_{rr}(t)$

Using the above two facts it follows that

$$E[|r(t)|^2 | r(t + \tau)|^2] = 4\phi_{rr}(0) + 4\phi_{rrr}(t) + 4\phi_{rqq}(t)$$

For isotropic scattering, $\phi_{rr}(t) = \frac{\Omega_p}{2} J_0(2\pi f_m t)$ and $\phi_{rqq}(t) = 0$. Therefore

$$E[|r(t)|^2 | r(t + \tau)|^2] = \Omega_p^2[1 + J_0^2(2\pi f_m t)]$$

(b)


Then

$$E[|r(t)|^2 | r(t + \tau)|^2] = 4\phi_{rr}(0) + 4\phi_{rrr}(t) + 4\phi_{rqq}(t) - 4S^2(\phi_{rr}(0) + \phi_{rrr}(t)) + 2S^4,$$

where $S^2 = E^2[X] + E^2[Y]$