Wireless Communications Technologies

Course No: 16:332:546

Solution to Homework 2

1. The series expansion for the Bessel function of order m of the first kind, $J_m(z)$ is

$$J_m(z) = (\frac{z}{2})^m \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(m+k+1)} (\frac{z}{2})^{2k}$$

2. In general,

$$J_m(z) = \frac{1}{\pi} \int_0^{\pi} \cos(mt - z\sin(t))dt, \ m = 0, 1, 2, \cdots$$

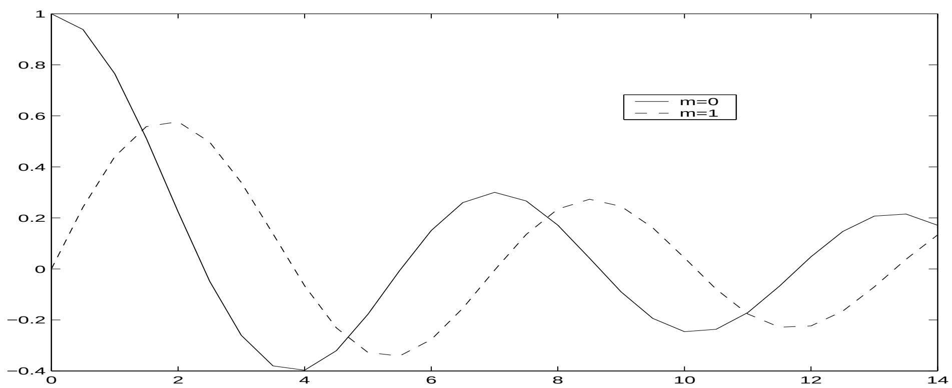

Figure 1: Bessel Functions

3. The modified Bessel function of order m, $I_m(z)$ is given as the solution to the following differential equation:

$$\frac{d^2w}{dw^2} + \frac{1}{z}\frac{dw}{dz} - (1 + \frac{m^2}{z^2})w = 0$$

The relation to the Bessel function of the first kind is given as

$$I_m(z) = i^{-m} J_m(iz)$$

4. (a)

Let $r(t) = r_I(t) + r_Q$. Then it follows by definition that

$$E[| r(t) |^2 | r(t+\tau) |^2] = E[r_I^2(t)[r_I^2(t+\tau)] + E[r_Q^2(t)[r_Q^2(t+\tau)] + E[r_I^2(t)[r_Q^2(t+\tau)] + E[r_Q^2(t)[r_I^2(t+\tau)]$$

Note the following two facts:

- If X and Y are Gaussian random variables with zero mean (see Papoulis or any basic probability book for details) :

$$E[X^2Y^2] = E[X^2] + E[Y^2] + 2E^2[XY]$$

- Since $r(t)$ is WSS, we have $\phi_{r_Q r_Q}(t) = \phi_{r_I r_I}(t)$ and $\phi_{r_I r_Q}(t) = -\phi_{r_Q r_I}(t)$

Using the above two facts it follows that

$$E[| r(t) |^2 | r(t+\tau) |^2] = 4\phi^2_{r_I r_I}(0) + 4\phi^2_{r_I r_I}(t) + 4\phi^2_{r_I r_Q}(t)$$

For isotropic scattering, $\phi_{r_I r_I}(t) = \frac{\Omega_p}{2} J_0(2\pi f_m t)$ and $\phi_{r_I r_Q}(t) = 0$ Therefore

$$E[| r(t) |^2 | r(t+\tau) |^2] = \Omega_p^2[1 + J_0^2(2\pi f_m t)]$$

(b)

When the Gaussian random process is not zero mean, then $E[X^2Y^2] = 2E^2[XY] + E[X^2]E[Y^2] - 4E[X]E[Y]E[XY] - E^2[X]E[Y^2] - E^2[Y]E[X^2] + 2E^2[X]E^2[Y]$

Then

$$E[| r(t) |^2 | r(t+\tau) |^2] = 4\phi^2_{r_I r_I}(0) + 4\phi^2_{r_I r_I}(t) + 4\phi^2_{r_I r_Q}(t) - 4S^2(\phi_{r_I r_I}(0) + \phi_{r_I r_I}(t)) + 2S^4,$$

where $S^2 = E^2[X] + E^2[Y]$