Wireless Communications Technologies

Course No: 16:332:546

Homework 1

1. Let $\lambda > 0$, and for each integer $n \geq \lambda$, $\{X_{i,n}\}$ are independent and identically distributed random variables with $P[X_{i,n} = 1] = \lambda/n$, and $P[X_{i,n} = 0] = 1 - \lambda/n$. Let $Y_n = \sum_{i=1}^{n} X_{i,n}$.

   (a) Find the characteristic function of Y_n.

   (b) Show that $\lim_{n\to\infty} Y_n$ is a Poisson random variable. Find its mean and variance.

2. Given $\{X_t, t \geq 0\}$ is a Wiener process with mean $\mu_X(t) = \mu t$ and correlation function $\phi_X(t,s) = \sigma^2 \min(t,s) + \mu^2 ts$. Let $\{Y_t, t \geq 0\}$ be such that $Y_t = X_{t+D} - X_t$, where D is a fixed positive number. Find the correlation function $\phi_Y(t,s)$ of Y_t. Is Y_t strictly stationary ?

3. An analog signal is sampled at a sampling rate of $8\,KHz$. It is then quantized using a total of 64 representation levels, followed by binary encoding. The digitized information is then transmitted over a baseband M-ary PAM system, i.e., M is the number of amplitude levels that the pulse amplitude modulator produces. Find the minimum bandwidth required for transmission for the following cases:

   (a) $M = 2$

   (b) $M = 4$

4. Consider a PCM system employing on-off keying. A bit 1 is represented by a pulse of height A for a duration of 1 second and a bit 0 is represented by sending no pulse for a duration of 1 second. The signals are transmitted over a AWGN channel with zero mean and power spectral density 1/2. A receiver is designed as shown in Figure 1 to decide if a 0 or 1 was transmitted.

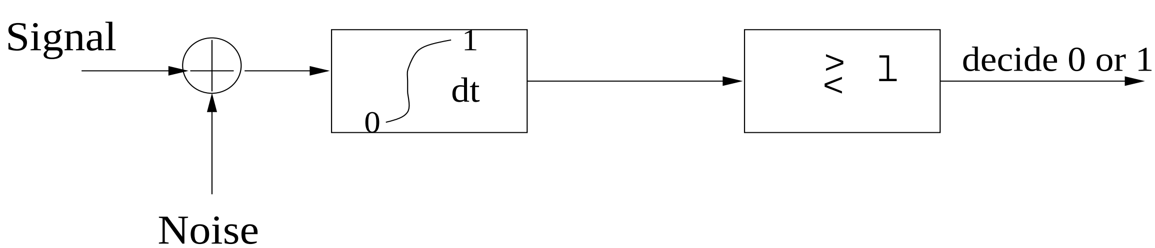


Figure 1: Receiver for the PCM System with On-off Keying

   (a) Assuming equiprobable bit-transmission, find the optimum threshold λ that minimizes the probability of error.

(b) Using the threshold in part (a), evaluate the average probability of error for this receiver in terms of the the complementary error function erfc(x) which is given as

$$\text{erfc(x)} = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-z^2)dz$$

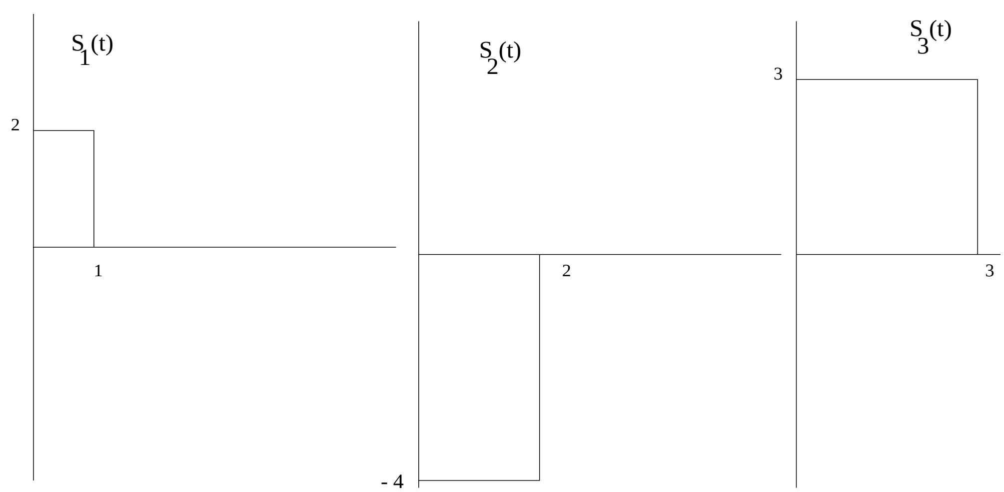
5. A binary PAM wave is to be transmitted over a baseband channel with an absolute maximum bandwidth of $60\,KHz$. The bit rate of the system is $100\,Kbps$. If we are to design a raised cosine spectrum that satisfies these requirements, find the *rolloff factor* α of the raised cosine pulse ?

6. Consider an AWGN process $w(t)$ with zero mean and spectral density $N_0/2$. The process $w(t)$ is projected onto a set of orthonormal basis functions $\{\phi_j(t)\}$, $j = 1, \cdots, N$, $0 \leq t \leq T$. Show that the projections along each basis function are i.i.d. Gaussian random variables.

   The projection of $w(t)$ onto the basis $\phi_j(t)$ is defined as

$$\int_0^T w(t)\phi_j(t)dt$$

7. Using the Gram-Schmidt orthogonalization procedure, find an expansion for the set of signals shown below:



8. Using the Gram-Schmidt orthogonalization procedure, find an expansion for the set of signals $\{s_i(t)\}_{i=1}^{i=4}$, where the signal $s_i(t)$ is of the form

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos(2\pi \frac{t}{T} + i\frac{\pi}{4}), & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$