ECE559:WIRELESS COMMUNICATION TECHNOLOGIES

LECTURE 16 AND 17

Digital signaling on frequency selective fading channels

Notes Prepared by: Abhishek Sood

1 OUTLINE

In section 2 we discuss the receiver design problem in presence of an unknown channel distortion and AWGN. The channel distortion causes inter symbol interference (ISI) which in turn causes high error rates. The solution is to use an equalizer for reducing ISI in the received signal. Section 3 deals in 3 types of equalization methods. The lecture concludes with an introduction to diversity systems, which are covered in subsequent lectures.

2 MODELLING OF ISI CHANNELS

We model the land mobile radio channel as a fading dispersive channel concentrating on the effects of,

1) delay spread that causes interference between adjacent symbols (ISI)
2) A large doppler spread that indicates rapid channel variations and necessitates a fast convergent algorithm.

Recall, the complex envelope of any modulated signal is represented by,

$$v(t) = A \sum_{K} b(t-kT, \underline{\mathbf{x}}_k) \quad (1)$$

We restrict to linear modulation schemes where,

$$b(t, \underline{\mathbf{x}}_k) = \underline{\mathbf{x}}_k \, \mathbf{h}_a(t) \quad (2)$$

where, $\mathbf{h}_a(t)$ = amplitude shaping pulse and $\{\underline{\mathbf{x}}_k\}$ = complex symbol sequence

The above signal (1), when transmitted over a channel c(t) results in received signal envelope given by: (refer figure 1)

$$w(t) = \sum^{\infty} \underline{\mathbf{x}}_k \, h(t-kT) + z(t) \quad (3)$$

K=0

Where z(t) is the zero mean AWGN with PSD N_0 watts/hertz and

$$h(t) = \int_{-\infty}^{\infty} \mathbf{h}_a(\delta)\, c(t-\delta)\, d\delta \qquad (4)$$

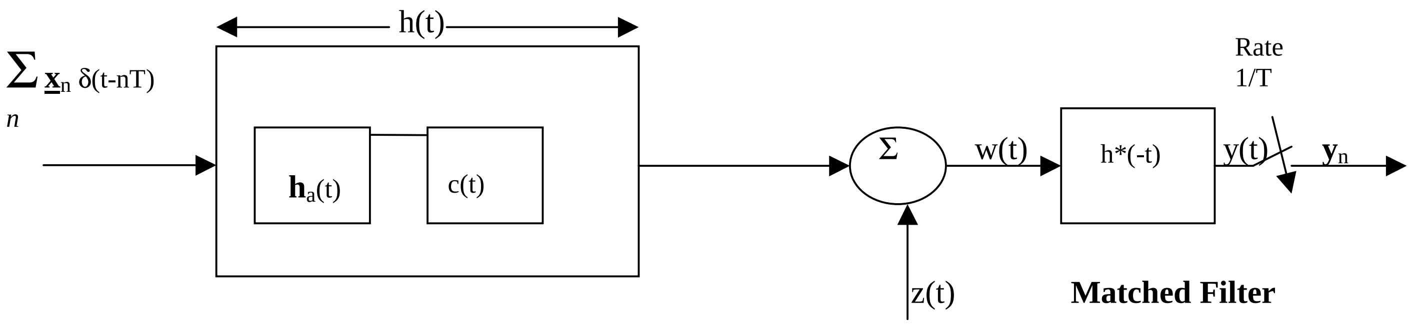
if we assume a causal channel(physical channel), the lower limit becomes zero:

$$h(t) = \int_{0}^{\infty} \mathbf{h}_a(\delta)\, c(t-\delta)\, d\delta \qquad t >= 0 \qquad (5)$$

finally assume some memory(finite duration) for pulse h(t):

$$h(t) = 0,\ t<=0$$
$$= 0,\ t>=LT$$

where L is some positive integer.



Complete model for an ISI channel with a matched filter receiver.
Figure 1

If we know h(t), a maximum likelihood receiver can be implemented, as a filter matched to h(t) as shown in Fig.1 and the complex low pass signal at the output of matched filter is:

$$y(t) = \sum_{K=-\infty}^{\infty} \underline{\mathbf{x}}_k\, f(t-kT) + v(t) \qquad (6)$$

Overall pulse response f(t) which accounts for transmit, channel and receive filter, is given by:

$$f(t) = \int^{\infty} h^*(\delta)\, h(t+\delta)\, d\delta \qquad \text{and} \qquad (7)$$

$-\infty$

$$v(t) = \int_{-\infty}^{\infty} h^*(\tau)\, z(t-\tau)\, d\tau \qquad (8)$$

is the filtered noise, which is gaussian but colored.

Sampling the matched filter output every T seconds gives the sequence:

$$\mathbf{y}_n = y(nT) = \sum_{k=-\infty}^{\infty} \underline{\mathbf{x}}_k\, f(nT-kT) + \Upsilon(nT) \qquad (9)$$

$$\mathbf{y}_n = \underbrace{\underline{\mathbf{x}}_n\, \mathbf{f}_0}_{\text{Desired symbol}} + \underbrace{\sum_{\substack{k=-\infty \\ k \neq n}}^{\infty} \underline{\mathbf{x}}_k\, f_{n-k}}_{\text{ISI}} + \Upsilon_n \qquad (10)$$

Conditions for ISI free transmission:

To achieve the same performance like an AWGN channel we need ISI free transmission. This can be achieved if (from 10),

$$\sum_{\substack{k=-\infty \\ k \neq n}}^{\infty} \underline{\mathbf{x}}_k\, f_{n-k} = 0$$

i.e. we can choose an $\mathbf{f}_k$ such that ISI = 0:

$$\mathbf{f}_k = \delta_{ko}\, \mathbf{f}_o \qquad (11)$$

and $\quad \mathbf{y}_n = \underline{\mathbf{x}}_n\, \mathbf{f}_0 + \Upsilon_n$

Equivalently in frequency domain we can write:

$$1/T \left[\sum_{n=-\infty}^{\infty} F(f + n/T) \right] = \mathbf{f}_o \qquad (12)$$

where $F(f) = \mathcal{F}\{f(t)\}$

(11) and (12) is the equivalent forms of conditions for ISI free transmission. The pulse f(t) can be a function with equally spaced zero crossings. To get rid of ISI we need to know F(t) and hence c(t). Thus, the whole design comes down to finding c(t) and then we know the receiver.

2.1 ML OPTIMUM RECEIVER FOR ISI CHANNELS WITH AWGN

Assuming that we know the channel c(t), use the karhunen loeve expansion to express the received signal as:

$$w(t) = \lim_{N\to\infty} \sum_{n=1}^{N} \mathbf{w}_n \phi_n(t) \qquad (13)$$

Where $\{\phi_n(t)\}$ form a complete set of orthonormal basis functions.

Then(refer to figure 1),

$$w_n = \sum_{k=-\infty}^{\infty} \underline{\mathbf{x}}_k \mathbf{h}_{nk} + \mathbf{z}_n \qquad (14)$$

where,

$$\mathbf{h}_{nk} = \int_0^T h(t-kT)\, \phi^*_n(t)\, dt \quad \text{and}$$

$$\mathbf{z}_n = \int_0^T z(t)\, \phi^*_n(t)\, dt \qquad (15)$$

since $\mathbf{z}_n$ is gaussian random variable, $\underline{w} = (w_1, w_2, \ldots\ldots, w_n)$ has a multivariate gaussian distribution.

$$P(\underline{w}|\underline{x},H) = \prod_{n=1}^{N} (1/\pi N_0) \exp\left\{(-1/N_0) \left| \mathbf{w}_n - \sum_{k=-\infty}^{\infty} \underline{\mathbf{x}}_k \mathbf{h}_{nk} \right|^2\right\} \qquad (16)$$

where $H = (\underline{h}_1, \underline{h}_2, \underline{h}_3, \ldots\ldots\ldots\ldots\ldots\ldots, \underline{h}_n)^T$, $\underline{h}_n = (\ldots\ldots, \underline{h}_{n,-3}, \underline{h}_{n,-2}, \underline{h}_{n,-1}, \underline{h}_{n,0}, \underline{h}_{n,1}, \underline{h}_{n,2}, \ldots\ldots\ldots)$

ML receiver decides in favour of the symbol sequence **x** that maximizes the Log likelihood function:

Choose $\underline{x}$ if

$$\text{Log } P\{\underline{w}|\underline{x},H\} > \log P\{\underline{w}|\underline{\hat{\mathbf{x}}},H\} \quad \text{for all } \hat{\mathbf{x}} \neq x \qquad (17)$$

For AWGN case we use (16) and the decision rule in (17) reduces to

$$\text{Arg}[\max_{\underline{x}} \ \mu(\underline{x}) = -\sum_{n=1}^{N} \left| \mathbf{w}_n - \sum_{k=-\infty}^{\infty} \underline{\mathbf{x}}_k \mathbf{h}_{nk} \right|^2] \qquad (18)$$

Expanding the square and ignoring the $|\mathbf{w}_n|^2$ term we get,
ML receiver chooses $\mathbf{x}$ to maximise:

$$\text{Arg}\{\max_{\underline{x}} \mu(\underline{x}) = 2\,\text{Re}\{ \sum_{k} \underline{\mathbf{x}}_k^* \sum_{n=1}^{N} \mathbf{w}_n h^*_{nk} \} - \sum_{k} \sum_{m} \underline{\mathbf{x}}_k \, x_m^* \sum_{n=1}^{N} \mathbf{h}_{nk} h_{nm}^* \}$$

where Re(y) denotes the real part of y. As $N \to \infty$, replace the 2 summations in 'n' by $\mathbf{y}_k$ and $\mathbf{f}_{m-k}$ respectively to rewrite the above equation as:

$$\text{Arg}\{\max_{\underline{x}} \mu(\underline{x}) = 2\,\text{Re}\,\{ \sum_{k} \underline{\mathbf{x}}_k^* \ \mathbf{y}_k \} - \sum_{k} \sum_{m} \underline{\mathbf{x}}_k \ \mathbf{f}_{m-k} \, \mathbf{x}_m^* \} \qquad (19)$$

$\{ \mathbf{f}_n \}$ are the ISI coefficients.

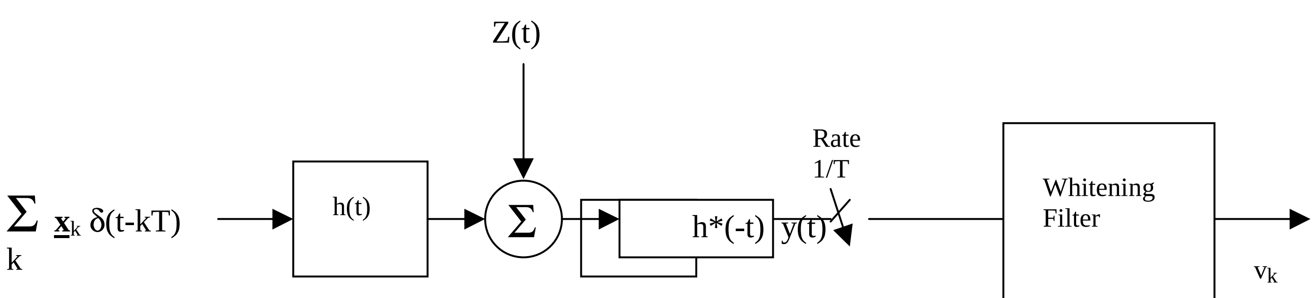
(1)Again the bottomline is that the knowledge of $\{f_n\}$ is required, which implies knowledge of channel response $\{c_n]$. Again, we need to estimate channel to equalize ISI.

(2)An additional problem results.

$$y(t) = \sum_{K=-\infty}^{\infty} \underline{\mathbf{x}}_k \, f(t-kT) + v(t)$$

Now v(t) is not white, the noise samples at the output are correlated and the equalization becomes complicated. To overcome this difficulty a noise whitening filter may be employed to process the sequence $\mathbf{y}_n$, resulting in discrete time white-noise channel model.

2.2 Discrete time white noise channel model:



Complete Model for an ISI channel with Whitening Filter
Figure 2

The discrete time samples at the output of the noise whitening filter are:

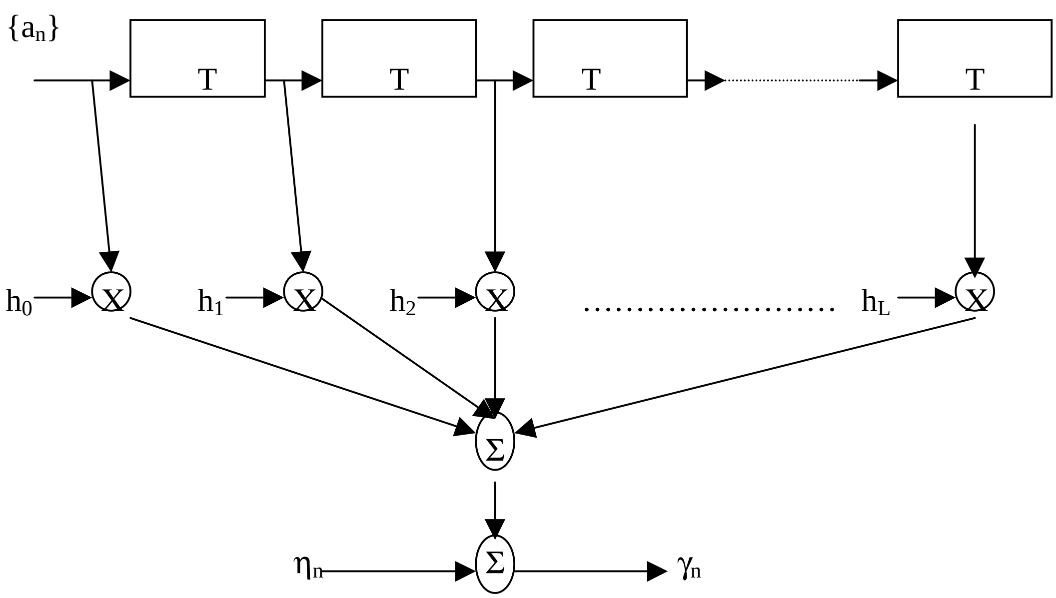
$$v_k = \sum_{n=0}^{L} g_n \underline{\mathbf{x}}_{k-n} + n_{kT} \text{ (now white)} \tag{20}$$

where g_n is the channel vector that describes the overall channel response.
We need to know the exact timings at which zero crossings occurs i.e.when zeros occur in frequency response.

(3)The design of equalizers is extremely sensitive to timing information.

Solutions:

(a)raised cosine pulse shaping (b) fractional sampling : normally we have filter taps spaced 'T' apart. Space them at say T/2 and oversample . Typically sample the output at a rate 2/T. We still have correlated noise and need whitening filter but oversampling makes the overall pulse shape less sensitive to timing errors. This leads to fractionally spaced receiver.



Discrete time white noise channel model

Figure 3

3 EQUALIZERS

The purpose of an equalizer is to mitigate the combined effects of ISI and noise.
There are 2 strategies to build equalizers :

(a)Symbol By Symbol Equalizers: Use a linear filter with adjustable coefficients or exploit the previously detected symbols to suppress ISI in the present symbol(DFE).

(b)Sequence Estimation Equalizers: Use maximum likelihood sequence detection from a probability of error viewpoint.

3.1 symbol by symbol equalizers:

They include a decision rule to make symbol by symbol decisions on received symbol sequences and are further split into 2 categories:
(1)Linear forward and (2) Non linear decision feedback equalizers

From figure 3:

$$\gamma_n = \sum_{k=0}^{L} h_k a_{n-k} + \eta_n \tag{21}$$

The equalizer estimates the nth transmitted symbols,

$$\hat{\mathbf{a}}_n = \sum_{j=-M}^{M} c_j \gamma_{n-j} \tag{22}$$

where,
$\{c_j\}$ = equalizer filter taps

N = 2M+1 =length of equalizer

Let $\underline{q} = (q_0\ q_1\ q_2 \ldots\ldots\ldots q_{n+L-1})^T$

q is the overall channel and equalizer sampled response, given as the discrete convolution of $\underline{h}$ and $\underline{c}$.

$$\text{i.e. } q_n = \sum_{j=0}^{N-1} c_j h_{n-j} = \underline{c}^T \underline{h}_n \tag{23}$$

$$\underline{h}_n = (h_n, h_{n-1}, h_{n-2}, \ldots\ldots\ldots\ldots h_{n-N+1})^T$$

where,

$h_i = 0 \quad i<0$

$\quad = 0 \quad i>L$

3.1.1 Linear Equalization

A linear forward equalizer consists of a filter with adjustable tap coefficients. It has the drawback of enhancing channel noise while eliminating ISI.

3.1.1.1 Zero forcing equalizer

It is based on the peak distortion criterion, which forces ISI to zero but ignores the effect of noise. With a zero forcing equalizer, choose the filter tap coefficients, $\underline{c}$ that minimize the peak distortion of the channel,

$$D_p = \{ 1/|q_d| \} \sum_{\substack{n=0 \\ n \neq d}}^{N+L-1} | q_n - \hat{q}_n | \qquad (24)$$

$\hat{\mathbf{q}} = (\hat{q}_0 \ldots\ldots\ldots\ldots\ldots\ldots \hat{q}_{n+l-1})^T$ is the desired equalized channel response and 'd' is the overall delay.

Perfect equalization demands that the combined channel and equalizer response is forced to zero, at all but one of the taps in the TDL filter, when $M \to \infty$, no ISI.

ZF equalizer is unsuitable for channels with severe ISI as the equalizer tries to compensate for nulls and introduces infinite gains at these frequencies. This enhances noise and is not a candidate for mobile radio applications where spectral nulls are often seen.

3.1.1.2 Mean square error equalizer

It is more robust than ZF equalizer in convergence properties and performance.

Define estimation error as $\varepsilon_n = a_n - \hat{a}_n$ (25)

Mean square error (MSE) is defined as J and to minimize the mean square error,

$$\min_{\underline{e}} J = E[\varepsilon_n^2] = \min_{\underline{e}} E[(a_n - \sum_{j=-m}^{m} c_j \gamma_{n-j})^2] \qquad (26)$$

Equalizer tap solution:

A mean squared error equalizer adjusts tap coefficients to minimize MSE.

Choose $\{c_j\}$ so as to make error sequence ε_n orthogonal to the signal sequence γ_{n-l} for $|l|<=m$.

$$E [\varepsilon_n \gamma_{n-l}] = 0 \qquad |l|<=m$$

$$\sum_{j=-M}^{M} c_j E [\gamma_{n-j} \gamma_{n-l}] = E [a_n \gamma_{n-l}] \qquad |L|<=m. \qquad (27)$$

$\underline{C}_{opt}$ (optimum coefficients) can be obtained by solving the matrix equality ,

$\underline{C}_{opt} = \Gamma^{-1} \underline{b}$

Γ is the matrix whose entries are $E [\gamma_{n-j} \gamma_{n-l}]$ $j,l = -M,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,M$

$$\underline{b} = \{ E [a_n \gamma_{n+m}] \; E [a_n \gamma_{n+m-1}] \; E [a_n \gamma_{n-m}] \}^T \qquad (28)$$

We need adaptive algorithms to implement the above criterion since the channel is not known readily.

Typically, we may use the steepest descent algorithm.

$$C_j(n+1) = c_j(n) - \tfrac{1}{2}\mu \frac{\partial}{\partial c_j} E[\varepsilon^2(n)], \quad j=0, \pm1, \pm2, \pm3, \ldots\ldots,\pm M \qquad \text{and } 0<\mu<1 \quad (28)$$

$$\frac{\partial}{\partial c_j} E[\varepsilon^2(n)] = -2\,E\left[\varepsilon(n)\frac{\partial}{\partial c_j}\varepsilon(n)\right] = -2\,E[\varepsilon(n)\,\gamma_{n-j}] = -2\,R_{\varepsilon\gamma}(j)$$

This algorithm is deterministic and assumes that E[.] is known. But the PDF's used to compute E[.] are not available. So, normally implement as stochastic gradient algorithm given by:

$$C_j(n+1) = c_j(n) + \mu\,\varepsilon(n)\,\gamma_{n-j}$$

γ_{n-j} is the input to the equalizer and $\varepsilon(n)$ is computed using training. This is employed in real systems like GSM. But a lot of effort is still devoted to finding fast convergent adaptive algorithms.

Although MSEE accounts for effects of noise, it doesn't work well with severe ISI channels or spectral nulls due to noise enhancement at the output.

3.1.2 NON LINEAR DECISION FEEDBACK EQUALIZERS (DFE)

It was proposed to mitigate the effects of noise enhancement. It consists of 2 sections; a feedforward section and a feedback section. The feedforward section is identical to linear forward equalizer discussed in 3.1.1 and reduces the precursor ISI. To eliminate the postcursor ISI, decisions made at the equalizer output are fed back through the feedback filter, which estimates ISI for these symbols. It introduces error propagation, which seriously degrades the performance of DFE and complicates performance analysis.

Recall,

$$y_n = \underline{\mathbf{x}}_n f_0 + \sum_{\substack{k=-\infty \\ k\neq n}}^{\infty} \underline{\mathbf{x}}_k f_{n-k} + \Upsilon_n$$

At a time n, all symbols till n have been decided and are used to cancel the ISI due all previous symbols and we have to deal with only the future ISI now. Thus we use previous decisions to eliminate ISI caused by previously detected symbols on the current symbol.

Output of the DFE equalizer is denoted by

$$\hat{a}_n = \sum_{j=-m}^{0} c_j^{(f|f)}\gamma_{n-j} + \sum_{j=1}^{m2} c_j^{(f|B)}\tilde{a}_{n-j} \qquad (29)$$

There are (m+1) taps in f|f part and m2 taps in the f|b part.

$\tilde{a}_{n-1}, \tilde{a}_{n-2},\ldots\ldots\ldots\ldots\ldots\tilde{a}_{n-m2}$ are the earlier detected symbols.

DFE is very effective in frequency selective fading channels.(better than linear equalizer)

Design says:

If we assume that the feedback decisions are correct then design of feed forward filter taps $c_0, c_1, c_2,\ldots c_{m+1}$ is given by

$$\text{Min } J = E\,[\,(a_n - \hat{a}_n)^2\,]$$

For the feedback part :

$$C_k = -\sum_{j=-m}^{0} c_j h_{k-j}\,,\quad c_k = \text{feedback coefficients and } c_j \text{ are the feedforward coefficient}\qquad k=1,2,\ldots m2$$

again the assumption is that the channel h_{k-j} is known. In reality we need adaptive algorithms for implementation in both FeedForward and FeedBack parts. This is not optimum, as non linear operation makes the distribution non-gaussian. [MSE is optimum for gaussian case only]

3.2 ML sequence estimation[MLSE] equalizers

These equalizers make decisions on a sequence of received symbols. They are more complex than Symbol by symbol equalizers but offer better performance. It is impractical for systems with large signal constellations and long channel response.

Consider the discrete time white noise channel model,

$$\gamma_n = \sum_{m=0}^{L} h_m a_{n-m} + \eta_n$$

Assume K symbols are transmitted over the channel, then after receiving $\{\gamma_n\}_{n=1}^{k}$, the ML receiver decides in favor of the sequence $\{a_n\}_{n=1}^{k}$ that maximizes the likelihood function.

Log-likelihood function is given by

$$\text{Log } P\,(\,\gamma_k, \gamma_{k-1}, \ldots, \gamma_1 \mid a_k, a_{k-1}, a_{k-2}, \ldots\ldots a_1\,)$$

Since the noise samples are independent and received signal γ_n depends only on L most recent transmitted symbols,

$$\text{Log } P\,(\,\gamma_k, \gamma_{k-1}, \ldots, \gamma_1 \mid a_k, a_{k-1}, a_{k-2}, \ldots\ldots a_1\,) =$$
$$\underbrace{\text{Log } P\,(\,\gamma_k \mid a_k, a_{k-1}, a_{k-2}, \ldots\ldots a_{k-L}\,)}_{\text{BRANCH- METRIC}} + \text{Log } P\,(\,\gamma_{k-1}, \gamma_{k-2}, \ldots, \gamma_1 \mid a_{k-1}, a_{k-2},\ldots\ldots a_1) \qquad (30)$$

$a_{k-l} = 0$ for $k-l \le 0$

If the second term has been previously calculated at epoch k-1 then, only the first need to be computed For each incoming signal γ_k at epoch k.

The first term as labeled is known as Branch metric.

When η_n is gaussian :

$$P(\gamma_k \mid a_k, a_{k-1}, a_{k-2}, \ldots\ldots a_{k-l}) = (1/\pi N_0)\ \exp\left\{ (-1/N_0) \left| \gamma_k - \sum_{I=0}^{L} h_i a_{k-i} \right|^2 \right\} \quad (31)$$

Taking logarithms implies the branch metric is given as:

$$\mu_k = \left| \gamma_n - \sum_{I=0}^{L} h_i a_{k-i} \right|^2 \quad (32)$$

(30) is recursive and (32) is the branch metric, ML receiver can be implemented using VITERBI algorithm by searching through the 2^{nl} state trellis for most likely transmitted sequence **x**. This search process is called MLSE. We still need the channel h_i to compute this branch metric. Any mobile phone with an equalizer implements viterbi algorithm. 2^n = digital constellation size

READING: Viterbi algorithm (pg. 295 –301)

3.2.1 Adaptive MLSE:

The viterbi algorithm uses channel vectors for computing branch metrics so that an adaptive channel estimator is required. Use training to establish the channel based on MSE. These estimates are passed onto the MLSE algorithm, which does the sequence estimation. It uses adaptive algorithms to estimate channel and then these estimates are fed to a viterbi equalizer for ML sequence estimation.(used in GSM).

Equalization was trying to undo the effects of fading/channel. Now we move on to diversity systems.

4 DIVERSITY SYSTEMS

To reduce the SNR penalty incurred due to fading we use diversity. It provides the receiver with multiple faded replicas of the same information signal and some may be better than others as far as the effects of fading are concerned. Diversity also improves the performance of data transmission over fading channels. Receiver is provided with multiple copies of information transmitted over 2 or more “independent” Transmission channels. The basic idea is “repetition of information and combination improves performance.” Diversity systems are classified into:

1)Macro diversity: It exploits combining of different base stations and mitigates long term fading effects.

2)Micro diversity : It handles short scale fading e.g. antenna elements

Diversity works well when different branches experience different fading. The diversity techniques are classified into 5 categories:

1)Frequency diversity : we transmit on 2 or more carriers separated by atleast the coherence bandwidth of the channel. It gobbles up bandwidth and is not efficient.

2)Time diversity : Transmit signals using multiple time slots separated by coherence time of the channel. it's not a good idea on slow fading channels. If fading rate is slow we need long intervals between transmission.

3)Polarization diversity : we manipulate Electro magnetic fields to obtain orthogonal polarization. Receiver antennas having different polarizations are used.

4)Angle diversity: Use multiple directional antennas to create independent copies of transmitted signals through multipath.

5)Space diversity:
Place multiple receiving antennas at different locations. It will result in different and possibly independent Signals.
Correlation between signals as a function of distance between antenna elements is given by

$\rho = J_0^2 (2\pi d/\lambda)$

If antennas are spaced at multiples of half wavelengths apart the received signals have zero correlation. We cannot do this at the mobile as the spacing required for zero correlation would be of the order of a foot .

So time/Frequency diversity uses more bandwidth while space diversity gives complex antennas.

5 References

1) Prof. Narayan Mandayam. Lecture notes from march 22'nd and 24'th,

2) Gordon L. Stuber, Principles of Mobile Communication, Second ed.

3) John G. Proakis, Digital Communications , Third edition.