Course: Wireless Communication Technologies

16:332:559 (Advanced Topics in Communication Engineering)
Lecture #9 and #10
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I *QPSK:*

In digital phase modulation, signal space diagram for M-ary modulation when M=4 is shown in figure 9.1. The dotted line represents the possible phase transition. This 4-phase PSK is also called Quadrature Phase Shift Keying (*QPSK*).

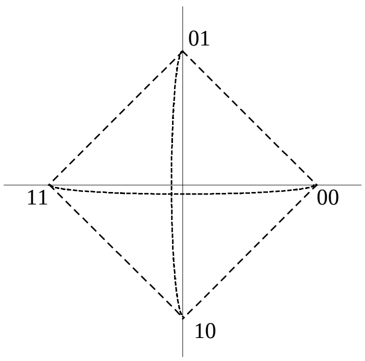


Fig 9.1 Phase transition of QPSK

When phase transition goes through the origin, abrupt phase reversal will happen in signal envelope. Any kind of hardlimiting or nonlinear amplification of the origin-crossings brings back the filtered sidelobes since the fidelity of the signal at small voltage levels is lost in transmission.

To prevent the regeneration of side-lobes and spectral widening, it is imperative that QPSK signals be amplified only using linear amplifiers, which are less efficient. A modified form of *QPSK*, called offset QPSK (*OQPSK*) is less susceptible to these deleterious effects and supports more efficient amplification.

II *OQPSK*

In *QPSK*, bit transitions are allowed that odd and even streams occur simultaneously. While in *OQPSK*, the bit stream is first broken into even and odd bit streams. And even and odd bitstreams are offset in alignment by one bit period. As is shown in the following figure:

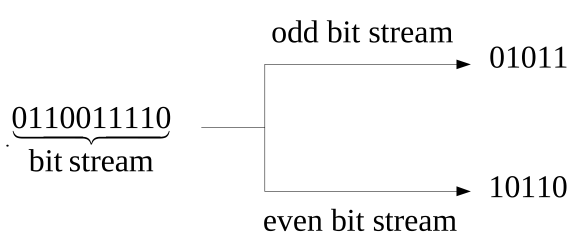


Fig 9.2 The bit stream is broken into odd and even bit stream

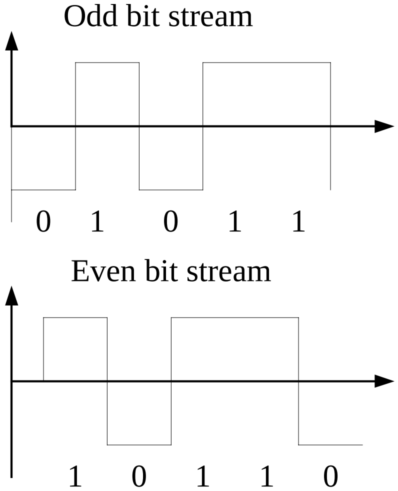


Fig 9.3 The time offset waveforms of odd and even bit steams

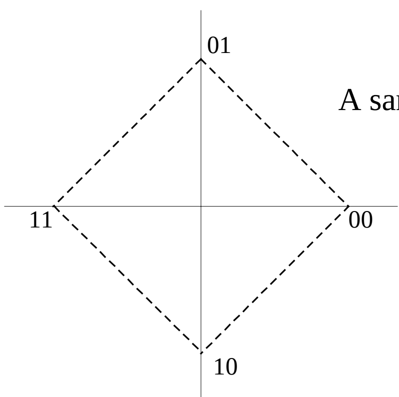
Since the transitions instants are offset for even and odd bit steams, at any given time only one of the two bit streams can change values. This implies that the maximum phase shift of the transmitted signal is limited to $\pm 90°$.

Power Spectrum Density of ***QPSK*** and ***OQPSK***.
The Power Spectrum Density remains the same for ***OQPSK*** compared to ***QPSK***. The offset does not change the Power Spectrum Density.

III $\pi/4$-*DQPSK* (used by IS-136 system)

A variant of ***DQPSK***, called $\pi/4$-***DQPSK*** is obtained by introducing an additional $\pi/4$ phase shift in the carrier phase in each symbol interval. It is a compromise between ***QPSK*** and ***OQPSK*** allowing maximum phase transition of $135°$. Phase transition is allowed at every symbol of either $\pm\pi/4$ or $\pm 3\pi/4$.

$$Q_k = Q_{k-1} + \phi_k$$



A sample phase transition is shown below for $\pi/4$ ***DQPSK***.

Information bits	ϕ_k
11	$\pi/4$
01	$3\pi/4$
00	$-\pi/4$
10	$-3\pi/4$

Table 9.1: One of the possible transitions for $\pi/4$ ***DQPSK***.

IV *Continuous Phase Modulation (CPM)*

When expressed in the next equation, the modulated signal becomes a continuous-phase modulated signal (*CPM*).

$$v(t) = A\exp\{j2\pi k_f \int_{-\infty}^{t} \sum_n x_n h_f(\tau - nT)d\tau\}$$

$$= A\exp\{j\phi(t)\}$$

Where:

- A Amplitude
- k_f Peak frequency deviation
- $h_f(t)$ Frequency shaping pulse
- $\{x_n\}$ Source symbol sequence
- T Symbol duration

The data sequence is $\{\pm1, \pm3, ..., \pm(N-1)\}$

$\phi(t)$ is called ***excess phase*** = $2\pi k_f \int_{-\infty}^{t} \sum_{n=-\infty}^{k-1} x_n h_f(\tau - nT)d\tau + 2\pi k_f x_k \int_{kT}^{t} h_f(\tau - kT)d\tau$

for interval $kT \le t \le (k+1)T$

If $h_f(t)$ represents a full response shaping function, it has a duration equal to *T*.

If $h_f(t)$ represents a partial response shaping function, it has a duration larger than *T*.

Standard form for *CPM* signal: $v(t) = A\sum_k b(t - kT, \underline{x}_k)$

$$b(t, \underline{x}_k) = \exp\{j(\beta(T)\sum_{n=-\infty}^{k-1} x_n + x_k \beta(t)\} U_T(t)$$

$\beta(T)\sum_{n=-\infty}^{k-1} x_n$ is accumulated excess phase, excess phase for current symbol

$$\beta(t) = \begin{cases} 0 & t < 0 \\ 2\pi k_t \int_0^t h_f(\tau)d\tau, & 0 \le t \le T \\ \beta(T) & t \ge T \end{cases}$$

Average Frequency Deviation: $\bar{k}_f = k_f \dfrac{1}{T}\int_0^T h_f(t)dt$

Modulation Index: $h = \dfrac{\beta(T)}{\pi} = 2\bar{k}_f T$

Various $h_f(t)$ give rise to various *CPM* signals, *h* and *M* do the same.

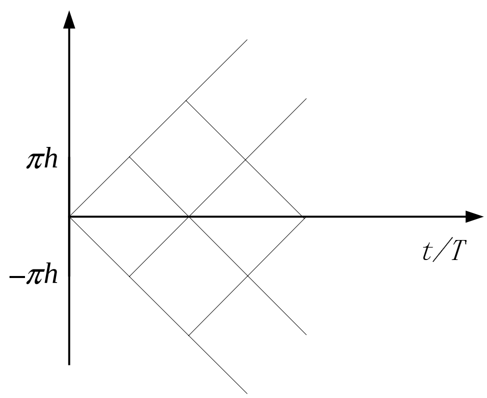
V *CPFSK* (Continuous Phase Frequency Shift Keying)

$$h_f(t) = U_T(t) \quad \bar{k}_f = k_f \quad h = 2k_f T$$

$$\beta(t) = \begin{cases} 0 & t < 0 \\ 2\pi k_f t = \dfrac{\pi h t}{T} & 0 \le t \le T \\ 2\pi k_f T = \pi h & t \ge T \end{cases}$$

CPM signals are often described by sketching the excess phase
$\phi(t) = \beta(T)\sum_{n=-\infty}^{k-1} x_n + x_k \beta(t - kT)$ for all possible symbol sequences $\{x_k\}$.

For $M = 2$, it is called ***Binary CPFSK***.
The so called ***Phase Trellis*** is depicted below, where $x_k \in \{+1,-1\}$.



Minimum shift keying (MSK)

MSK is a special case of binary *CPFSK*, with $h = \dfrac{1}{2}$,

$$\beta(t) = \begin{cases} 0 & t < 0 \\ \pi t / 2T & 0 \le t \le T \\ \pi / 2 & t \ge T \end{cases}$$

So carrier phase is given as (during $kT \le t \le (k+1)T$)

$$\phi_1(t) = 2\pi f_c t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} x_n + \frac{\pi}{2} x_k \frac{t - kT}{T}$$

$$= (2\pi f_c + \frac{\pi x_k}{2T})t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} x_n - \frac{\pi k}{2} x_k$$

The *MSK* waveform:

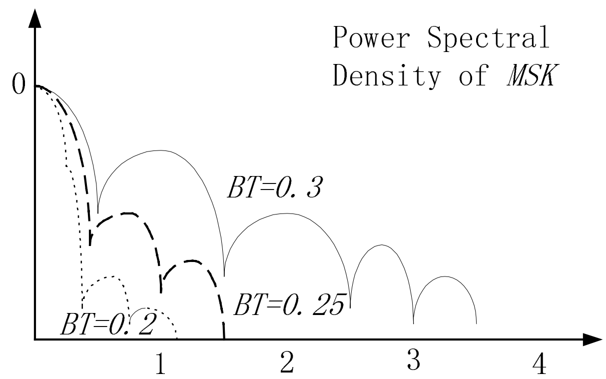
$$s(t) = A\cos[(2\pi f_c + \frac{\pi x_k}{2T})t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} x_n - \frac{\pi k}{2} x_k],$$

$$= A\cos[2\pi(f_c + x_k / 4T)t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} x_n - \frac{\pi k}{2} x_k] \qquad kT \le t \le (k+1)T$$

$$f_l = f_c - \frac{1}{4T} \text{ and } f_u = f_c + \frac{1}{4T}$$

$$\Delta f = f_u - f_l = \frac{1}{2T}$$

This minimum frequency separation ensures orthogonality between 2 sinusoids of duration *T*. The power spectral density for *MSK* is shown below.



Partial response signaling:

Make $h_f(t)$ of duration greater than T:

$$h_f(t) = h_f(t)U_{kT}(t) \quad \text{has duration } kT$$

$$U_{kT} = \sum_{k=0}^{k-1} U_T(t - kT)$$

Advantage of Partial Response *CPM* is that its narrower main lobe and faster roll-off of side lobe.

Gaussian MSK (GMSK, used in GSM)

Rectangular pulse through a pre-modulation filter:

$$H(f) = \exp\{-(\frac{f}{B})^2 \frac{h^2}{2}\},$$

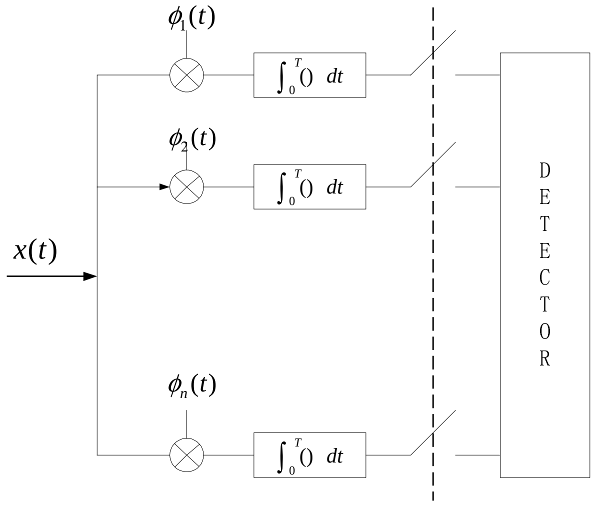
(B is the bandwidth of filter. $H(f)$ is bell-shaped about $f = 0$), it becomes “Gaussian”.
When *BT* decreases, bandwidth efficiency increases. Then Inter-Signal-Interference(ISI) also increases which requires sophisticated equalizer. For *GSM* cellular system, it uses *BT*=0.3.

Coherent Detection of Signals in Noise

One of M signals $s_1(t), s_2(t), ... s_M(t)$ is transmitted with equal probability $1/M$.
Observe signal in AWGN

$$x(t) = s_i(t) + w(t)$$

The received signal is applied to a bank of correlators from N basis functions. The demodulation scheme is shown below:



Since $f_{\underline{x}}(\underline{x} \mid m_k)$ is nonnegative log monotonic transformation. Set $\hat{m} = m_i$ if $\ln[f_{\underline{x}}(\underline{x} \mid m_k)]$ is maximum for $i = k$. Observation vector $\underline{x}$ lies in region Z_i if $\ln[f_{\underline{x}}(\underline{x} \mid m_k)]$ is maximum for $i = k$.

Z is the obervation space. $\bigcup_{i=1}^{m} Z_i = Z$.

For AWGN channel,

$$f_{\underline{x}}(\underline{x} \mid m_k) = (\pi N_0)^{-\frac{N}{2}} \exp[-\frac{1}{N_0} \sum_{j=1}^{N} (x_j - s_{kj})^2] \qquad k = 1,2,...M$$

The item $-\frac{1}{N_0} \sum_{j=1}^{N} (x_j - s_{kj})^2$ reaches maximum when $k = i$.

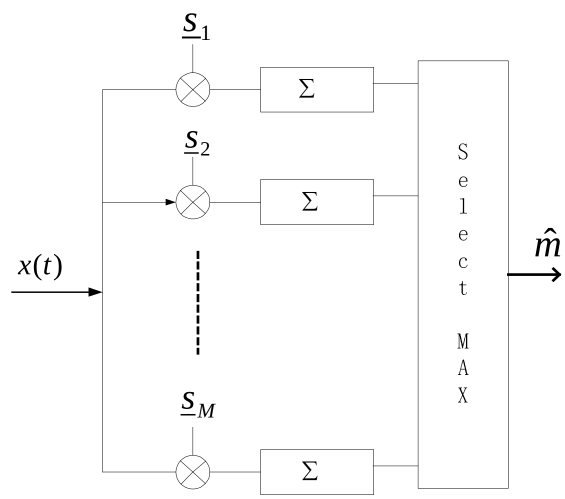
We have the conclusion that observation vector lies in Z_i if $\left\| \underline{x} - \underline{s_k} \right\|^2$ is minimum for $k = j$.

$$-\frac{1}{N_0} \sum_{j=1}^{N} (x_j - s_{kj})^2 = \sum_{j=1}^{N} x_j^{\ 2} - 2\sum_{j=1}^{N} x_j s_{kj} + \sum_{j=1}^{N} S_{kj}^2$$

Hence, the observation vector $\underline{x}$ lies in Z_i if $\sum_{j=1}^{N} x_j x_{kj} - \frac{1}{2} E_k$ is maximal.

$$\max_k \{x^T s_k - \frac{1}{2} E_k\})\text{for } k = i.$$

The demodulation scheme is:



Average Probability of Error

If symbol m_i is transmitted, error occurs if $\underline{x}$ lies outside Z_i .

$$P_e = \sum_{i=1}^{M} P(\underline{x} \text{ does not lie in } Z_i \text{ and } m_i \text{ sent})$$

$$= \sum_{i=1}^{M} P(\underline{x} \notin Z_i \mid m_i \text{ sent}) \times P(m_i \text{ sent})$$

$$= \frac{1}{M} \sum_{i=1}^{M} P(\underline{x} \notin Z_i \mid m_i \text{ sent})$$

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} P(\underline{x} \in Z_i \mid m_i \text{ sent})$$

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i} f_{\underline{x}}(\underline{x} \mid m_i) d\underline{x}$$

Using union bound,

$$P_e(m_i) \leq \frac{1}{2} \sum_{\substack{k=1 \\ k \neq i}}^{M} erfc\left(\frac{d_{ik}}{2\sqrt{N_0}}\right) \qquad \forall i$$

erfc is the error function $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-Z^2} dZ$

$$d_{ik} = \left\| \underline{s}_i - \underline{s}_k \right\|$$

Another expression is

$$P_e(m_i) \leq \sum_{\substack{k=1 \\ k \neq i}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right) \qquad \forall i$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$

Coherent Binary Phase Shifting Keying(*BPSK*)

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

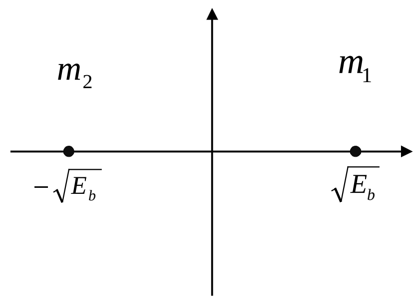
$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

The probability of error is

$$P_e = \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}}) = Q(\sqrt{\frac{2E_b}{N_0}})$$

The signal transition diagram is:



Coherent *QPSK*

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + (2i-1)\pi/4); & 0 \le t \le T \quad i = 1,2,3,4 \\ 0 & t < 0 \text{ or } t > T \end{cases}$$

E, Transmitted energy per symbol

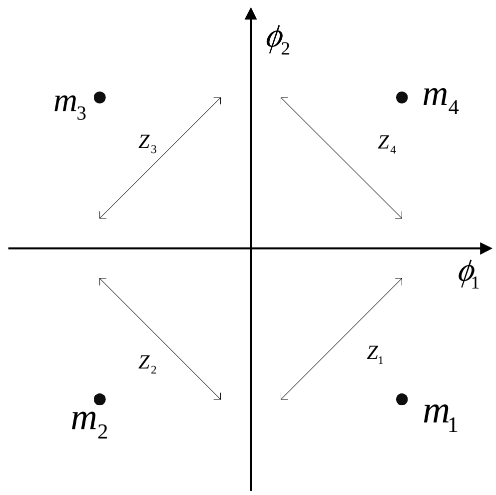
T, Symbol duration

Each signal $s_i(t)$ corresponds to a pair of bits: 00, 01, 10, 11.

$$\phi_1 = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad s_i = \begin{bmatrix} s_{i,1} \\ s_{i,2} \end{bmatrix} = \begin{bmatrix} \sqrt{E} \cos((2i-1)\frac{\pi}{4}) \\ -\sqrt{E} \sin((2i-1)\frac{\pi}{4}) \end{bmatrix}$$

$$\phi_2 = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

The signal transition diagram is as follows:



Average Probability of bit errors

$$P_e = \frac{1}{2} erfc(\sqrt{\frac{E/2}{N_0}}) = \frac{1}{2} erfc(\sqrt{\frac{E}{2N_0}}) = Q(\sqrt{\frac{E}{N_0}}) = Q(\sqrt{\frac{2E_b}{N_0}})$$

$$P_c = (1-p)^2 = 1 - erfc(\sqrt{\frac{E}{2N_0}}) + \frac{1}{4} erfc^2(\sqrt{\frac{E}{2N_0}})$$

$$P_e = erfc(\sqrt{\frac{E}{2N_0}}) \quad \text{(for large } \frac{E}{2N_0} >> 1\text{)}$$

Spectral efficiency and Probability of error related to power efficiency

M-ary *PSK*:

In M-ary *PSK*, the signal is represented by,

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t + \frac{2\pi}{M}(i-1)), \qquad 0 \le \text{t} \le \text{T}_\text{s}$$

$E_s = E_b \log_2 M$ is energy per symbol.

$T_s = T_b \log_2 M$

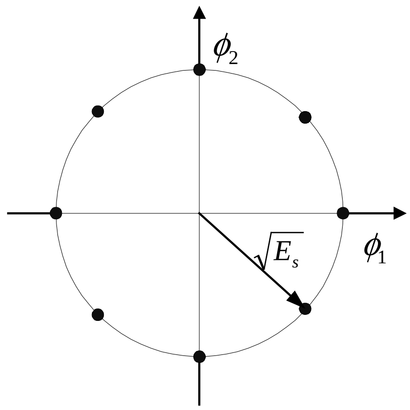
T_s Symbol period

T_b Bit period

$$\phi_1 = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad \phi_2 = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

$$s_i = \begin{cases} \sqrt{E_s} \cos((i-1)\frac{2\pi}{M}) \\ -\sqrt{E_s} \sin((i-1)\frac{2\pi}{M}) \end{cases}$$

The signal transition diagram is as follows:



The probability of error is:

$$p_e = 1 - \frac{1}{M}\sum_{i=1}^{M}\int_{Z_i} f(\underline{x} \mid m_i)$$

$$p_e = \sum_{k=1}^{M} Q(\frac{d_{ik}}{\sqrt{2N_0}})$$

As M increases, p_e decreases since distance gets smaller.

Below is a table about power efficiency, say $\frac{E_b}{N_0}$ for $P_e = 10^{-6}$

	$M = 2$	$M = 4$	$M = 8$	$M = 16$	$M = 32$	$M = 64$
η_p	10	10.5	14	18.5	23.4	28.5
η_B	0.5	1.0	1.5	2.0	2.5	3.0

Coherent Binary FSK

In Binary *FSK* binary symbol '1' is represented by

$$\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t),$$

'0' is represented by

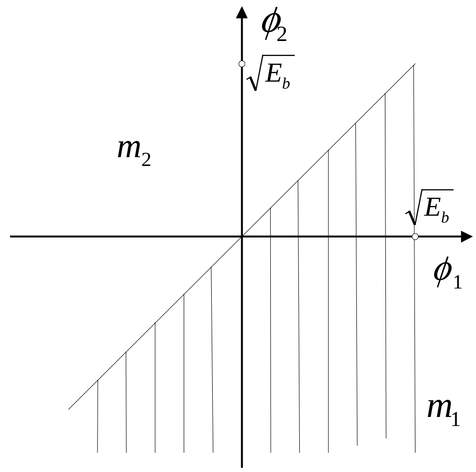
$$\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t).$$

$$f_i = \frac{n_c + i}{T_b} \text{ for some integer } n_c \text{ and } i = 1,2.$$

The probability of error is given by:

$$P_e = \frac{1}{2} erfc(\sqrt{\frac{E_b}{2N_0}}) = Q(\sqrt{\frac{E_b}{N_0}})$$

The signal transition diagram is:



References

[1] Theodore S. Rappaport, *Wireless Communications: Principles and Practic*e, Prentice-Hall, NJ, 1996.
[2] John G. Proakis, *Digital Communications*: *Fourth Edition* McGraw Hill, 2001
[3] John M. Wozencraft and Irwin Mark Jacobs, *Principles of Communication Engineering* Reissued by Waveland Press, 1990