Wireless Communication Technologies

Rutgers University – Dept. of Electrical and Computer Engineering (ECE)
Course No. 16:332:559:01 (Advanced Topics in Communications Engineering)
Spring 2002
Professor Narayan Mandayam

Lecture #5 – Wednesday February 6, 2002

Summary by Hamsini Bhaskaran and Weiliang Liu

Frequency Selective Fading Channels

In the last lecture, two terms Average Delay and RMS Delay Spread were defined as below

$$\mu_\tau = \frac{\int_0^\infty \tau \Phi(\tau) d\tau}{\int_0^\infty \Phi(\tau) d\tau} \tag{1}$$

$$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \Phi_c(\tau) d\tau}{\int_0^\infty \Phi_c(\tau) d\tau}} \tag{2}$$

If the power density is discrete like Figure 1, the Average Delay and RMS Delay Spread for the multipath profile could be written in the following way:

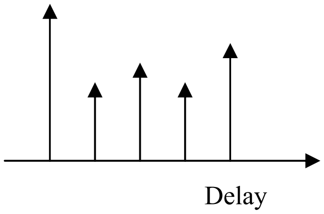


Figure1: Discrete multipath profile

$$\overline{\tau} = \frac{\sum_k \tau_k p(\tau_k)}{\sum_k p(\tau_k)} \tag{3}$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\overline{\tau})^2} \tag{4}$$

Where,

$$\overline{\tau^2} = \frac{\sum_k \tau_k^2 p(\tau_k)}{\sum_k p(\tau_k)} \tag{5}$$

Coherence Bandwidth (B_c)

Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat" (e.g. a channel that passes all spectral components with approximately equal gain and phase). Equivalently speaking, coherence bandwidth is the range of frequencies over which two frequency components have a strong potential for amplitude correlation. It is known that the coherence bandwidth is inversely proportional to the RMS delay spread: ($B_c \propto \frac{1}{\sigma_\tau}$).

It is important to note that an exact relationship between coherence bandwidth and RMS delay spread does not exit. In general, spectral analysis techniques and simulation are required to determine the exact impact that time varying multipath has on a particular transmitted signal.

Doppler Spread (B_d), **Coherence Time** (T_c)

RMS delay spread σ_τ and coherence bandwidth B_c are parameters which describe the time dispersive nature of the channel in a local area and they do not offer any information about the time varying nature of the channel due to the relative motion between the mobile station and base station.

Doppler Spread B_d is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel and is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero. In other words, if the baseband signal bandwidth is much greater than B_d, the effects of Doppler spread are negligible at the receiver. This is also called slow fading.

Coherence time T_c is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain. The Doppler spread and coherence time are inversely proportional to one another: $T_c \approx \frac{1}{B_d}$.

Coherence Time T_c is actually a statistical measure of the time duration over which the channel impulse response is essentially invariant, and quantifies the similarity of the channel response at different times. In other words, coherence time is the time duration over which two received signals have a strong potential for amplitude correlation. Thus, if the inverse bandwidth of signal is greater than T_c of the channel, the channel changes during the transmission of a symbol (or say, baseband message), causes distortion at the receiver. If coherence time is defined as the time over which the time correlation function is above 0.5, then it is approximately given by:

$$T_c \approx \sqrt{\frac{9}{16\pi f_m^2}} \tag{6}$$

Where, $f_m = \frac{v}{\lambda}$, is the maximum Doppler frequency. This is an empirical equation.

Example 1: A vehicle's velocity is 60mph and the carrier frequency is 900MHz.

Solution to Example 1: By using the equation (6), we can get T_c=6.77ms and B_d=150bps, so if the symbol rate in such environment is greater than 150bps, there would be no distortion due to motion.

Similarly, if defined coherence bandwidth as the bandwidth over which the frequency correlation is above 0.5, we have the approximation:

$$B_c \approx \frac{1}{5\sigma_\tau} \tag{7}$$

Again, this is an empirical relationship and there is no exactly relationship between coherence bandwidth and RMS delay spread.

Classification of small-scale fading

From the discussion above, we know that the type of fading experienced by a signal propagating through a mobile radio channel depends on the nature of the transmitted signal with respect to the characteristics of the channel. Depending on the relation between the signal parameters (such as bandwidth, symbol period, etc) and the channel parameter (such as RMS delay spread and Doppler spread), different transmitted signals will undergo different types of fading. The time dispersion and frequency dispersion mechanisms in a mobile radio channel lead to four possible distinct effects, which are manifested depending on the nature of the transmitted signal, the channel, and the velocity. We will discuss them one by one below.

1. Flat Fading

If the mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater that the bandwidth of the transmitted signal, which means

$$B_s \ll B_c \quad or \quad T_s \gg \sigma_\tau \tag{8}$$

Then the received signal will undergo flat fading. In flat fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath. Figure 2 shows the characteristics of a flat fading channel. Flat fading channels are also known as amplitude varying channels and are sometimes referred to as narrowband channels, since the bandwidth of the applied signal is narrow as compared to the channel flat fading bandwidth. Detailed description could be found in [1].

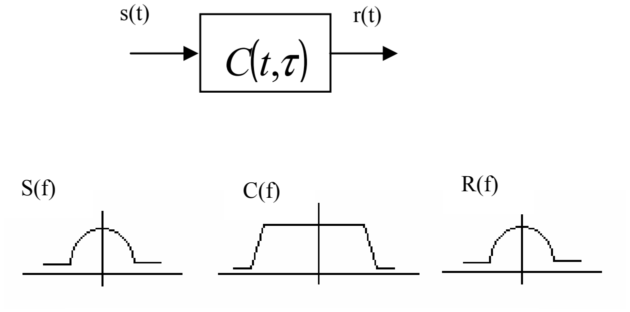


Figure 2: Flat fading channel characteristics

2. Frequency Selective Fading

If the channel possesses a constant-gain and linear phase response over a bandwidth that is smaller than the bandwidth of transmitted signal, the channel creates frequency selective fading on the received signal, which means

$$B_s > B_c \quad or \quad T_s < \sigma_\tau \tag{9}$$

Under such conditions the channel impulse response has a multipath delay spread which is greater than the reciprocal bandwidth of the transmitted message waveform. When it occurs, the received signal includes multiple versions of the transmitted waveform that are attenuated and delayed, and hence the received signal is distorted. Figure 3 illustrates the characteristics of the frequency selective fading channel. For instance, the fading type in GSM system is frequency selective.

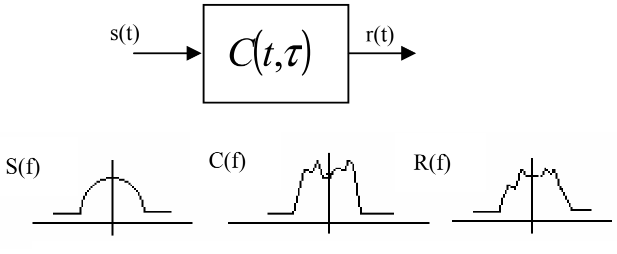


Figure 3: Frequency Selective fading channel characteristics

3. Fast Fading

In a fast fading channel, the channel impulse response changes rapidly within the symbol duration. That is, the coherence time of the channel is smaller that the symbol period of the transmitted signal. Viewed in the frequency domain, signal distortion due to fast fading increases with increasing Doppler spread relative to the bandwidth of the transmitted signal. Therefore, a signal undergoes fast fading if

$$T_s > T_c \quad or \quad B_s < B_d \tag{10}$$

4. Slow Fading

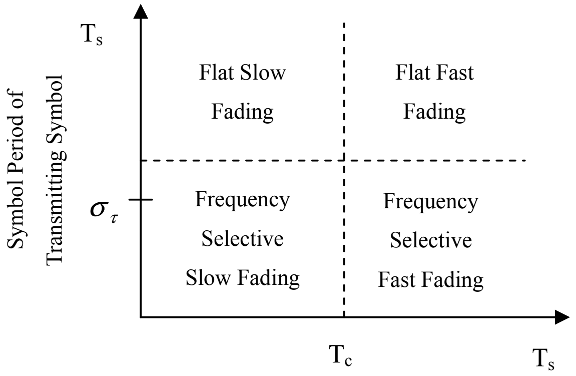
In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal S(t). In the frequency domain, this implies that the Doppler spread of the channel is much less than the bandwidth of the baseband signal. There fore, a signal undergoes slow fading if

$$T_s << T_c \quad or \quad B_s >> B_d \tag{11}$$

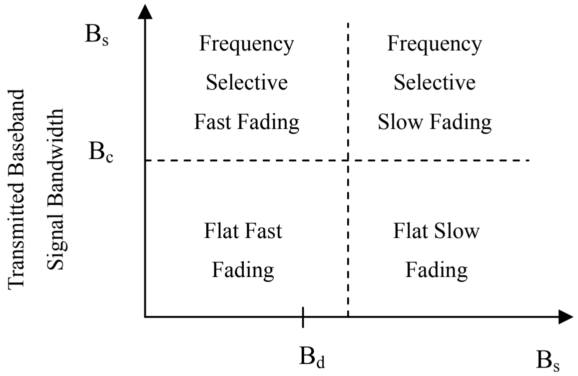
It should be clear that the velocity of the mobile (or velocity of objects in the channel) and the baseband signaling determine whether a signal undergoes fast fading or slow fading.

Summary of small-scale fading

It should also be clear that when a channel is specified as a fast or slow fading channel, it does not specify whether the channel is flat fading or frequency selective in nature. Fast and slow fading deal with the relationship between the time rate of change in the channel and the transmitted signal, and not with propagation path loss models. Shown below, is a matrix illustrating type in both time and frequency domains to show the relationships among the four types of fading. Usually, the fast and frequency selective fading rarely occur and the fading behavior is the function of the transmitted signal.



(a) Symbol Period



(b) Baseband signal bandwidth

Figure 4: Matrix illustration type of fading

Shadowing

Recall from small scale fading models, the envelope of the transmitted signal, Z(t) is either a Rayleigh or a Ricean faded signal. Let's define the mean of it as:

$$\Omega_v = E[Z(t)] \tag{12}$$

Ω_v is the mean envelope level of Z(t) and is also called "local mean" since it represents the envelope level averaged over a distance of a few wavelengths. Actually, Ω_v itself is a random variable due to shadow variations that caused by large terrain features such as buildings, hills etc. between the mobile station and base station. The distribution of Ω_v is purely based on empirical observations. It is given as

$$p(\Omega_v) = \frac{\xi}{\Omega_v \cdot \sigma_\Omega \cdot \sqrt{2\pi}} \exp\left\{ \frac{-\left(10\log_{10}\Omega_v - \mu_{\Omega_v}\right)^2}{2\sigma_\Omega{}^2} \right\} \tag{13}$$

Where

$$\mu_{\Omega_v} = E[\Omega_v(dB)]$$

$$\xi = \frac{10}{\ln 10}$$

Consider $\Omega_v(dB) = 10\log_{10}\Omega_v$

Then

$$P(\Omega_v(dB)) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ \frac{-\left(\Omega_v(dB) - \mu_{\Omega_v}\right)^2}{2\sigma_\Omega{}^2} \right\} \tag{14}$$

From (13) we know that Ω_v is a random variable with log normal distribution and

Ω_v (dB) is a random variable with Gaussian distribution as shown in (14). σ_Ω is about 8dB in microcellular application and its range is usually from 5dB to 12dB. The path loss is always the mean value of Ω_v . Since Ω_v is averaged over a few wavelengths, it does not vary over the duration of several bits. The empirical evaluation of Ω_v is important for power control, handoff in cellular system on the base station side. The relationships among the pass loss, shadowing and small-scale fading are illustrated in Figure 5.

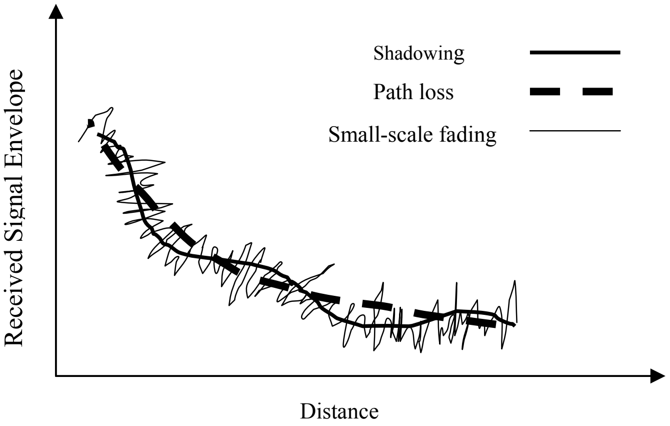


Figure 5: Effects of pass loss, shadowing and small-scale fading on the received signal envelope

Wireless Communication Technologies

Rutgers University – Department of Electrical and Computer Engineering
Course No: 16:332:559:01 (Advanced Topics in Wireless Communication)
Spring 2002
Professor Narayan Mandayam

Lecture #6 – Monday February 11, 2002

Summary by Hamsini Bhaskaran and Weiliang Liu

Composite Shadow-Fading Distribution

It is sometimes desirable to know the composite distribution due to shadowing and multipath fading, especially for a slow-moving or stationary MS where the receiver is unable to average over the effects of fading and composite distribution is necessary to evaluate the link performance. One could express the envelope conditioned on the "local mean" Ω_v and then integrate the conditional P.D.F of the envelope over the density of Ω_v.

$$P_{Z_C}(x) = \int_0^\infty P_{Z/\Omega_v}(x/\omega) P_{\Omega_v}(\omega) d\omega \qquad (1)$$

For Rayleigh fading

$$P_{Z/\Omega_v}(x/\omega) = \frac{\pi x}{2\omega^2} \exp(\frac{-\pi x^2}{4\omega^2}) \qquad (2)$$

Hence

$$P_{Z_c}(x) = \int_0^\infty \frac{\pi x}{2\omega^2} \exp(-\frac{\pi x^2}{4\omega^2}) \frac{\xi}{\omega \sigma_\Omega \sqrt{2\pi}} \exp(-(\frac{10\log_{10}\omega - \mu_{\Omega_v}}{2{\sigma_\Omega}^2})) d\omega \quad (3)$$

This distribution is called the "Susuki distribution" after the original work of Susuki.

The Effect of Co-Channel Interference

In wireless cellular communication the radio link is affected more by co-channel interference than by the noise in the media and hence the probability of co-channel-interference (CCI) is of primary concern. Also, many system level issues such as cell size, reuse distance, handoffs and power control are limited by co-channel interference between cells. Calculations of the probability of CCI for signals with composite log-normal shadowing and fading show that shadowing has a more significant effect on the probability of CCI than small-scale fading. The analysis of CCI for the log-normally shadowed signals typical in cellular frequency reuse systems requires the probability distribution of the interference power that is accumulated from several log-normal signals. Although there is no exact expression for this distribution, several approximations have been derived by various authors.

Multiple Log Normal Interferers

Consider N_I interferers each lognormally shadowed. In channelized systems such as TDMA and FDMA, the number of interferers (cells using the same frequency channels) is limited by their spatial separation; hence N_I is typically not a large number.

$$L = \sum_{k=1}^{N_I} L_k = \sum_{k=1}^{N_I} 10^{\Omega_K (dB)/10} = 10^{Z(dB)/10} = \widetilde{L} \quad (4)$$

where L_k ($k=1,2\ldots N_I$) are lognormal random variables and Ω_k are Gaussian random variables with mean μ_{Ω_K} and variance $\sigma_{\Omega_K}^{2}$. As N_I is not a large number we do not employ the central-limit-theorem approximation for the sum but adopt the general consensus that the sum of lognormal random variables will be a lognormal variable. The accuracy of the approximation varies with N_I and the range of σ_Ω. There are three well-known approaches to determine the mean and variance of Z (dB) i.e. $\mu_z and \sigma_z{}^2$.

Fenton-Wilkinson Method

μ_z and σ_Z^2 are obtained by matching the first and second moments of the power sum L with the first two moments of $\widetilde{L}$. Rewriting the earlier equation

$$L_k = 10^{\Omega_k (dB)/10} = e^{\xi\Omega_k (dB)} = e^{\widetilde{\Omega}_k} \quad (5)$$

where $\xi = \ln 10/10 = 0.2306$ and $\widetilde{\Omega}_k$ is a Gaussian random variable with mean $\mu_{\widetilde{\Omega}_k} = \xi\mu_{\Omega_k}$ and variance $\sigma_{\widetilde{\Omega}_k}{}^2 = \xi^2\sigma_{\Omega_k}{}^2$. The r^th^ moment of L_k can be obtained from the moment generating function of $\widetilde{\Omega}_k$.

$$E[L_k^r] = E[e^{r\widetilde{\Omega}_k}] = e^{r\mu_{\widetilde{\Omega}_k} + (1/2)r^2\sigma^2_{\widetilde{\Omega}_k}} \quad (6)$$

To find the appropriate moments of the approximation, we equate moments on both sides of the equation

$$L = e^{\hat{z}} = \widetilde{L} \quad (7)$$

Let $\widetilde{\Omega}_1, \widetilde{\Omega}_2 \ldots\ldots \widetilde{\Omega}_{N_I}$ be independent random variables with means

$\mu_{\tilde{\Omega}_1}, \mu_{\tilde{\Omega}_2} .... \mu_{\tilde{\Omega}_{N_I}}$ respectively and identical variance $\sigma_{\tilde{\Omega}}^{\;2}$. Identical variances are often assumed because the standard deviation of log-normal shadowing is largely independent of the radio path length.

$$\mu_L = \sum_{k=1}^{N_I} E[L_k] = (\sum_{k=1}^{N_I} e^{\mu_{\tilde{\Omega}_k}}) e^{\frac{1}{2}\sigma_{\tilde{\Omega}}^{\;2}} = \text{L.H.S of equation (7)} \tag{8}$$

$$E[e^{\hat{Z}}] = e^{\mu_{\hat{Z}} + \frac{1}{2}\sigma_{\hat{Z}}^2} = \text{R.H.S of equation (7)} \tag{9}$$

$$\text{L.H.S=R.H.S} \Rightarrow \; (\sum_{k=1}^{N_I} e^{\mu_{\tilde{\Omega}_k}}) e^{\frac{1}{2}\sigma^2_{\tilde{\Omega}}} = e^{\mu_{\hat{Z}} + \frac{1}{2}\sigma_{\hat{Z}}^2} \tag{10}$$

Similarly for second moments (or variances)

$$\text{L.H.S} = (\sum_{k=1}^{N_I} e^{2\mu_{\tilde{\Omega}_k}})(e^{\sigma^2_{\tilde{\Omega}}})(e^{\sigma^2_{\tilde{\Omega}}} - 1) = \text{R.H.S} = e^{2\mu_{\hat{Z}}} e^{\sigma^2_{\hat{Z}}} (e^{\sigma^2_{\hat{Z}}} - 1) \tag{11}$$

Squaring equation (10) and dividing by (11) yields

$$\mu_{\hat{Z}} = \frac{\sigma^2_{\tilde{\Omega}} - \sigma^2_{\hat{Z}}}{2} + \ln(\sum_{k=1}^{N_I} e^{2\mu_{\tilde{\Omega}_k}}) \tag{12}$$

$$\sigma^2_{\hat{Z}} = \ln[(e^{\sigma^2_{\tilde{\Omega}}} - 1) \frac{\sum_{k=1}^{N_I} e^{2\mu_{\tilde{\Omega}_k}}}{(\sum_{k=1}^{N_I} e^{\mu_{\tilde{\Omega}_k}})^2} + 1] \tag{13}$$

It has been found that the Fenton-Wilkinson method breaks down for $\sigma_{\Omega} > 4\text{dB}$ while the standard-deviation of log-normal shadowing for cellular radio applications typically ranges from 5 to 12dB. The approximation does, however, work well for evaluating “tail functions” for the fading distributions.

$$P[L > x] \approx \Pr ob[e^{\hat{Z}} \geq x] = Q(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}) \tag{14}$$

Such probabilities are used to determine the outage which will be described in a following section.

Schwartz Yeh’s Method

This method equates the L.H.S and R.H.S of equation (7) above by evaluating the exact expression for the first two moments of the sum of two lognormal random variables. Then a recursive method is employed for a general N_I number of interferers. This yields more accurate results than the Fenton-Wilkinson’s method.

Farley's Method

This method uses the central limit theorem approximation by assuming that N_I is large. Under the assumption that the Ω_K s are independent and identically distribute the approximation yields the following result for the power sum

$$\Pr ob[L \le x] \approx \left[1 - Q(\frac{\ln x - \mu_{\tilde{\Omega}}}{\sigma_{\tilde{\Omega}}})\right]^{N_I} \tag{15}$$

How are the above models of co-channel interference used to evaluate system performance?

For cellular radio systems the transmission quality will be acceptable provided the average received *carrier-to-interference (SIR)* exceeds a receiver threshold λ_{th} , also known as the target SIR We define the probability of outage as

$$P_{out} = \Pr ob[SIR < \lambda_{th}(dB)] \tag{16}$$

The designers of cellular systems usually aim at achieving a probability of outage of about 1%.

Let the MS be at a distance d_o from the base station and distance $\{d_k\}$ (k=1, 2… N_I) from the co-channel base stations.

Let d = $\{d_0, d_1 \ldots d_{N_I}\}$ be the vector that completely characterizes the system. Let $\lambda(dB)(\bar{d})$ be the SIR achieved at the base-station. Then

$$\lambda(dB)(\bar{d}) = \lambda(dB)(d_0) - 10\log_{10}\sum_{k=1}^{N_I} 10^{\lambda(dB)d_k/10} \tag{17}$$

$$P_{out} = prob[\lambda(dB)(\bar{d}) < \lambda_{th}(dB)] \tag{18}$$

Using log-normal approximation

$$\sum_{k=1}^{N_I} 10^{\lambda(dB)d_k/10} = \sum_{k=1}^{N_I} 10^{Z(dB)\ /10} = e^{\hat{Z}} \tag{19}$$

where $\mu_{\hat{Z}}$ and $\sigma_{\hat{Z}}^2$ are the mean and the variances of the approximation.

$$Z(dB) = \frac{\hat{Z}}{\xi} \qquad \mu_Z = \frac{\mu_{\hat{Z}}}{\xi} \qquad \sigma_Z^2 = \frac{\sigma_{\hat{Z}}^2}{\xi^2}$$

$$\lambda(dB)(\bar{d}) = \lambda(dB)(d_0) - Z(dB)(\mathrm{d}_0, \mathrm{d}_1 \ldots \mathrm{d}_{\mathrm{N_I}})$$

$\lambda(dB)$ can be approximated to a Gaussian where

$$\mu_{\lambda}(\bar{d}) = \mu_{\Omega(d_0)} - \mu_Z \qquad \sigma^2_{\lambda(dB)} = \sigma^2_{\tilde{\Omega}} + \sigma^2_Z$$

The probability of outage is then given by

$$P_{out} = Q\left(\frac{\mu_{\Omega(d_0)} - \mu_Z - \lambda_{th}(dB)}{\sigma^2_{\tilde{\Omega}} + \sigma^2_Z}\right) \tag{20}$$

A restriction that the probability of outage should be less than, say 0.01, will have a direct bearing on the cell-size and reuse-distance of the cellular system. A similar analysis on the uplink would yield the restrictions on cell-capacity.

References

[1] Theodore S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall, 1996

[2] John G. Proakis, *Digital Communications*, Third Edition, McGraw-Hill, 1995

[3] Savo G. Glisic and Pentti A. Leppanen, *Wireless Communications*, Kluwer, 1997

[4] Mahrotra A., *Cellular Radio Performance Engineering*, Artech House, 1994

[5] Vijay K. Garg, *IS-95 CDMA and cdma2000 Cellular/PCS System Implementation*, Prentice-Hall, 2000

[6] L. F. Fenton, "The Sum of Log-normal Probability distributions in Scatter transmission Systems," IRE Trans. Comm., Vol. 8, pp. 57-67, March 1960.

[7] S. Schwartz and Y. S. Yeh, "On the Distribution Function and Moments of Power Sums with Log-normal components," Bell System Tech. J., Vol.61, pp. 1441-1462, September 1982.

[8] Narayan Mandayam, *Lecture Notes*, February 2002