Wireless Communication Technologies

Lectures 23 & 24 (April 22 & April 24)

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Optimum Linear Detectors

Optimum linear detectors construct linear transformations of $\underline{y}$ to optimize one of the following criteria:

1.) Maximum asymptotic efficiency

Consider $\underline{y}$ to be the normalized vector of MF outputs. Let $\underline{v}_k$ be the linear transformation of user k. Then the decision rule is:

$$b_k = \text{sgn}(\underline{v}_k{}^T \underline{y})$$
$$\underline{v}_k{}^T \underline{y} = \underline{v}_k{}^T (RA\underline{b} + \underline{h})$$
$$\Rightarrow \underline{v}_k{}^T \underline{y} = \sum_{j=1}^{k} (A_j b_j \underline{v}_k{}^T \underline{r}_j) + \underline{v}_k{}^T \underline{h}$$

The asymptotic efficiency of the kth user is:

$$h_k(\underline{v}_k) = \frac{1}{\underline{v}_k{}^T R \underline{v}_k} \max{}^2\{0, \underline{v}_k{}^T \underline{r}_k - \sum_{j \neq k} \frac{A_j}{A_k} |\underline{v}_k{}^T \underline{r}_j|\}$$

This is a non-linear optimization problem. There is no closed form solution. Lupus and Verdu proposed an algorithm to solve it.

For the two user case, k = 2, without loss of generality,

$$\underline{v}_1 = [1 \quad x]^T$$
$$\Rightarrow h_1(\underline{v}_1) = \max{}^2\{0, f(x, r, \frac{A_2}{A_1})\} \qquad \text{(I)}$$
$$\text{where } f(x, r, \frac{A_2}{A_1}) = \frac{1 + xr - \frac{A_2}{A_1}|x + r|}{\sqrt{1 + 2rx + x^2}}$$

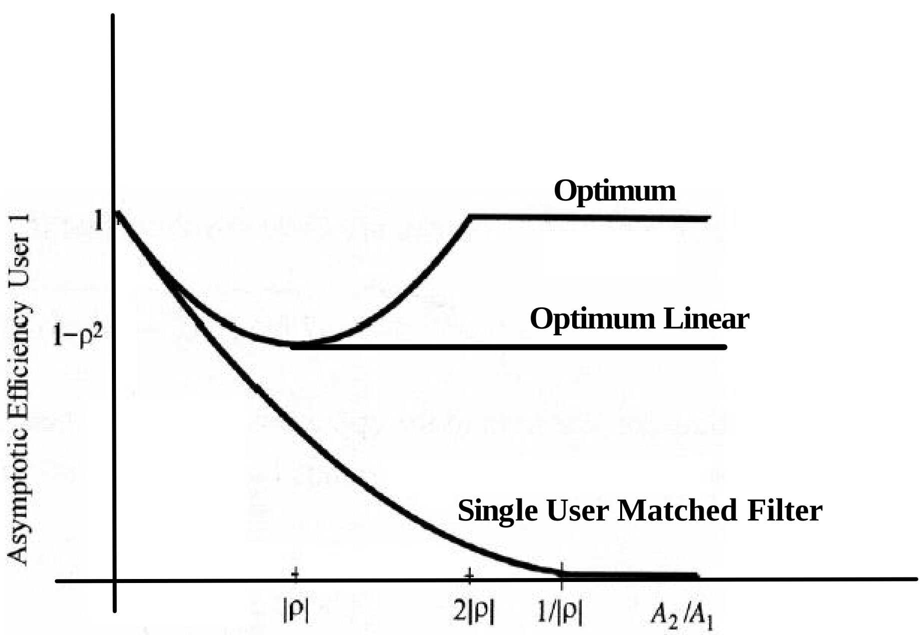
The argument x^* that maximizes f (.) is given by

$$x^* = \begin{cases} -\dfrac{A_2}{A_1}\operatorname{sgn}(r)\,; if\ \dfrac{A_2}{A_1} < \dfrac{1}{|r|} \\ -r\ ; otherwise \end{cases} \qquad \text{(II)}$$

The result says if the interferer is strong enough ($A_1 < A_2 |r|$), the decorrelator maximizes asymptotic efficiency otherwise the received signal is correlated with $s_1(t) - \dfrac{A_2}{A_1}\operatorname{sgn}(r) s_2(t)$. The maximum asymptotic efficiency linear detector is a compromise between the decorrelator and the single-user matched filter.

The asymptotic efficiency of this detector is obtained by substituting (II) in (I).

$$h_1(\underline{v}_1^*) = \begin{cases} 1 + \dfrac{A_2^2}{A_1^2} - 2|r|\dfrac{A_2}{A_1}\,; if\ \dfrac{A_2}{A_1} < \dfrac{1}{|r|} \\ 1 - r^2\ ; otherwise \end{cases}$$



2.) Minimum probability of error (*Mandayam-Aazhang*)

$$\min_{\underline{v}_k} \underline{P}_k^{\underline{v}_k} = \min_{\underline{v}_k} E[Q(\frac{A_k \underline{v}_k{}^T \underline{r}_k + \sum_{j \neq k} A_j b_j \underline{v}_k{}^T r_j}{s\sqrt{\underline{v}_k{}^T R_k \underline{v}_k}})]$$

$$\textit{where the expectation is with respect to } b_j \;\; j \neq k.$$

Infinitesimal Perturbation Analysis (IPA) is used to estimate the sensitivity of average probability of bit-error to different parameters. These estimates are shown to be unbiased. A stochastic gradient algorithm to solve this optimization problem is proposed and shown to converge almost surely when near – far resistance is strictly positive, that is, when eye is open. (*Refer to Appendix 1*)

3.) Minimum mean square error (*Madhow- Honig*)

The approach here is to turn linear multi-user detection problem into a linear estimation problem.

Idea: Require MSE between k[th] bit and output of the linear transformation $\underline{v}_k{}^T \underline{y}$ to be minimized. This approach does not minimize the bit error rate.

For the k[th] user solve: $\min_{\underline{v}_k} E[(b_k - v_k{}^T \underline{y})^2]$; *for* $k = 1,2,3,.......K$ *and* $\underline{v}_k \in \Re^k$

Combining the K equations,

$$\min_{M \in \Re^{k \times k}} E[\| b_k - M\underline{y} \|^2]$$

where the expectation is with respect to bits and noise.

Since $\| x \|^2 = trace\{xx^T\}$ *this problem is equivalent to*

$$\min_{M \in \Re^{k \times k}} trace\{E[(\underline{b} - M\underline{y})(\underline{b} - M\underline{y})^T]\}$$

Solution to this problem is given by $M^* = A^{-1}[R + s^2 A^{-2}]^{-1}$. *(Refer to Appendix 2 for proof.)*

Hence the MMSE linear detector outputs decisions as:

$$\hat{b}_k = \mathrm{sgn}\{\frac{1}{A_k}([R + s^2 A^{-2}]^{-1} \underline{y})_k\}$$
$$= \mathrm{sgn}\{([R + s^2 A^{-2}]^{-1} \underline{y})_k\}$$

A^{-1} is only a scaling factor and can be dropped without affecting the decision rule. Hence the optimum linear transformation for an MMSE receiver is

$$L = [R + s^2 A^{-2}]^{-1}$$

MMSE detector is a compromise between the conventional receiver (which is optimized to fight only background noise) and the decorrelator (which is optimized to fight only interference). It takes into account the relative importance of each interfering user and the background noise.

If $A_{2,}\ A_{3,\ldots,}A_k \rightarrow 0$ with A_1 being fixed, then the first row of $[R + s^2 A^{-2}]^{-1}$ becomes $[\frac{A_1}{A_1^2 + s^2} \quad 0 \quad 0 \quad \ldots\ldots \quad 0]$ which is the same as a conventional receiver (matched-filter) for user 1. Instead, if $s \rightarrow 0$ then $[R + s^2 A^{-2}]^{-1} \rightarrow R^{-1}$ which is equivalent to a decorrelator. Hence, the MMSE receiver converges to the decorrelator at very high SNRs and has same asymptotic efficiency and near-far resistance as a decorrelator.

Successive Cancellation

Idea: If a decision has been made about an interfering user's bit, then that interfering signal can be recreated at the receiver and subtracted from the received waveform. This can be done at the uplink as uplink has information about all interferers. If the decision on interferer's bit is correct this perfectly cancels out interference else interference is doubled. Receiver employs this scheme with the optimistic view that the resulting signal contains one fewer user and hence the process can be repeated with another interferer to cancel out interference completely.

In order to fully describe the receiver we just need to specify how the intermediate decisions are obtained. In its simplest form, successive cancellation uses decisions produced by single user matched filters. This scheme requires ordering of users and the order in which users are demodulated affects performance. Ordering based on received powers ignores cross-correlations, instead order users based on matched filter outputs:

$$E[(\int_0^T y(t)s_k(t)dt)^2] = s^{\ 2} + A_k^2 + \sum_{j \neq k} A_j^2 r_{jk}^{\ 2}$$

Let us consider the synchronous two-user case. Suppose user 2 is demodulated first:

$$\begin{aligned} \hat{b}_2 &= \text{sgn}(\int_0^T y_2(t)s_2(t)dt) \\ &= \text{sgn}(y_2) \end{aligned}$$

Re-modulating signal of user 2 with $\hat{b}_2$, we get $A_2\hat{b}_2 s_2(t)$, subtracting it from y(t) yields

$$\begin{aligned} \hat{y}(t) &= y(t) - A_2\hat{b}_2 s_2(t) \\ &= A_1\hat{b}_1 s_1(t) + A_2(b_2 - \hat{b}_2)s_2(t) + h(t) \end{aligned}$$

Processing $\hat{y}$ with matched filter for s_1 gives

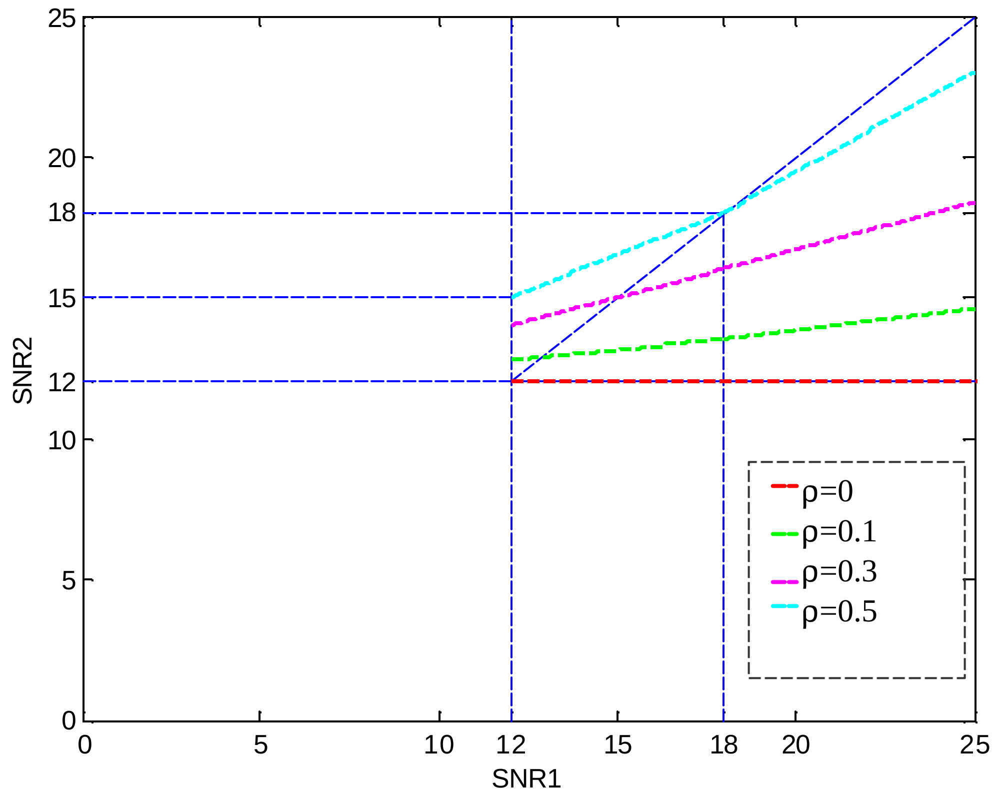
$$\begin{aligned} \hat{b}_1 &= \text{sgn}(< \hat{y}, s_1 >) \\ &= \text{sgn}(y_1 - A_2\hat{b}_2 r) \\ &= \text{sgn}(y_1 - A_2 r \,\text{sgn}(y_2)) \\ &= \text{sgn}(A_1 b_1 + A_2(b_2 - \hat{b}_2) r + s < h, s_1 >) \end{aligned}$$

General expression for a K-user system,

$$\hat{b}_k = \text{sgn}(y_k - \sum_{j=k+1}^{K} A_j r_{jk} \hat{b}_j)$$

This receiver is not near far resistant because of single user matched filters.

Power trade off

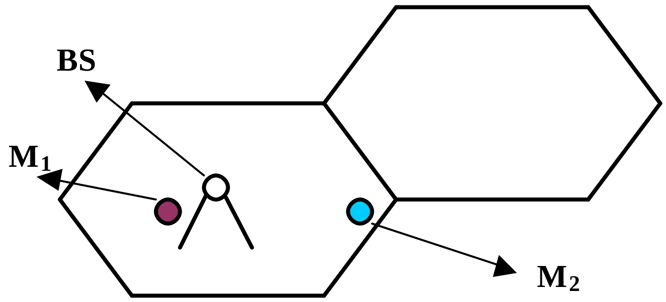


SNRs to achieve BER $\leq 3 * 10^{-5}$

Inferences

1.) Power tradeoff regions are asymmetric.
2.) Using equal powers is disadvantage. If $\rho = 0.5$, then $SNR_1 = 12dB$ and $SNR_2 = 15dB$ are sufficient to stay below the specified BER, whereas if received powers are identical, then $SNR_1 = SNR_2 = 18dB$.

A Design Example: Consider two users M_1 and M_2, M_1 is close to the base station and M_2 is close to the edge of the cell. Who should be given higher transmit power?



Strong interferers are not a problem as they can be easily canceled out. M_1 should be given high power and M_2 should be given low power. M_1 has a strong signal and hence can be easily detected and interference to M_2 from M_1 can be easily canceled out. Interference from M_2 to M_1 and users in adjacent cells will be small because of low power.

RADIO RESOURCE MANAGEMENT

The objective of a cellular communication system is to provide communication services anytime and anywhere and also to maintain connection quality regardless of user motion.

In a cellular communication system, there are sets of mobiles, radio access ports (RAP) or base stations and mobile switching centers (MSC). The base stations have radio link connections to mobiles and wired connections to the MSC. MSC connects the cellular communication system to other communication systems. The main purpose of the base station is to process the received signals from mobiles and to transfer information between mobiles and MSC. Due to user motion, the link quality will vary with time. The main issue for *Radio resource management* (RRM) is managing resources to support every user's quality requirement.

Terminology

Service area: Geographical area where we wish to provide communication services to mobile users. It is the entire area served by a communication service provider.

Coverage area: It is the specific area or region around a RAP where transmission conditions are favorable enough to maintain a connection of required quality.

Coverage area depends on propagation conditions and current interference from other users in the system. Coverage areas are not perfect hexagons and the shape of a coverage area keeps changing. This is called *cell breathing*.

Range limited systems: Systems where range of a RAP is smaller than inter RAP distance.

Bandwidth or interference-limited systems: Systems where number of transmitters is large compared to available bandwidth.

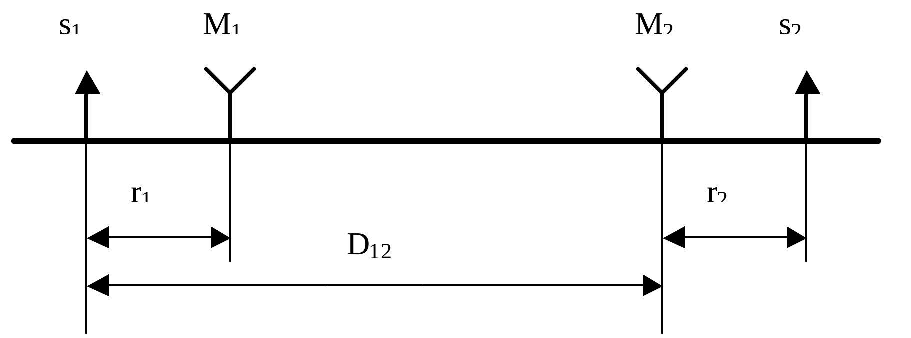
If users are orthogonal, there is no interference in the system. If orthogonality constraint is removed, more users can be accommodated and the users can also be asynchronous. This is usually the case in CDMA systems. Signature waveforms used are not orthogonal but have low cross correlations to keep interference under check.

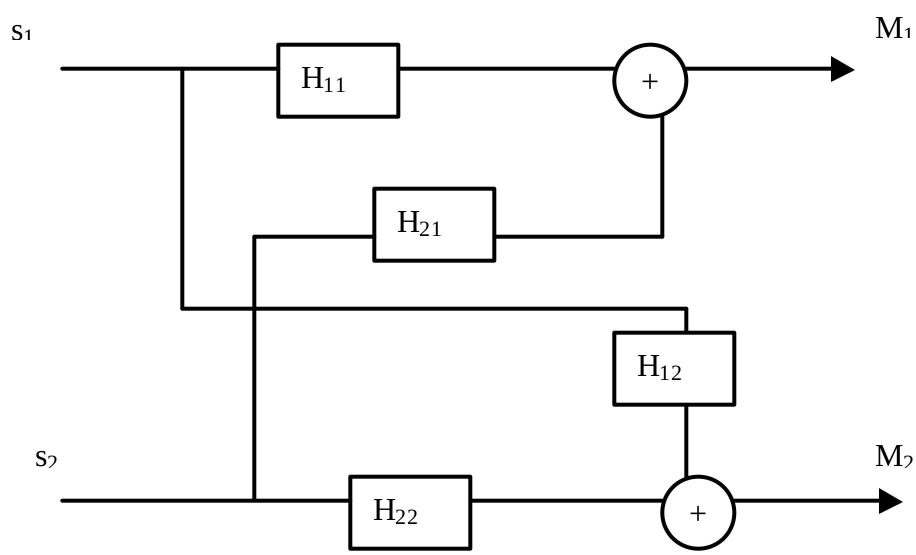
<u>Definition of RRM</u>: Radio resource management is the study of interference-limited systems where the number of simultaneous connections is larger than the number of orthogonal signals that the available bandwidth may produce. This involves the following issues:

1. *Power control*: Regulation of transmitter powers to limit attenuation of transmitted signal and interference seen by other users.
2. *Channel allocation*: Selection of frequency or channel for transmission.
3. *Handoff*: Assigning Base stations/Radio Access Ports (RAP) based on mobile user location.

<u>An example of reuse distance and its impact on capacity</u>

Consider two transmitters s_1 and s_2 separated by certain distance as shown in the figure below. M_1 and M_2 are intended receivers for s_1 and s_2 respectively. The transmitters use identical modulation schemes and transmitter powers are P_1 and P_2. Let path loss exponent be α. We require SIR $\geq$ 10db in each case.





(a) No shadowing

Here we neglect shadowing and background noise. We assume link gains H_{ij} depend only on path loss. Received power for this model is given by:

$$P_{Rx} = P_{Tx} c D^{-a}$$
$$\textit{where } c \textit{ is a constant.}$$

SIR at receiver M_2

$$g_2 = \frac{cP_2 r_2^{-a}}{cP_1 D_{12}^{-a}} = \frac{P_2}{P_1}\left(\frac{D_{12}}{r_2}\right)^a$$

For equal powers $P_1 = P_2$

$$g_2 = \left(\frac{D_{12}}{r_2}\right)^a$$
$$\textit{SIR requirement} \Rightarrow g_2 > 10db$$
$$\Rightarrow D_{12} > r_2(10)^{1/a}$$

$$\textit{This gives } D_{12} \approx 3.2 r_2 \textit{ if } a = 2$$
$$\approx 1.8 r_2 \textit{ if } a = 4$$

(b) With shadow fading

Small scale fading is ignored in this model since timescales are such that it is averaged out. Received power with shadow fading for this model is

$$P_{Rx} = cD^{-a} G P_{Tx}$$

where G is a lognormal random variable which models large scale fading effects. Assume G is zero mean and standard deviation 6.

Require $P[g_2 > 10dB] > 0.9$

$$g_2 = \frac{cr_2^{-a} G_{22} P_2}{cD_{12}^{-a} G_{12} P_1} = \frac{G_{22} P_2}{G_{12} P_1}\left(\frac{D_{12}}{r_2}\right)^a$$

For equal powers

$$g_2 = \frac{G_{22}}{G_{12}}\left(\frac{D_{12}}{r_2}\right)^a$$
$$= G\left(\frac{D_{12}}{r_2}\right)^a$$

Ratio of two lognormal variables is again a lognormal random variable. Hence G is lognormal with mean zero and standard deviation 8dB(approx.)

$$P[g_2 > 10] = \Pr\{10\log_{10} G > 10 - 10a\log_{10}\left(\frac{D_{12}}{r_2}\right)\}$$
$$= Q\left(\frac{10 - 10a\log_{10}\left(\frac{D_{12}}{r_2}\right)}{s_G}\right) > 0.9$$
$$\Rightarrow D_{12} > r_2 10^{2/a}$$

This gives

$$D_{12} \approx 11 r_2 \ \ if \ \ a = 2$$
$$\approx 3.3 r_2 \ \ if \ \ a = 4$$

Inferences

1.) Shadowing increases reuse distance over a simple path loss model, almost by a factor of 2.

2.) SIR depends only on the ratio $\frac{D_{12}}{r_2}$ and not on absolute distances.

3.) SIR increases rapidly with reuse distance.

4.) Increase in α allows smaller reuse distances.

5.) If powers are not equal, they affect SIR too.

Resource Allocation in RRM

RRM is mainly concerned with solving resource allocation problems. We examine the formulation of a typical resource allocation problem in RRM here.

1.) Interference in the system is mapped to SIR and performance is mapped to a minimum SIR requirement.
2.) Flat and slow fading is assumed. Signal quality is assumed to depend only on local mean SIR.
3.) Orthogonal signaling is assumed, that is all transmitters choose orthogonal signals from the same set of orthogonal signals.

Let *M* = {1, 2, 3,….,M} be the set of active mobiles. Typically *M* is a random variable. Let *B* = {1, 2, 3,….,B} be the set of RAPs and let C = {1, 2, …….. ,C} be the set of orthogonal channels available for establishing links between RAPs and mobiles.

Resource allocation algorithm

Problem: For each mobile we have to assign:

(a) An access port from set *B*.
(b) A channel pair from set *C*.
(c) A transmitter power for mobile and RAP.

such that all assigned links meet their minimum SIR requirement.

Generalizing from the two-user case and under the assumption of flat and slow fading channels, the relevant link gains of the associated system are characterized by the following link gain matrix –

$$G = \begin{bmatrix} G_{11} & G_{12} & . & . & . & G_{1M} \\ G_{21} & G_{22} & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ G_{B1} & . & . & . & . & G_{BM} \end{bmatrix}$$

Given that a mobile *j* has been assigned to port *i* on a channel pair *c*, the following must hold for constant transmit power.

For the uplink:

$$g_j^u = \frac{PG_{ij}}{\sum_{m \in M^{(c)}} PG_{im} + N_b} \geq g_0$$

For the downlink:

$$g_j^d = \frac{PG_{ij}}{\sum_{b \in B^{(c)}} PG_{bj} + N_m} \geq g_0$$

$M^{(c)} = \{m$: mobile m has been assigned channel $c\}$
$B^{(c)} = \{b$: base station b has been assigned channel $c\}$
Where c = 1,2, ……., C

This problem is NP complete, there is no algorithm to solve it in polynomial time.

REFERENCES

1.) Lecture notes, Wireless Communication Technologies, Spring 2002 – Dr. Narayan B. Mandayam
2.) “Multiuser Detection” – Sergio Verdu, Cambridge University Press.
3.) “Gradient Estimation for Sensitivity Analysis and Adaptive Multiuser Interference Rejection in Code Division Multiple Access Systems” – Narayan B. Mandayam and Behnaam Aazhang.

APPENDIX

1. Stochastic Gradient Algorithms using IPA

$$\min_{\underline{v}_k} \underline{P}_k^{\underline{v}_k} = \min_{\underline{v}_k} E[\,Q(\frac{A_k \underline{v}_k^{\,T} \underline{r}_k + \sum_{j \neq k} A_j b_j \underline{v}_k^{\,T} r_j}{s\,\sqrt{\underline{v}_k^{\,T} R_k \underline{v}_k}})].$$

The general form of this problem is $\min_{q} \underline{P}_e = \min_{q} E[\,L(q, y)]$.

Using IPA, the derivative of average probability of error with respect to the parameter θ is estimated as

$$\frac{\partial}{\partial q}\hat{\underline{P}}_e(q) = \frac{\partial}{\partial q} L(q, y)$$

where the derivative of L is computed as

$$\frac{\partial}{\partial q} L(q, y) = \lim_{\Delta q \to 0} \frac{L(q + \Delta q, y) - L(q, y)}{\Delta q}$$

where Δq *is a small perturbation in* q.

If this estimate is unbiased then

$$E[\frac{\partial}{\partial q} L(q, y)] = \frac{\partial}{\partial q} E[L(q, y)]$$

Instead of taking derivatives of average probability of error, which is a formidable task, we can take derivatives of its argument function *L*. Derivative of L with respect to θ is computed using IPA as shown above.

The optimization problem is solved using the following algorithm:

$$q_{i+1} = q_i - g_i \nabla_q L(q_i, y_i)$$

where q_{i+1} *is the* i^{th} *update of the parameter* q,

$\nabla_q L(q_i, y_i)$ *is the gradient evaluted using IPA*

and g_i *is the step size*

2. Solution to $\min_{M \in \Re^{k\times k}} trace\{E[(\underline{b} - M\underline{y})(\underline{b} - M\underline{y})^T]\}$

$$E[(\underline{b} - M\underline{y})(\underline{b} - M\underline{y})^T] = E[\underline{b}\underline{b}^T] - E[\underline{b}\underline{y}^T]M^T - ME[\underline{y}\underline{b}^T] + ME[\underline{y}\underline{y}^T]M^T$$

Since $\underline{y} = RA\underline{b} + \underline{h}$ and noise is uncorrelated with data,

$$E[\underline{b}\underline{b}^T] = I$$
$$E[\underline{b}\underline{y}^T] = E[\underline{b}\underline{b}^T AR] = AR$$
$$E[\underline{y}\underline{b}^T] = E[RA\underline{b}\underline{b}^T] = RA$$
$$E[\underline{y}\underline{y}^T] = E[RA\underline{b}\underline{b}^T AR] + E[hh^T] = RA^2R + s^2R$$

Using these results,

$$E[(\underline{b} - M\underline{y})(\underline{b} - M\underline{y})^T] = I + M(RA^2R + s^2R)M^T - ARM^T - MRA$$
$$= [I + s^{-2}ARA]^{-1} + (M - \hat{M})(RA^2R + s^2R)(M - \hat{M})^T$$
$$where\ \hat{M} = A^{-1}[R + s^2A^{-2}]^{-1}$$
$$trace\{E[(\underline{b} - M\underline{y})(\underline{b} - M\underline{y})^T]\} = trace\{[I + s^{-2}ARA]^{-1}\} + trace\{(M - \hat{M})(RA^2R + s^2R)(M - \hat{M})^T\}$$

The matrix $RA^2R + s^2R$ is non-negative definite. Hence the trace of second term is always non-negative. Therefore the solution is given by:

$$M = \hat{M} = A^{-1}[R + s^2A^{-2}]^{-1}$$