## Wireless Communication Technologies

16:332:559 (Advanced Topics in Communication Engineering)
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## Decision Feedback Equalizers (Nonlinear Equalizers)

The basic idea behind decision feedback equalization is that when a decision has been made on an information symbol, the intersymbol interference that is induces on further symbols can be estimated and subtracted out before detection of the successive symbols. In principle, the decision feedback equalizers use previous decision to eliminate ISI caused by previously detected symbols on arrival symbol. Output signal from equalizer expressed as:

$$\hat{a}_n = \sum_{j=-M_1}^{0} c_j x_{n-j} + \sum_{j=1}^{M_2} c_j \tilde{a}_{n-j}$$

$\sum_{j=-M_1}^{0} c_j x_{n-j}$ are feedforward $(M_1+1)$ taps in FF filter.

$\sum_{j=1}^{M_2} c_j \tilde{a}_{n-j}$ are feedback $M_2$ tapes in FB filter.

$\tilde{a}_{n-1}, \tilde{a}_{n-2,} .......\tilde{a}_{n-M_2}$ are earlier detected symbols.

DFE is very effective in frequency selective fading channels.
If we assume feedback decisions are correct, then FF filter tapes $c_0, c_1, ....c_{M+1}$ are by

$$\min J = E\left[(a_n - \hat{a}_n)^2\right]$$

For FB part, $c_k = -\sum_{j=-M_1}^{0} \lambda_j h_{k-j} \qquad k = 1,2,.....M_2$

## Sequence Estimation (MLSE)

Consider the discrete-time while noise channel model

$$r_n = \sum_{n=0}^{L} h_m a_{n-m} + \eta_k$$

Assume k symbols are transmitted over channel. After receiving the $\{x_n\}_{n=1}^{k}$, the ML receiver finds the sequence $\{a_n\}_{n=1}^{k}$ that maximize the likelihood function

$$\log P(r_k, r_{k-1}, \ldots r_1 | a_k, a_{k-1}, \ldots a_1)$$

Since noise samples are independent and $r_n$ depends only on L most recent symbols

$$\log P(r_k, r_{k-1}, \ldots r_1 | a_k, a_{k-1}, \ldots a_1) = \log P(r_k | a_k, a_{k-1}, \ldots a_{k-L}) + \log P(r_{k-1}, r_{k-2}, \ldots r_1 | a_{k-1}, a_{k-2}, \ldots a_1)$$

$$a_{k-L} = 0, \quad for \quad k - L \leq 0$$

If the 2nd term has been previously calculated at epoch (k-1), then only first term needs to be computed for each incoming signal $r_k$ at time k.
When $\eta_n$ is Gaussian

$$\log P(r_k | a_k, a_{k-1}, \ldots a_{k-L}) = \frac{1}{\pi N_0} \exp\{-\frac{1}{N_0} \left| r_k - \sum_{i=0}^{L} h_i a_{k-i} \right|^2 \}$$

$\log P(r_k | a_k, a_{k-1}, \ldots a_{k-L})$ yields branch metric

$$\mu_k = -\left| r_k - \sum_{i=0}^{L} h_i a_{k-i} \right|^2$$

Note receiver requires channel $\{h_i\}$ to compute branch metric. The Viterbi algorithm can be used to implement receiver through trellis diagram corresponding to $N_S = 2^{nL}$ state, where $2^n$ is the constellation size. However, the complexity increases exponentially in channel memory length. To solve this problem, a DDFSE is used.

Adaptive MLSE
Use training sequences to estimate channel response for use in MLSE algorithm.

## Delayed Decision Feedback Sequence Estimation (DDFSE)

This technique reduce the receiver complexity by truncating the effective channel memory to $\mu$ term, where $\mu$ is an integer that can be varied from 0 to L. Thus, a suboptimum decoder is obtained with complexity controlled parameter $\mu$. This is the basic principle of DDFSE.

## Diversity Systems

Improve performance over fading channels. Receiver is provided with multiple copies of information transmitted over 2 or more independent transmission channels.
Basic ideal: repetition + combination => improve performance.
Macroscopic Diversity – long term or scale fading
=> distances between base stations, e.g. repetition at 2 different BSs.
Microscopic Diversity – short scale fading
=> distance of order of wavelengths, e.g. antenna elements.
Works well when different signals (branches) experience independent fading (at least uncorrelated fading)

1. Frequency Diversity: transmit signal on 2 or more carriers (related to coherent bandwidth)
2. Time Diversity: transmit signal repeatedly at intervals (related to coherent time). If fading rate is slow, you need long interval between repetitions.
3. Polarization Diversity: manipulate electric and magnetic fields to obtain orthogonal polarizations.
4. Angle Diversity: use directional antennas to create independent copies of transmitted signals through multiple paths.
5. Space Diversity: placing of receiving antennas at different locations will result in different (and possible independent) signals.

Frequency diversity and time diversity use up more bandwidth. Others require more complex antennas.

Correlation between signals is a function of distance between antenna elements:

$$\rho = J_0^{\,2}\left(\frac{2\pi d}{\lambda}\right)$$

where $J_0(x)$ = Bessel function of zero order
$d$ = distance between two antenna
$\lambda$ = transmitted wave length

If we want to achieve zero correlation, the distance between two antennas is approximately half wavelength separating to get zero cancellation.

For example: $f_c = 5MHz \quad \Rightarrow \quad d = 30m$

$f_c = 500MHz \quad \Rightarrow \quad d = 30cm$

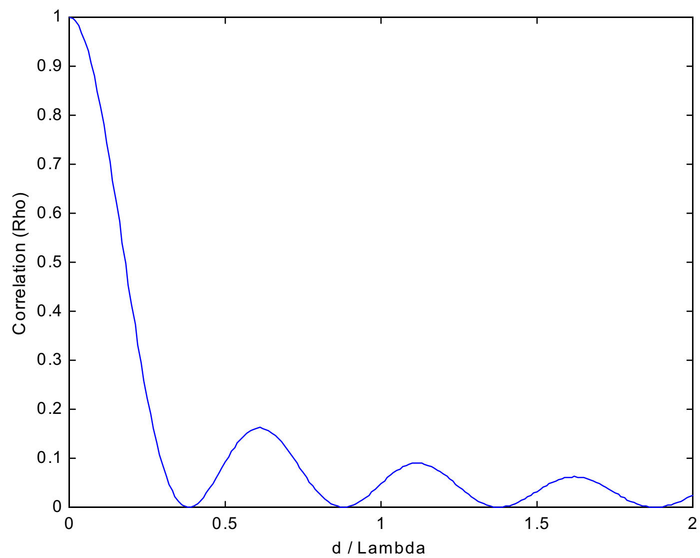



Figure 1. Correlation for different values of $d/\lambda$

# Combining methods

We will assume that there are M independent copies of the transmitted signal are available from M independent paths (branches) in a diversity system.

## 1. Selection Diversity

Consider M diversity branches where the SNR achieve on each branches is $\gamma_i, i = 1,2,...M$ . Let us assume received signal on each branch is independent Rayleigh distributed with mean power $2\sigma^2$. $\gamma_i$ is exponentially distributed as:

$$P(\gamma_i) = \frac{1}{\gamma_0}\exp(\frac{-\gamma_i}{\gamma_0}), \quad \gamma_i \geq 0$$

$$\gamma_0 = 2\sigma^2 \frac{E_b}{N_0}$$

$$\frac{E_b}{N_0} = \text{SNR without fading}$$

The CDI of $\gamma_i$ is

$$P[\gamma_i < \gamma] = \int_0^{\gamma} P(\gamma_i) d\gamma_i = 1 - \exp(\frac{-\gamma_i}{\gamma_0})$$

The selection diversity scheme always selects the branch with the highest SNR, i.e., $= \max_i \{\gamma_i\}$. Then the probability that this selected SNR is less than $\gamma$, is equal to the probability that SNR in all M branches, simultaneously, is less than or equal to $\gamma$, that is

$$P(\gamma) \equiv P_r[\max_i \{\gamma_i\} \le \gamma] = P_r[\gamma_1 \le \gamma, \gamma_2 \le \gamma, \ldots \gamma_M \le \gamma] = [1 - \exp(\frac{-\gamma}{\gamma_0})]^M$$

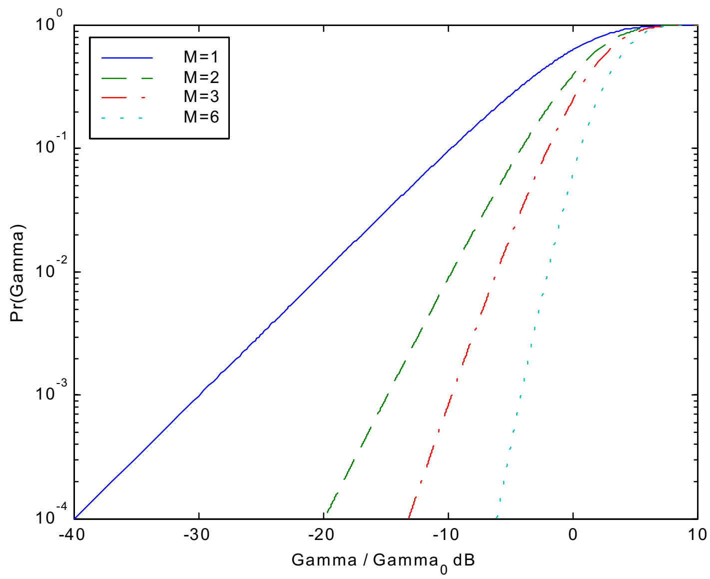



Figure 2. CDF for Selection Diversity with M=1, 2, 3, 6

Note: the probability of very low SNR decrease rapidly when diversity is used. For SNR far below average $\gamma_0$, we may use the approximation $\exp(-x) \approx 1 - x$, which

$$P(\gamma) \approx (\frac{\gamma}{\gamma_0})^M \quad , \quad \gamma << \gamma_0$$

For the logarithmic scale, there correspond to straight lines with slop M.

What is the average SNR due to selection diversity?

$$E\{\max_i [\gamma_i]\} = \int_0^{\infty} \gamma p(\gamma) d\gamma$$

$$P(\gamma) = \frac{dP(\gamma)}{d\gamma} = \frac{M}{\gamma_0} \exp(\frac{-\gamma}{\gamma_0})[1 - \exp(\frac{-\gamma}{\gamma_0})]^{M-1}$$

$$E\{\max_i[\gamma_i]\} = \gamma_0 \sum_{k=1}^{M} \frac{1}{k}$$

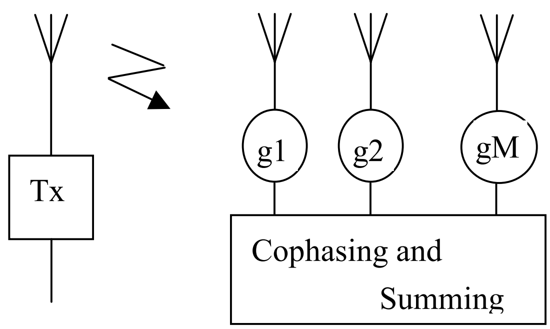
M=1 to M=2 => 1.8 dB improvement

M=2 to M=3 => 0.9 dB improvement

Disadvantage: signals need be monitored at a rate further than the fading.

## 2. Maximum Ratio Combining (MRC)

All branches are used simultaneously. Each of these branch signals is weighted with a gain factor proportional to its SNR.




Requiring adding up signals after making the same phase. If $a_i$ is the signal envelope in each branch, then the combined signal envelope is

$$a = \sum_{i=1}^{M} g_i a_i$$

Assume noise components are i.i.d. in each branch. Then total noise power

$$N_t = N_0 \sum_{i=1}^{M} {g_i}^2$$

$N_0$ is the noise power in each branch.

The resulting SNR is

$$\gamma = \frac{a^2 E_b}{N_t} = \frac{E_b}{N_0} \frac{(\sum_{i=1}^{M} {a_i}^2 {g_i}^2)}{\sum_{i=1}^{M} {g_i}^2}$$

Using Schwarz inequality

$$\gamma \le \frac{E_b}{N_0} \frac{(\sum_{i=1}^{M} a_i^{\ 2} g_i^{\ 2})}{\sum_{i=1}^{M} g_i^{\ 2}}$$

Equality is obtained in Schwarz inequality, if $g_i = ka_i$, k is some constant.

The maximum value of output SNR is given as

$$\gamma = \frac{E_b}{N_0} \sum_{i=1}^{M} a_i^{\ 2} = \sum_{i=1}^{M} \frac{E_b}{N_0} a_i^{\ 2} \sum_{i=1}^{M} \gamma_i$$

The resulting SNR is sum of SNRs.

To obtain distribution of combined signals, observe

$$\gamma_i = \frac{E_b}{N_0} a_i^{\ 2} = \frac{E_b}{N_0} (x_i^{\ 2} + y_i^{\ 2})$$

Where $x_i$ and $y_i$ are independent Gaussian random variables with zero mean and variance of $\sigma^2$. The sum of two squared Gaussian random variables is Chi-square distributed with degree of two. Since we have M branches and each branch has two degrees, thus the total degree of freedom for $\gamma$ is 2M.

$$\gamma = \sum_{i=1}^{M} \gamma_i$$

$$P(\gamma) = \frac{1}{(M+1)!} \frac{\gamma^{M-1}}{\gamma_0^{\ M}} \exp(\frac{-\gamma}{\gamma_0}), \qquad \gamma \ge 0$$

Where $\gamma_0$ is the mean SNR in each branch.

$$\gamma_0 = 2\sigma^2 \frac{E_b}{N_0}$$

CDF is

$$P(\gamma) = \int_0^{\gamma} p(x)dx = \frac{1}{(M-1)!} \frac{1}{\gamma_0^{\ M}} \int_0^{M} x^{M-1} \exp(\frac{-x}{\gamma_0}) dx = 1 - \exp(\frac{-\gamma}{\gamma_0}) \sum_{i=1}^{M} \frac{1}{(i-1)!} (\frac{\gamma}{\gamma_0})^{i-1}$$

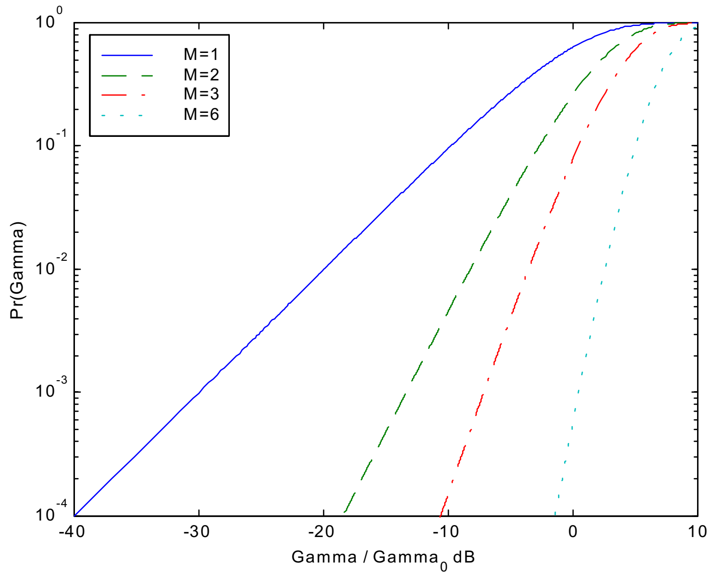



Figure 4. CDF for Maximum Ratio Combining with M=1, 2, 3, 6

We can note that the mean value increases considerably faster for maximal ratio combining compared with selection diversity.

E.g. 10 dB below $\gamma_0$, M=2, $p(\gamma) = 0.005$, mean SNR of combined signals

$E[\sum_{i=1}^{M} \gamma_i] = M\gamma_0$ => mean value increases linearly with M

## 3. Equal Gain Combining

Set $g_i = 1, \quad for \quad i = 1,2,...M$

$$a = \sum_{i=1}^{M} a_i$$

$$\gamma = \frac{a^2 E_b}{MN_0} = \frac{E_b}{MN_0} (\sum_{i=1}^{M} a_i)^2$$

SNR is the square of a sum of M Rayleigh distributed random variables and no closed form. Performance is a little worse than MRC.

$$E[\gamma] = \frac{E_b}{MN_0} E[(\sum_{i=1}^{M} A_i)^2] = \frac{E_b}{MN_0} \sum_{i=1}^{M} \sum_{j=1}^{M} E[A_i A_j]$$

For uncorrelated branches $E[A_i A_j] = E[A_i]E[A_j]$, and for Rayleigh signals $E[A_i^2] = 2\sigma^2$

and $E[A_i] = \sqrt{\frac{\pi}{2}\sigma^2}$

$$E[\gamma] = \frac{E_b}{MN_0}[2M\sigma^2 + M(M-1)\frac{\pi\sigma^2}{2}] = \gamma_0[1 + (M-1)\frac{\pi}{4}] \approx \frac{\pi}{4}M\gamma_0$$

For large M, EGC is worse than MRC by $\frac{\pi}{4} \approx 0.8$

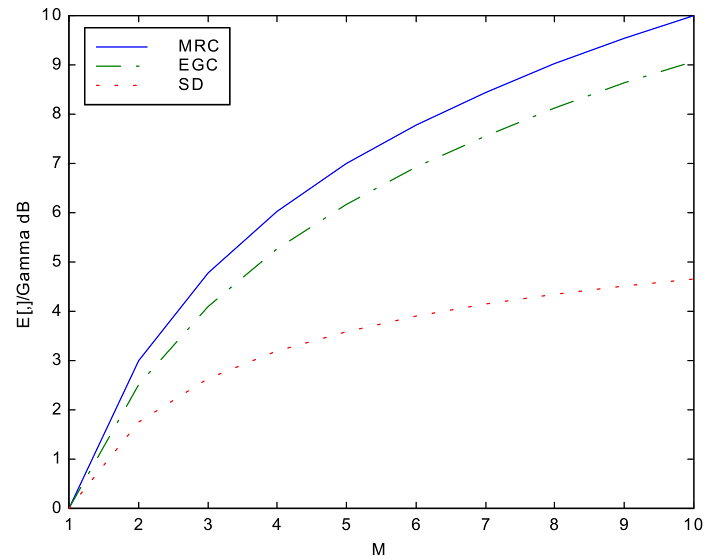



Figure 5. Comparison of average SNR at different branches, M, for Maximum Ratio Combining, Equal Gain Combining and Selection Diversity