Probability and Random Processes

Course No: 14:332:321 (Fall 2000)

Exam 2

Maximum Marks : 30 **Total Time : 1hour & 10minutes**

Instructions : *Answer all questions. The points for each question are listed below in parentheses.*

1. Fill in the blanks **(5)**

   (a) If random variable X is Bernoulli with parameter 0.5, $E[X] =$ _____

   (b) If random variable X is Bernoulli with parameter 0.5, $Var[X] =$ _____

   (c) If random variable X is geometric with parameter 0.5, $E[X] =$ _____

   (d) If random variable X is geometric with parameter 0.5, $Var[X] =$ _____

   (e) If random variable X is Poisson with parameter 0.5, $Var[X] =$ _____

2. X is a uniform random variable taking values over the interval $[1, 5]$. Consider $K = \lceil X \rceil$. **(8)**

   (a) What is the probability mass function (PMF) of K ?

   (b) What is E[K] ?

   (c) What is $Var[K]$?

3. Flip a fair coin until heads occurs twice. Let X_1 equal the number of flips up to and including the first head. Let X_2 equal the number of additional flips up to and including the second head. **(6)**

   (a) What is $P_{X_1}(x_1) = ?$

   (b) What $P_{X_2}(x_2) = ?$

   (c) Are X_1 and X_2 independent ? Give reasons for your answer.

   (d) What is $P_{X_1,X_2}(x_1, x_2) = ?$

4. If X and Y are random variables such that $Y = aX + b$, prove that **(6)**

$$\rho_{X,Y} = \begin{cases} -1 & a < 0 \\ 0 & a = 0 \\ 1 & a > 0 \end{cases}$$

5. X is a random variable with PDF given as **(5)**

$$f_X(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{(x-3)^2}{18}} \quad -\infty \leq x \leq \infty$$

   Find the second moment of X ?