Probability and Random Processes

Course No: 14:332:321 (Fall 2001)

Solutions to Exam 1

Maximum Marks : 40   **Total Time : 1hour & 10minutes**

Instructions : *Answer all questions. The points for each question are listed below in parentheses.*

1. Read the following statements and state if they are "true" or "false"   **(5)**

   (a) True

   (b) False

   (c) True

   (d) False

   (e) False

2. Consider the type of pizza (thin-crust or deep-dish) purchased by each of five   **(8)** different customers at a certain pizzeria.

   Let us denote a customer buying a thin-crust pizza as a "success" and a customer buying a deep-dish pizza is a "failure".

   Let the random variable K denote the number of successes and let $P_K(k)$ denote the probability of k successes.

   (a) It is given that the probability that at most one of the customers purchases a thin-crust [**4**] pizza is 0.2, i.e., the probability of atmost one success is

   $$P_K(0) + P_K(1) = 0.2$$

   Therefore, the probability that at least two customers purchase a thin crust pizza is the probability of at least two successes and is given as

   $$P_K(2) + P_K(3) + P_K(4) + P_K(5) = 1 - [P_K(0) + P_K(1)] = 0.8$$

   (b) It is given that $P(all\ five\ customers\ purchase\ deep-dish\ pizzas) = P_K(0) = 0.15$ and [**4**] $P(all\ five\ customers\ purchase\ thin-crust\ pizzas) = P_K(5) = 0.05$.
   The probability that at least one of each type is purchased is the probability of at least one success and one failure and is given as

   $$P_K(1) + P_K(2) + P_K(3) + P_K(4) = 1 - [P_K(0) + P_K(5)] = 0.8$$

3. You repeatedly toss an unbiased coin until you observe 5 heads. What is the probability the **(6)** you tossed the coin a total of 7 times including the time that the fifth head appears ?

   The total number of tosses X required to observe 5 heads is a Pascal random variable. The PMF of X is given as

   $$P_X(x) = \begin{cases} \binom{x-1}{5-1}(1/2)^5(1/2)^{x-5} & x = 5, 6, \cdots \\ 0 & \text{otherwise} \end{cases}$$
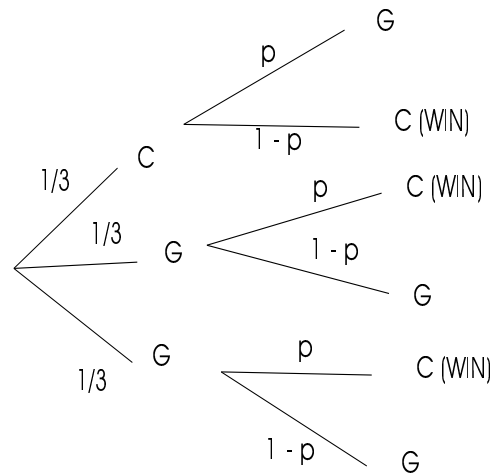
Therfore,

$$P_X(7) = \binom{7-1}{5-1}(1/2)^5(1/2)^{7-5} = 0.117$$

4. Assume you are a contestant appearing on the Monty Hall TV show. To refresh your **(20)** memory regarding the details of the show, there are three doors that are closed and behind them are 2 goats and 1 car (only one item is behind each door). You are asked to select one of the doors without opening it. Then, your host Monty Hall opens one of the other doors to reveal a goat. You are then asked if you would like to switch your selection to the other unopened door. Since you are not sure, you toss a coin and decide on the following strategy:

Heads shows on toss $\Rightarrow$ You switch to the other door

Tails shows on toss $\Rightarrow$ You do not switch to the other door

The tree diagram for the procedure is given as

(a) The probability that you win the car if the probability of observing heads on the coin toss is p is $P[W] = \frac{1}{3}(1-p) + \frac{1}{3}p + \frac{1}{3}p = \frac{1}{3} + \frac{1}{3}p$ [15]

(b) Using the result in (a), the probability of winning if $p = 1/2$ is $P[W] = 1/2$ [5]

5. How old do you think is the instructor for this course ? **(1)**

Does'nt matter how old I am .... you got your 1 point anyway! :)