Probability and Random Processes

Course No: 14:332:321 (Fall 2000)

Solutions to Exam 1

Maximum Marks : 30   **Total Time : 1hour & 10minutes**

Instructions : *Answer all questions. The points for each question are listed below in parentheses.*

1. Read the following statements and state if they are "true" or "false"   **(5)**

    (a) False

    (b) False

    (c) False

    (d) True

    (e) False

2. At the end of regulation time in a basketball game, the L.A. Lakers are trailing by one point and Shaquille "Shaq" O'Neal (a pretty lousy free throw shooter) goes to the line for two free throws. If he makes exactly one free throw, the game goes into overtime. The probability that the first free throw is good is 1/3. Further, if the first attempt is good, Shaq relaxes and the second attempt is good with probability 3/4. However, if Shaq misses the first attempt, the added pressure reduces the success probability on the second attempt to 1/4.   **(14)**

    (a) What is the probability that the game goes into overtime ?

   Let G denote the event of good shot and B denote event of bad shot.

    Overtime happens, if Shaq makes only one shot and misses the other. There are two possibilities

    (I) Player making first shot and missing second $P[GB] = 1/3 \times 1/4 = 1/12$

    (II) Player missing first shot and making second shot is $P[BG] = 2/3 \times 1/4 = 2/12$

    The probability that the game goes into overtime is

    $$P[O] = P[GB] + P[BG] = 1/4$$

First shot   Second Shot

1
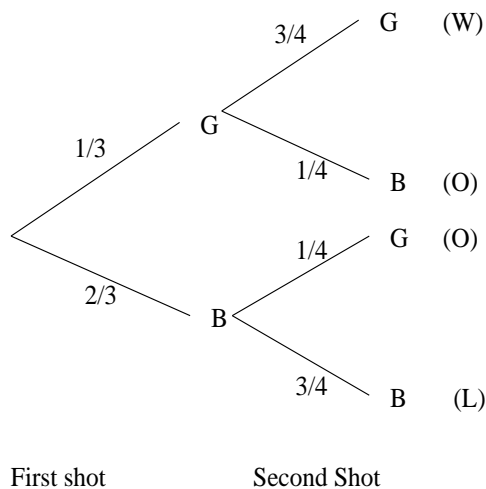
(b) Given that the game goes into overtime, what is the probability that Shaq made the first shot ?

We need to find

$$P[\text{Shaq makes first shot} \mid \text{game goes into overtime}]$$

This is given as

$$\frac{P[\text{Shaq makes first shot } and \text{ game goes into overtime}]}{P[\text{game goes into overtime}]} = \frac{P[GB]}{P[O]} = \frac{1/12}{1/4} = 1/3$$

3. You and I decide to make a wager for a dollar. We toss an unbiased coin repeatedly until one of the two things happens: **(10)**

   (a) Two tails (TT) appear in a row and we stop and you win a dollar.
   (b) A head appears immediately followed by a tail (HT) and we stop and I win a dollar

   If we repeat this wager an infinite number of times, who do you think is going to win more money ? In fact, what is the probability of my winning ?

   From the tree diagram it is clear that you can win only if the first two tosses are tails, i.e., TT is the sequence observed after the first two tosses. Therefore the probability of your winning is

   $$P[You\ Win] = 1/2 \times 1/2 = 1/4$$

   The probability of my winning is

   $$P[I\ Win] = 1 - P[You\ Win] = 3/4$$
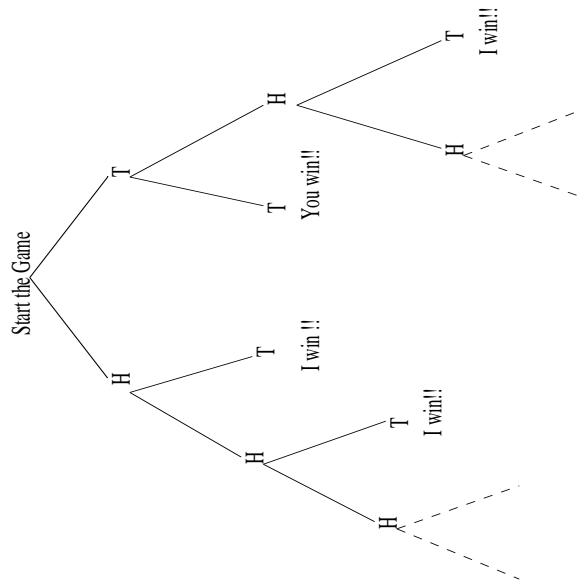
4. How old do you think is the instructor for this course ? **(1)**

   Does'nt matter how old I am .... you got your 1 point anyway! :)

   Good luck!