Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers

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Problem Solutions : Yates and Goodman,2.5.6 2.5.7 2.5.9 and 2.5.10

Problem 2.5.6

random variable X has PMF

$$P_X(x) = \begin{cases} \binom{4}{x}(1/2)^4 & x = 0,1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

The expected value of X is

$$\begin{aligned} E[X] = \sum_{x=0}^{4} x P_X(x) &= 0\binom{4}{0}\frac{1}{2^4} + 1\binom{4}{1}\frac{1}{2^4} + 2\binom{4}{2}\frac{1}{2^4} + 3\binom{4}{3}\frac{1}{2^4} + 4\binom{4}{4}\frac{1}{2^4} \\ &= [4+12+12+4]/2^4 = 2 \end{aligned}$$

Problem 2.5.7

random variable X has PMF

$$P_X(x) = \begin{cases} \binom{5}{x}(1/2)^5 & x = 0,1,2,3,4,5 \\ 0 & \text{otherwise} \end{cases}$$

The expected value of X is

$$\begin{aligned} E[X] &= \sum_{x=0}^{5} x P_X(x) \\ &= 0\binom{5}{0}\frac{1}{2^5} + 1\binom{5}{1}\frac{1}{2^5} + 2\binom{5}{2}\frac{1}{2^5} + 3\binom{5}{3}\frac{1}{2^5} + 4\binom{5}{4}\frac{1}{2^5} + 5\binom{5}{5}\frac{1}{2^5} \\ &= [5+20+30+20+5]/2^5 = 2.5 \end{aligned}$$

Problem 2.5.9

In this ”double-or-nothing” type game, there are only two possible payoffs. The first is zero dollars, which happens when we lose 6 straight bets, and the second payoff is 64 dollars which happens unless we lose 6 straight bets. So the PMF of Y is

$$P_Y(y) = \begin{cases} (1/2)^6 = 1/64 & y = 0 \\ 1-(1/2)^6 = 63/64 & y = 64 \\ 0 & \text{otherwise} \end{cases}$$

The expected amount you take home is

$$E[Y] = 0(1/64) + 64(63/64) = 63$$

So, on the average, we can expect to break even, which is not a very exciting proposition.

Problem 2.5.10

By the definition of the expected value,

$$\begin{aligned} E[X_n] &= \sum_{x=1}^{n} x \binom{n}{x} p^x (1-p)^{n-x} \\ &= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-1-(x-1))!} p^{x-1} (1-p)^{n-1-(x-1)} \end{aligned}$$

With the substitution $x' = x - 1$, we have

$$E[X_n] = np \underbrace{\sum_{x'=0}^{n-1} \binom{n-1}{x'} p^{x'} (1-p)^{n-x'}}_{1} = np \sum_{x'=0}^{n-1} P_{X_{n-1}}(x) = np$$

The above sum is 1 because it is he sum of a binomial random variable for $n-1$ trials over all possible values.