Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
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Problem Solutions : Yates and Goodman,2.2.1 2.2.6 2.3.1 2.3.2 2.4.1 2.4.3 2.5.1 and 2.5.2

Problem 2.2.1

(a) We wish to find the value of c that makes the PMF sum up to one.

$$P_N(n) = \begin{cases} c(1/2)^n & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Therefore, $\sum_{n=0}^{2} P_N(n) = c + c/2 + c/4 = 1$, implying $c = 4/7$.

(b) The probability that $N \leq 1$ is

$$P[N \leq 1] = P[N = 0] + P[N = 1] = 4/7 + 2/7 = 6/7$$

Problem 2.2.6

The probability that a caller fails to get through in three tries is $(1-p)^3$. To be sure that at least 95% of all callers get through, we need $(1-p)^3 \leq 0.05$. This implies $p = 0.6316$.

Problem 2.3.1

(a) If it is indeed true that Y, the number of yellow M&M's in a package, is uniformly distributed between 5 and 15, then the PMF of Y, is

$$P_Y(y) = \begin{cases} 1/11 & y = 5, 6, 7, \ldots, 15 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$P[Y < 10] = P_Y(5) + P_Y(6) + \cdots + P_Y(9) = 5/11$$

(c)

$$P[Y > 12] = P_Y(13) + P_Y(14) + P_Y(15) = 3/11$$

(d)

$$P[8 \leq Y \leq 12] = P_Y(8) + P_Y(9) + \cdots + P_Y(12) = 5/11$$

Problem 2.3.2

(a) Each paging attempt is an independent Bernoulli trial with success probability p. The number of times K that the pager receives a message is the number of successes in n Bernoulli trials and has the binomial PMF

$$P_K(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & k = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

(b) Let R denote the event that the paging message was received at least once. The event R has probability

$$P[R] = P[B > 0] = 1 - P[B = 0] = 1 - (1-p)^n$$

To ensure that $P[R] \geq 0.95$ requires that $n \geq \ln(0.05)/\ln(1-p)$. For $p = 0.8$, we must have $n \geq 1.86$. Thus, $n = 2$ pages would be necessary.

Problem 2.4.1
Using the CDF given in the problem statement we find that

(a) $P[Y < 1] = 0$

(b) $P[Y \leq 1] = 1/4$

(c) $P[Y > 2] = 1 - P[Y \leq 2] = 1 - 1/2 = 1/2$

(d) $P[Y \geq 2] = 1 - P[Y < 2] = 1 - 1/4 = 3/4$

(e) $P[Y = 1] = 1/4$

(f) $P[Y = 3] = 1/2$

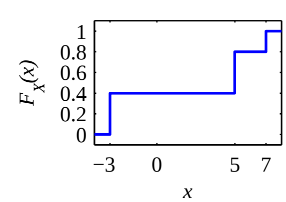
(g) From the staircase CDF of Problem 2.4.1, we see that Y is a discrete random variable. The jumps in the CDF occur at at the values that Y can take on. The height of each jump equals the probability of that value. The PMF of Y is

$$P_Y(y) = \begin{cases} 1/4 & y = 1 \\ 1/4 & y = 2 \\ 1/2 & y = 3 \\ 0 & \text{otherwise} \end{cases}$$

Problem 2.4.3

(a) Similar to the previous problem, the graph of the CDF is shown below.

$$F_X(x) = \begin{cases} 0 & x < -3 \\ 0.4 & -3 \leq x < 5 \\ 0.8 & 5 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$$



(b) The corresponding PMF of X is

$$P_X(x) = \begin{cases} 0.4 & x = -3 \\ 0.4 & x = 5 \\ 0.2 & x = 7 \\ 0 & \text{otherwise} \end{cases}$$

Problem 2.5.1

For this problem, we just need to pay careful attention to the definitions of mode and median.

(a) The mode must satisfy $P_X(x_{\text{mod}}) \geq P_X(x)$ for all x. In the case of the uniform PMF, any integer x' between 1 and 100 is a mode of the random variable X. Hence, the set of all modes is

$$X_{\text{mod}} = \{1, 2, \ldots, 100\}$$

(b) The median must satisfy $P[X < x_{\text{med}}] = P[X > x_{\text{med}}]$. Since

$$P[X \leq 50] = P[X \geq 51] = 1/2$$

we observe that $x_{\text{med}} = 50.5$ is a median since it satisfies

$$P[X < x_{\text{med}}] = P[X > x_{\text{med}}] = 1/2$$

In fact, for any x' satisfying $50 < x' < 51$, $P[X < x'] = P[X > x'] = 1/2$. Thus,

$$X_{\text{med}} = \{x | 50 < x < 51\}$$

Problem 2.5.2

Voice calls and data calls each cost 20 cents and 30 cents respectively. Furthermore the respective probabilities of each type of call are 0.6 and 0.4.

(a) Since each call is either a voice or data call, the cost of one call can only take the two values associated with the cost of each type of call. Therefore the PMF of X is

$$P_X(x) = \begin{cases} 0.6 & x = 20 \\ 0.4 & x = 30 \\ 0 & \text{otherwise} \end{cases}$$

(b) The expected cost, $E[C]$, is simply the sum of the cost of each type of call multiplied by the probability of such a call occurring.

$$E[C] = 20(0.6) + 30(0.4) = 24 \text{ cents}$$