# Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers

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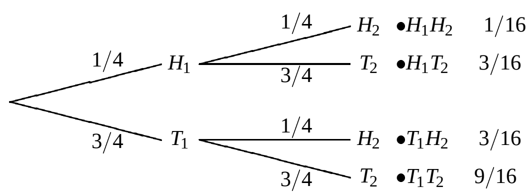
**Problem Solutions** : Yates and Goodman,1.7.1 1.7.5 1.8.1 1.8.2 1.8.4 and 1.9.1

**Problem 1.7.1**

A sequential sample space for this experiment is




(a) From the tree, we observe

$$P[H_2] = P[H_1H_2] + P[T_1H_2] = 1/4$$

This implies

$$P[H_1|H_2] = \frac{P[H_1H_2]}{P[H_2]} = \frac{1/16}{1/4} = 1/4$$

(b) The probability that the first flip is heads and the second flip is tails is $P[H_1T_2] = 3/16$.

**Problem 1.7.5**

The $P[-|H]$ is the probability that a person who has HIV tests negative for the disease. This is referred to as a false-negative result. The case where a person who does not have HIV but tests positive for the disease, is called a false-positive result and has probability $P[+|H^c]$. Since the test is correct 99% of the time,

$$P[-|H] = P[+|H^c] = 0.01$$

Now the probability that a person who has tested positive for HIV actually has the disease is

$$P[H|+] = \frac{P[+,H]}{P[+]} = \frac{P[+,H]}{P[+,H] + P[+,H^c]}$$

We can use Bayes' formula to evaluate these joint probabilities.

$$\begin{aligned} P[H|+] &= \frac{P[+|H]P[H]}{P[+|H]P[H] + P[+|H^c]P[H^c]} \\ &= \frac{(0.99)(0.0002)}{(0.99)(0.0002) + (0.01)(0.9998)} \\ &= 0.0194 \end{aligned}$$

Thus, even though the test is correct 99% of the time, the probability that a random person who tests positive actually has HIV is less than 0.02. The reason this probability is so low is that the a priori probability that a person has HIV is very small.

**Problem 1.8.1**

There are $2^5 = 32$ different binary codes with 5 bits. The number of codes with exactly 3 zeros equals the number of ways of choosing the bits in which those zeros occur. Therefore there are $\binom{5}{3} = 10$ codes with exactly 3 zeros.

**Problem 1.8.2**

Since each letter can take on any one of the 4 possible letters in the alphabet, the number of 3 letter words that can be formed is $4^3 = 64$. If we allow each letter to appear only once then we have 4 choices for the first letter and 3 choices for the second and two choices for the third letter. Therefore, there are a total of $4 \cdot 3 \cdot 2 = 24$ possible codes.

**Problem 1.8.4**

In this case, the designated hitter must be chosen from the 15 field players. We can break down the experiment of choosing a starting lineup into two sub-experiments. The first is to choose 1 of the 10 pitchers, the second is to choose the remaining 9 batting positions out of the 15 field players. Here we are concerned about the ordering of our selections because a new ordering specifies a new starting lineup. So the total number of starting lineups when the DH is selected among the field players is

$$\binom{10}{1}(15)_9$$

Where $(15)_9$ is read as 15 ”permute” 9 and is equal to

$$(15)_9 = 15!/9! = 1,816,214,400$$

This gives a total of $N_1 = 18,162,144,000$ different lineups

**Problem 1.9.1**

(a) Since the probability of a zero is 0.8, we can express the probability of the code word 00111 as 2 occurrences of a 0 and three occurrences of a 1. Therefore

$$P[00111] = (0.8)^2(0.2)^3 = 0.00512$$

(b) The probability that a code word has exactly three 1’s is

$$P[\text{three 1's}] = \binom{5}{3}(0.8)^2(0.2)^3 = 0.0512$$