Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers Roy D. Yates and David J. Goodman

Problem Solutions : Yates and Goodman, 1.2.1 1.2.2 1.2.3 1.2.5 1.3.1 1.3.3 1.3.5 1.4.1 1.4.3 and 1.4.4

Problem 1.2.1

(a) An outcome specifies whether the fax is high (h), medium (m), or low (l) speed, and whether the fax has two (t) pages or four (f) pages. The sample space is

$$S = \{ht, hf, mt, mf, lt, lf\}$$

- (b) The event that the fax is medium speed is $A_1 = \{mt, mf\}$.
- (c) The event that a fax has two pages is $A_2 = \{ht, mt, lt\}$.
- (d) The event that a fax is either high speed or low speed is $A_3 = \{ht, hf, lt, lf\}$.
- (e) Since $A_1 \cap A_2 = \{mt\}$ and is not empty, A_1, A_2 , and A_3 are not mutually exclusive.
- (f) Since

 $A_1 \cup A_2 \cup A_3 = \{ht, hf, mt, mf, lt, lf\} = S,$

the collection A_1, A_2, A_3 is collectively exhaustive.

Problem 1.2.2

(a) The sample space of the experiment is

$$S = \{aaa, aaf, afa, faa, ffa, faf, aff, fff\}$$

(b) The event that the circuit from Z fails is

$$Z_F = \{aaf, aff, faf, fff\}$$

The event that the circuit from *X* is acceptable is

$$X_A = \{aaa, aaf, afa, aff\}$$

- (c) Since $Z_F \cap X_A = \{aaf, aff\} \neq \phi, Z_F \text{ and } X_A \text{ are not mutually exclusive.}$
- (d) Since $Z_F \cup X_A = \{aaa, aaf, afa, aff, faf, fff\} \neq S$, Z_F and X_A are not collectively exhaustive.

(e) The event that more than one circuit is acceptable is

$$C = \{aaa, aaf, afa, faa\}$$

The event that at least two circuits fail is

$$D = \{ffa, faf, aff, fff\}$$

- (f) Inspection shows that $C \cap D = \phi$ so *C* and *D* are mutually exclusive.
- (g) Since $C \cup D = S$, C and D are collectively exhaustive.

Problem 1.2.3

The sample space is

The event *H* is the set

$$H = \{A\heartsuit, \ldots, K\heartsuit\}$$

Problem 1.2.5

this problem. Here are four event spaces.

- 1. We can divide students into engineers or non-engineers. Let A_1 equal the set of engineering students and A_2 the non-engineers. The pair $\{A_1, A_2\}$ is an event space.
- 2. We can also separate students by GPA. Let B_i denote the subset of students with GPAs *G* satisfying $i-1 \le G < i$. At Rutgers, $\{B_1, B_2, \ldots, B_5\}$ is an event space. Note that B_5 is the set of all students with perfect 4.0 GPAs. Of course, other schools use different scales for GPA.
- 3. We can also divide the students by age. Let C_i denote the subset of students of age *i* in years. At most universities, $\{C_{10}, C_{11}, \dots, C_{100}\}$ would be an event space. Since a university may have prodigies either under 10 or over 100, we note that $\{C_0, C_1, \dots\}$ is always an event space
- 4. Lastly, we can categorize students by attendance. Let D_0 denote the number of students who have missed zero lectures and let D_1 denote all other students. Although it is likely that D_0 is an empty set, $\{D_0, D_1\}$ is a well defined event space.

Problem 1.3.1

The sample space of the experiment is

$$S = \{LF, BF, LW, BW\}$$

From the problem statement, we know that P[LF] = 0.5, P[BF] = 0.2 and P[BW] = 0.2. This implies P[LW] = 1 - 0.5 - 0.2 - 0.2 = 0.1. The questions can be answered using Theorem 1.5.

(a) The probability that a program is slow is

$$P[W] = P[LW] + P[BW] = 0.1 + 0.2 = 0.3.$$

(b) The probability that a program is big is

$$P[B] = P[BF] + P[BW] = 0.2 + 0.2 = 0.4$$

(c) The probability that a program is slow or big is

$$P[W \cup B] = P[W] + P[B] - P[BW] = 0.3 + 0.4 - 0.2 = 0.5$$

Problem 1.3.3

A reasonable probability model that is consistent with the notion of a shuffled deck is that each card in the deck is equally likely to be the first card. Let H_i denote the event that the first card drawn is the *i*th heart where the first heart is the ace, the second heart is the deuce and so on. In that case, $P[H_i] = 1/52$ for $1 \le i \le 13$. The event *H* that the first card is a heart can be written as the disjoint union

$$H = H_1 \cup H_2 \cup \cdots \cup H_{13}$$

Using Theorem 1.1, we have

$$P[H] = \sum_{i=1}^{13} P[H_i] = 13/52$$

This is the answer you would expect since 13 out of 52 cards are hearts. The point to keep in mind is that this is not just the common sense answer but is the result of a probability model for a shuffled deck and the axioms of probability.

Problem 1.3.5

Let s_i equal the outcome of the student's quiz. The sample space is then composed of all the possible grades that she can receive.

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Since each of the 11 possible outcomes is equally likely, the probability of receiving a grade of *i*, for each i = 0, 1, ..., 10 is $P[s_i] = 1/11$. The probability that the student gets an A is the probability that she gets a score of 9 or higher. That is

$$P[\text{Grade of A}] = P[9] + P[10] = 1/11 + 1/11 = 2/11$$

The probability of failing requires the student to get a grade less than 4.

$$P[\text{Failing}] = P[3] + P[2] + P[1] + P[0] = 1/11 + 1/11 + 1/11 + 1/11 = 4/11$$

Problem 1.4.1

From the table we look to add all the disjoint events that contain H_0 to express the probability that a caller makes no hand-offs as

$$P[H_0] = P[LH_0] + P[BH_0] = 0.1 + 0.4 = 0.5$$

In a similar fashion we can express the probability that a call is brief by

$$P[B] = P[BH_0] + P[BH_1] + P[BH_2] = 0.4 + 0.1 + 0.1 = 0.6$$

The probability that a call is long or makes at least two hand-offs is

$$P[L \cup H_2] = P[LH_0] + P[LH_1] + P[LH_2] + P[BH_2]$$

= 0.1 + 0.1 + 0.2 + 0.1 = 0.5

Problem 1.4.3

The first generation consists of two plants each with genotype yg or gy. They are crossed to produce the following second generation genotypes, $S = \{yy, yg, gy, gg\}$. Each genotype is just as likely as any other so the probability of each genotype is consequently 1/4. A pea plant has yellow seeds if it possesses at least one dominant y gene. The set of pea plants with yellow seeds is

$$Y = \{yy, yg, gy\}$$

So the probability of a pea plant with yellow seeds is

$$P[Y] = P[yy] + P[yg] + P[gy] = 3/4$$

Problem 1.4.4

consequence of part 4 of Theorem 1.4.

- (a) Since $A \subset A \cup B$, $P[A] \leq P[A \cup B]$.
- (b) Since $B \subset A \cup B$, $P[B] \leq P[A \cup B]$.
- (c) Since $A \cap B \subset A$, $P[A \cap B] \leq P[A]$.
- (d) Since $A \cap B \subset B$, $P[A \cap B] \leq P[B]$.