Communications Engineering

Course No: 16:332:421 - (Fall 2007)

Homework 1-a

1. X is a random variable with PDF given as

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty \le x \le \infty,\ \sigma > 0$$

   Using the definition of the mean of a random variable, find the mean of X

2. Consider a random vector

$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

   whose entries X_1, X_2, X_3 are Gaussian random variables. The covariance matrix C of $\underline{X}$ is given as

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

   and the correlation matrix R of $\underline{X}$ is given as

$$R = \begin{bmatrix} 4 & 2\sqrt{2} & \sqrt{6} \\ 2\sqrt{2} & 8 & 2\sqrt{3} \\ \sqrt{6} & 2\sqrt{3} & 6 \end{bmatrix}$$

   (a) Find the PDF of X_1

   (b) Find $P[0 < X_2 \le 2]$

   (c) Find the correlation coefficient ρ_{X_2,X_3}

   (d) Find the PDF of X_1 conditioned on X_3, i.e., $f_{X_1|X_3(x_1|x_3)}$

3. Consider the random process

$$X(t) = A\cos(\omega t + \theta)$$

   where A is an exponential random variable with parameter 1 and θ is a continuous uniform random variable over the interval $[0, 2\pi]$. Assume that A and θ are independent of each other and ω is a constant carrier frequency.

   (a) Find the mean of the stochastic process, E[X(t)]

   (b) Find the autocorrelation function $R_X(t, \tau)$.

   (c) Using answers in (a) and (b) above, determine if $X(t)$ wide-sense stationary ?