Communications Engineering

Course No: 16:332:421 - (Fall 2007)

Homework 1

1. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} Ce^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is C?

   (b) What is $E[X]$?

   (c) Let A be the event that $X > a$. What is $P\{A\}$? what is the conditional PDF $f_{X|A}(x|A)$? Does $f_{X|A}(x|A)$ depend on a?

2. Let X be a Gaussian random variable with mean 3 and variance 4. Let $Y = 2X - 6$.

   (a) What are the mean $E[Y]$ and the variance σ_Y^2 of Y?

   (b) What is the PDF $f_Y(y)$?

   (c) Are X and Y independent?

3. Let X be a random variable with PDF

$$f_X(x) = \begin{cases} 1/4 & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

   Let $Y = X^2$.

   (a) Find the cumulative distribution function (CDF) $F_Y(y) = P\{Y \leq y\}$. Note that you must find $F_Y(y)$ for every value of y.

   (b) What is the PDF $f_Y(y)$?

4. You are given a pile of m coins. The probability of "heads" of the i^{th} coin is p_i. A coin is randomly selected from this pile and it is tossed n times. It is observed that "heads" shows k times out of n tosses. Show that the probability, q_r, that the r^{th} coin was selected is

$$q_r = \frac{p_r^k(1-p_r)^{n-k}}{\sum_{j=1}^m p_j^k(1-p_j)^{n-k}}$$

   Hint : Use Bayes Rule

5. Let $\lambda > 0$, and for each integer $n \geq \lambda$, $\{X_{i,n}\}$ are independent and identically distributed random variables with $P[X_{i,n} = 1] = \lambda/n$, and $P[X_{i,n} = 0] = 1 - \lambda/n$. Let $Y_n = \sum_{i=1}^n X_{i,n}$.

   (a) Find the characteristic function of Y_n.

   (b) Show that $\lim_{n\to\infty} Y_n$ is a Poisson random variable. Find its mean and variance.

6. Show that the following are true for a wide-sense stationary process :

   (a) $R_X(\tau)$ is an even function

   (b) $\mid R_X(\tau) \mid \leq R_X(0)$

7. Given $\{X_t, t \geq 0\}$ is a random process with mean $\mu_X(t) = \mu t$ and correlation function $R_X(t,s) = \sigma^2 \min(t,s) + \mu^2 ts$. Such a process is called as a Wiener process. Let $\{Y_t, t \geq 0\}$ be such that $Y_t = X_{t+D} - X_t$, where D is a fixed positive number. Find the correlation function $R_Y(t,s)$ of Y_t. Is Y_t strictly stationary?