Forwarding Strategies for Gaussian Parallel-Relay Networks

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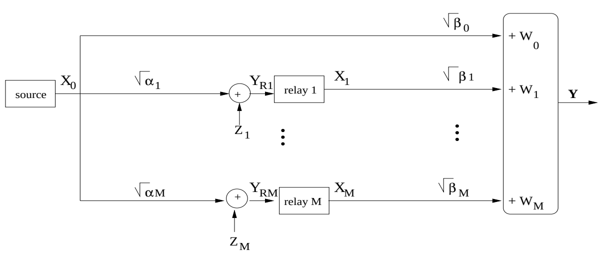
Abstract — **For reliable and unreliable forwarding in a parallel-relay network that allows orthogonal transmissions, we maximize the achievable rate under the total power constraint over all nodes. In such a network, the energy cost per information bit [1] during the reliable forwarding is minimized in the wideband regime. For the wideband decode-and-forward (DF) strategy, we show that the optimum parallel-relay solution is to send the data through one relay that is in the "best" position. On the other hand, as observed in [2], the benefit of unreliable amplify-and-forward (AF) strategy diminishes in the wideband regime. We characterize the optimum bandwidth for AF and show that transmitting in the optimum bandwidth allows the network to operate in the linear regime where the achieved rate increases linearly with transmit power. We identify the best subset of AF relay nodes and characterize the optimum power allocation per dimension among relays.**

We consider a Gaussian parallel-relay channel, that consists of a single source-destination pair and M relays, as shown:



Each transmission occurs in an orthogonal AWGN channel of bandwidth W. We consider two transmission strategies at the relays:

- **Decode-and-Forward:** A relay reliably decodes the source message, re-encodes it and transmits in an orthogonal channel.
- **Amplify-and-Forward:** The received signal at relay m is amplified and forwarded in an orthogonal channel. For amplification gain $b_m \geq 0$, relay m transmits $X_m = \sqrt{b_m} Y_{Rm} = \sqrt{b_m}\,(\sqrt{\alpha_m} X_0 + Z_m)$.

For a power allocation $\mathbf{p} = [p_0, p_1 \ldots p_M]^T$, the upper bounds on the capacity follow from the cut-set bound [3]. The parallel-Gaussian network is a Gaussian vector channel and thus [4] determines the achievable rate for the DF strategy, I_{DF}, for a given set of reliable nodes as well as the achievable rate for the AF strategy. The following observation [2, 5] motivates the problem formulation.

Lemma 1 *For given powers* $\mathbf{p}$*,* $W I_{DF}(\mathbf{p}/W)$ *is increasing in* W*. For AF, there exists finite* W^* *that maximizes* $W I_{AF}(\mathbf{p}/W)$*.*

With this observation in mind, we maximize the achievable rate under the constraint on the total power in the network $\mathbf{1}^T\mathbf{p} \leq p$. This problem has its dual with the objective to communicate to the destination at rate r bits/s using minimum power p^*. We present the solution to the latter problem.

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Following Lemma 1, we consider the wideband DF relay problem by allowing W to be large. The solution will specify the best choice of relays and the optimum power allocation among them.

Theorem 1 *The wideband DF relay problem admits an optimal solution in which no more than one relay node transmits.*

By Theorem 1, it is sufficient to consider only policies that employ one relay k. It is easy to show that the relay k is used only if it belongs to the set of useful relays $U = \{i|\alpha_i > \beta_0, \beta_i > \beta_0\}$ and is chosen so as to minimize the total power:

Theorem 2 *If the set* U *of useful relays is non-empty, the optimal solution to the wideband DF relay problem is to employ relay*

$$k^* = \arg\min_{k\in U}\left[\frac{1}{\alpha_k} + \frac{1}{\beta_k} - \frac{\beta_0}{\alpha_k\beta_k}\right] \qquad (1)$$

otherwise, if U *is empty, then direct transmission is optimal.*

The solution to the AF relay problem will, in addition to the best choice of K relays, $K \leq M$ and the transmit powers, specify the optimum bandwidth W^*.

Theorem 3 *The AF relay problem has an optimum solution in which the bandwidth* W^**, rate* r *and the total transmit power* p^* *have a linear relationship. The optimum power allocation per dimension* $(P_0^*, \ldots, P_K^*)$ *is independent of* r *and* W^**.*

For a given source power and bandwidth, the relay powers are

$$P_m^* = \frac{\alpha_m}{\gamma_m}\left[\frac{1}{\sqrt{\eta}} - \frac{1}{\gamma_m}\right]^+ \quad m = 1, \ldots K \qquad (2)$$

where η is found from the boundary constraint and $\gamma_m = \sqrt{\alpha_m\beta_m/(\alpha_m P_0 + 1)}$. It can be observed from (2) that the best choice of relays depends on the source transmit power and that the AF strategy in general employs a set of relays, unlike DF. In fact, in a Gaussian parallel-relay network that allows transmissions to share the bandwidth, the achievable rate behaves as $\Theta(\log(M))$ achieving the capacity in the limit of large M [6]. The asymptotic behavior of AF in our model is different: for large M, the AF achievable rate is of the order of $\Theta(1)$, since the orthogonal transmissions preclude from the gain due to the coherent combining of the relay signals at the destination.

References

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