The Strong Interference Channel with Unidirectional Cooperation

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***Abstract*— We consider the interference channel in which messages sent at one encoder are known to the other encoder, but not vice versa. Such channel model allows for unidirectional node cooperation: the encoder that knows both messages can exploit that information to improve the achievable rates. We derive conditions under which the capacity region of this channel coincides with the capacity region of the channel in which both messages are decoded at both receivers. We compare the obtained conditions with the strong interference conditions in the interference channel with independent messages, as well as in the interference channel with both private and common messages.**

I. INTRODUCTION

A discrete memoryless interference channel [1] consists of two input alphabets $\mathcal{X}_1, \mathcal{X}_2$, two output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$ and the conditional probability distribution $p(y_1, y_2|x_1, x_2)$. This channel model assumes that the two channel inputs are independent, precluding any form of cooperation between encoders. However, cooperation among encoders can improve the achievable rates. For a Gaussian network with two transmitters and two receivers, improvements in the achievable rates due to node cooperation were shown in [2]–[6].

In a discrete memoryless channel, a problem in which encoders partially cooperate was proposed by Willems for a multiple access channel (MAC) [7]. To model the transmitter cooperation, two communication links with finite capacities are introduced between the two encoders. When cooperating over links with finite capacities, encoders obtain partial information about each other's messages. This information is referred to as a *common* message as it is known to both encoders after cooperation. In addition, each encoder will still have independent information referred to as a *private* message, unknown to the other encoder.

In this paper, we consider the interference channel in which full information about messages sent at one encoder is available to the other encoder, but not vice versa. Such channel model allows the encoder that knows both messages to exploit that information to improve the achievable rates. The achievable rates for this channel model have been determined in [8]. The communication system is shown in Figure 1. Without cooperation, this channel reduces to the interference channel [1], [9] for which the capacity region is known in the case of *strong interference* [10] satisfying

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2) \tag{1}$$

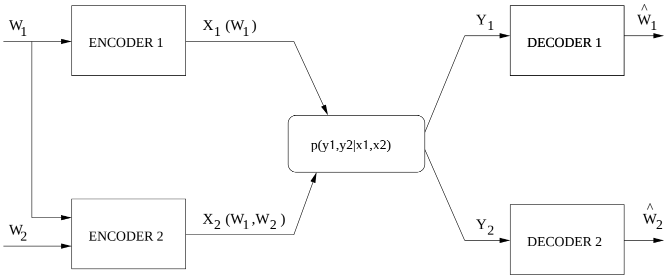

Fig. 1. Interference channel with unidirectional cooperation.

$$I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1) \tag{2}$$

for all product distributions on the inputs X_1 and X_2. The capacity region in this case coincides with the capacity region of the two-sender, two-receiver channel in which both messages are decoded at both receivers, as determined by Ahlswede [11].

In this paper, we derive conditions equivalent to (1)-(2) under which there is no penalty in decoding both messages at both decoders in the interference channel with unidirectional cooperation. We compare the obtained conditions to the strong interference conditions determined in [10] and [12].

II. CHANNEL MODEL

We consider a memoryless interference channel [1] that consists of finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$ and a conditional probability distribution $p(y_1, y_2|x_1, x_2)$. Symbols $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ are the corresponding channel outputs. Each encoder t, $t = 1, 2$, wishes to send a message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. Indexes W_1 and W_2 are independently generated. It is assumed that message W_1 is also known at encoder 2, thus allowing for unidirectional cooperation. The channel is memoryless and time-invariant in the sense that

$$p(y_{1,n}, y_{2,n}|\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{y}_1^{n-1}, \mathbf{y}_2^{n-1}) = p(y_{1,n}, y_{2,n}|x_{1,n}, x_{2,n}) \tag{3}$$

where $\mathbf{x}_t^n = \begin{bmatrix} x_{t,1}, & \ldots, & x_{t,n} \end{bmatrix}$. Here we follow the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables. To simplify notation, we drop the superscript when $n = N$.

An (M_1, M_2, N, P_e) code for the channel consists of two encoding functions generating codewords

$$\mathbf{x}_1 = f_1(W_1) \tag{4}$$

$$\mathbf{x}_2 = f_2(W_1, W_2) \tag{5}$$

two decoding functions

$$\hat{W}_t = g_t(\mathbf{Y}_t) \quad t = 1, 2 \tag{6}$$

and a maximum error probability

$$P_e = \max\{P_{e,1}, P_{e,2}\} \tag{7}$$

where, for $t = 1, 2$

$$P_{e,t} = \sum_{(w_1,w_2)} \frac{1}{M_1 M_2} P\left[g_t(\mathbf{Y}_t) \neq (w_t) | (w_1, w_2) \text{ sent}\right]. \tag{8}$$

A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_1, M_2, N, P_e) code such that

$$P_e \leq \epsilon \text{ and } M_i \geq 2^{NR_i} \quad i = 1, 2.$$

The capacity region of the *interference channel with unidirectional cooperation* is the closure of the set of all achievable rate pairs (R_1, R_2).

The next theorem is the main result of this paper. It gives conditions under which the capacity region coincides with the capacity region of the channel in which both messages are required at both receivers.

Theorem 1: For an interference channel $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ with unidirectional cooperation satisfying

$$I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1) \tag{9}$$

$$I(X_1, X_2; Y_1) \leq I(X_1, X_2; Y_2) \tag{10}$$

for all joint input distributions on X_1 and X_2, the capacity region $\mathcal{C}$ is given by

$$\begin{aligned} \mathcal{C} = \bigcup \Big\{ (R_1, R_2) : & \\ R_2 &\leq I(X_2; Y_2|X_1) \qquad (11) \\ R_1 + R_2 &\leq I(X_1, X_2; Y_1) \Big\} \qquad (12) \end{aligned}$$

where the union is over joint distributions $p(x_1, x_2, y_1, y_2)$. In the following section, we discuss how the achievability of the $\mathcal{C}$ rate region follows from the compound MAC with common information. In Section IV, we show the converse to complete the proof of Theorem 1.

III. ACHIEVABILITY: THE COMPOUND MAC WITH COMMON INFORMATION

The encoder cooperation results in a common message known to both encoders, in addition to the private messages. The capacity region of the interference channel with encoder cooperation is then closely related to the capacity region of the multi-access channel in which private and common messages are transmitted, refered to as the MAC with common information [13]. Unlike the MAC, the interference channel model assumes two receivers. When each receiver wishes to decode both private and common messages, the resulting channel becomes a compound MAC with common information. We wish to determine the conditions under which the capacity region of the interference channel with unidirectional cooperation coincides with the capacity region of the compound MAC with common information, denoted $\mathcal{C}_{\text{CMAC}}$, and given by [14]

$$\begin{aligned} \mathcal{C}_{\text{CMAC}} = \bigcup \Big\{ (R_0', R_1', R_2') : & \\ R_1' &\leq \min_t I(X_1; Y_t|X_2, U) \\ R_2' &\leq \min_t I(X_2; Y_t|X_1, U) \\ R_1' + R_2' &\leq \min_t I(X_1, X_2; Y_t|U) \\ R_0' + R_1' + R_2' &\leq \min_t I(X_1, X_2; Y_t) \Big\} \qquad (13) \end{aligned}$$

where the union is over all $p(u, x_1, x_2, y_1, y_2)$ that factor as $p(u)p(x_1|u)p(x_2|u)p(y_1, y_2|x_1, x_2)$.

For the interference channel with unidirectional cooperation, encoder 2 knows the entire message W_1, and thus we view R_1 as the common rate, $R_0' = R_1$. For the same reason, user 1 in the compound MAC has private message rate $R_1' = 0$. We can choose $U = X_1$ and the region (13) becomes

$$\begin{aligned} \mathcal{C}_{\text{MAC}} = \bigcup \Big\{ (R_1, R_2) : & \\ R_2 &\leq \min_t I(X_2; Y_t|X_1) \\ R_1 + R_2 &\leq \min_t I(X_1, X_2; Y_t) \Big\} \qquad (14) \end{aligned}$$

where the union is over all $p(x_1, x_2, y_1, y_2)$. When conditions (9)-(10) are satisfied, region (14) reduces to region (11)-(12) in Theorem 1. Consider next the strong interference channel with unidirectional cooperation. The achievability of the rates of Theorem 1 in the case in which both messages are required at the receivers guarantees that these rates are also achieved when a weaker constraint of decoding of a single message is imposed at the receivers. Hence the proof of achievability in Theorem 1 is immediate. We next prove the converse and determine the strong interference conditions.

IV. CONVERSE: STRONG INTEFERENCE CONDITIONS

Consider a code (M_1, M_2, N, P_e) for the interference channel with unidirectional cooperation. Applying Fano's inequality results in

$$H(W_1|\mathbf{Y}_1) \leq P_{e1} \log(M_1 - 1) + h(P_{e1}) \triangleq N\delta_{1,N}, \tag{15}$$

$$H(W_2|\mathbf{Y}_2) \leq P_{e2} \log(M_2 - 1) + h(P_{e2}) \triangleq N\delta_{2,N}. \tag{16}$$

where $\delta_{t,N} \to 0$ as $P_{et} \to 0$ (or as $P_e \to 0$). For notational convenience, we define $\delta_N \triangleq \delta_{1,N} + \delta_{2,N}$ and $R_s = R_1 + R_2$. We now derive the R_s bound (12) for receiver $t = 1$.

From independence of W_1 and W_2 and the Fano's inequalities (15) and (16), we have

$$\begin{aligned} NR_s &= H(W_1) + H(W_2|W_1) && (17) \\ &= I(W_1; \mathbf{Y}_1) + I(W_2; \mathbf{Y}_2|W_1) \\ &\quad + H(W_1|\mathbf{Y}_1) + H(W_2|\mathbf{Y}_2, W_1) && (18) \\ &\leq I(W_1; \mathbf{Y}_1) + I(W_2; \mathbf{Y}_2|W_1) + N\delta_N. && (19) \end{aligned}$$

With the assumption that (4) defines $\mathbf{X}_1$ as a deterministic one-to-one function of W_1, it follows that

$$\begin{aligned} NR_s &\le I(W_1, \mathbf{X}_1; \mathbf{Y}_1) + I(\mathbf{X}_2; \mathbf{Y}_2 | W_1) + N\delta_N && (20)\\ &\le I(W_1, \mathbf{X}_1; \mathbf{Y}_1) + I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1, W_1) + N\delta_N. && (21)\end{aligned}$$

Therefore, if the condition

$$I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1, W_1) \le I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, W_1) \qquad (22)$$

holds, then it follows from (21) that

$$\begin{aligned} NR_s &\le I(W_1, \mathbf{X}_1; \mathbf{Y}_1) + I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, W_1) + N\delta_N && (23)\\ &= I(\mathbf{X}_1, \mathbf{X}_2; \mathbf{Y}_1) + N\delta_N && (24)\\ &\le \sum_{n=1}^{N} I(X_{1n}, X_{2n}; Y_{1n}) + N\delta_N. && (25)\end{aligned}$$

And therefore, we obtain the sum-rate outer bound (12). Per-letter conditions follow from next Lemma.

Lemma 1: If per-letter conditions

$$I(X_2; Y_2 | X_1) \le I(X_2; Y_1 | X_1) \qquad (26)$$

are satisfied for joint distributions $p(x_1, x_2)$, then

$$I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1, W_1) \le I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, W_1). \qquad (27)$$

Lemma 1 is similar to the Lemma by Costa and El Gamal [10]. One difference is that the condition (26) in our case has to be satisfied for all the input distributions not only for product ones, as cooperation introduces dependence between inputs.

The bound (11) is an immediate single-link outer bound and hence the Theorem follows. □

In the case when (10) does not hold, showing that (14) is indeed the capacity region would require proving an outer bound of the form $R_S \le I(X_1, X_2; Y_2)$. Due to the asymmetry of the problem, the approach (17)-(25) does not apply.

V. GAUSSIAN CHANNEL

We next consider the Gaussian interference channel in the standard form [1], [15]

$$\begin{aligned} y_{1i} &= x_{1i} + h_{21}x_{2i} + z_{1i} && (28)\\ y_{2i} &= h_{12}x_{1i} + x_{2i} + z_{2i} && (29)\end{aligned}$$

where the Z_t are independent, zero-mean, unit-variance Gaussian random variables. The code definition is the same as given in Section II with the addition of the power constraints

$$\frac{1}{N}\sum_{i=1}^{N} E[X_{ti}^2] \le P_t, \qquad t = 1, 2. \qquad (30)$$

From the maximum-entropy theorem [16, Thm. 9.6.5] it follows that Gaussian inputs are optimal.

Corollary 1: When the strong interference conditions

$$\begin{aligned} h_{21} &\ge 1 && (31)\\ h_{12} &\ge \frac{1}{\alpha}\left(\sqrt{\alpha^2 + h_{21}^2 - 1 + 2\rho\alpha h_{21} + \rho^2} - \rho\right) && (32)\end{aligned}$$

hold, where ρ is a correlation coefficient for X_1, X_2 and $\alpha = \sqrt{P_1/P_2}$, the capacity region of the Gaussian interference channel with unidirectional cooperation is given by

$$\begin{aligned} \mathcal{C} = \bigcup_{\rho} \{ &(R_1, R_2) : \\ &R_2 \le C\left((1-\rho^2)P_2\right) && (33)\\ &R_1 + R_2 \le C\left(P_1 + h_{21}^2 P_2 + 2\rho\sqrt{h_{21}^2 P_1 P_2}\right)\}. && (34)\end{aligned}$$

One can show that satisfying conditions (31)-(32) for all possible values of ρ is more demanding than the conditions (1)-(2) which in Gaussian case reduce to $h_{21} \ge 1, h_{12} \ge 1$. Still, we observe there will always be some set of values h_{21}, h_{12} that satisfy conditions (31)-(32), except when $P_1 = 0$. For $P_1 = 0$, the channel reduces to the broadcast channel from encoder 2. As the channel is degraded, there can be no strong interference conditions.

VI. FUTURE WORK

It would be interesting to investigate if there are weaker strong interference conditions for the considered channel. The strong interference conditions for more general channel models with cooperation are still an open problem.

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