Two-Dimensional Orthogonal Variable-Spreading-Factor Codes for Multichannel DS-UWB

Di Wu, Predrag Spasojević, and Ivan Seskar
WINLAB, Rutgers University
73 Brett Road, Piscataway, NJ 08854
{diwu,spasojev,seskar}@winlab.rutgers.edu

***Abstract*— We propose a recursive algorithm to construct ternary two-dimensional orthogonal variable-spreading-factor (2D OVSF) codes with a tree-structure for multirate multichannel DS-UWB system. The proposed ternary 2D OVSF codes, including binary codes as a special case, have a significantly reduced restriction on code lengths and ensure orthogonality between different code layers. 2D OVSF codes modulated by the same information bit are transmitted over orthogonal channels. We study the BER performance of 2D OVSF code based UWB system in Nakagami fading multipath channels. The results show that, if the orthogonal channels have identical channel coefficients, the system is with zero multiple access interference (MAI). Furthermore, if the sequence length is larger than the number of multipath components, there is no multipath interference.**

***Index Terms*— ternary sequence, ultra-wide band (UWB), orthogonal variable-spreading-factor (OVSF), complementary sets**

I. Introduction

In an OVSF-CDMA system, each user is assigned a single orthogonal variable-spreading-factor code and a higher date rate access is achieved by using a lower spreading factor code [1]. These OVSF codes are designed for single carrier or single channel systems and here classified as one-dimensional (1D) codes. It is impossible to construct a set of 1D spreading codes with ideal correlation properties (i.e. zero autocorrelation sidelobes and zero cross-correlation functions). Thus, in an asynchronous system or a multipath scenario, the orthogonality of 1D OVSF codes is lost. However, as shown in [2]–[4], it is possible to construct a class of two-dimensional (2D) spreading codes with such correlation properties, resulting in multipath and multiuser interference suppression.

In order to meet the natural tree-structure of OVSF codes [1], we need a recursive algorithm to generate 2D OVSF codes from smaller size to a larger one. Each 2D OVSF code is an either binary $\{+1, -1\}$ or ternary $\{+1, 0, -1\}$ entry M by N matrix, where the number of rows M and the number of columns N correspond to the number of multichannel and the spreading sequence length, respectively.

Former methods which focus on binary OVSF codes design can be found in [4]–[6]. In [4], the authors generate the 2D OVSF codes with $M = 2^i$ and $N = 2^l$, where both i and l are positive integers with $i \leq l$, and show that these codes have less restrictions on M and N than codes in [5], [6]. However, approaches in [4]–[6] have a significant number of spreading sequence length constraints. In addition, because of a lack of tree-structure in [5], [6], the orthogonality between the smaller size codes and larger size codes remains fuzzy and, hence, additional implementation complexity might be incurred. Here, we propose a general recursive algorithm to construct ternary 2D OVSF codes, including binary codes as a special case, which has a significantly reduced restriction on code lengths and ensures orthogonality between different code layers.

This work was supported by the NSF Grant 0434854.

The recursive approach can be initialized using any ternary complementary pair (TCP) with an arbitrary sequence length $N^{(1)}$ [7]. At step i, we obtain 2D OVSF codes with $M = 2^i$ and $N = 2^{i-1}N^{(1)}$. In particular, if we start from binary complementary pair (Golay pair) [8], binary 2D OVSF codes as the subset of ternary codes are obtained. The sequence length in binary 2D OVSF codes is limited by $N^{(1)} = 2^\alpha 10^\beta 26^\gamma$, where α,β and γ are non-negative integers. Note that, the proposed construction includes [4] as a special case when Golay length $N^{(1)}$ with parameters $\beta = 0$ and $\gamma = 0$. Thus, the construction in this paper improves the flexibility in choosing the number of multichannel and the sequence lengths.

Akin to the 1D OVSF code assignment, the 2D OVSF code (or matrix) is properly selected based on the tree-structure and assigned to each user in the multichannel DS-CDMA system. The 2D codes may have different spreading factors (sequence length) to achieve different data rate, but the orthogonality is always preserved between any two codes. Different rows of the matrix carrying the same information bit are transmitted over orthogonal channels. We consider a UWB system, where the multiple channels are enabled by orthogonal chip pulses [3]. We analyze the BER performance of multirate multichannel UWB system employing ternary 2D OVSF codes in a multipath environment and get the numerical results in Nakagami fading channels. The results show that, if the orthogonal channels are with identical channel coefficients, the system is with zero multiple access interference (MAI). Furthermore, if the sequence length N is larger than the number of multipath components, the multipath interference is zero.

II. Design of 2D OVSF codes

In this section, the correlation properties of 2D OVSF code sets are given, followed by the comparison with that of mutually orthogonal (MO) complementary sets. We prove that MO complementary sets satisfy all the correlation requirements of 2D OVSF code sets. Thus, the construction of 2D OVSF codes converts to the design of MO complementary sets.

A. Definition of 2D OVSF codes

Spreading sequence set $\boldsymbol{T}$, a collection of M by N matrices, is said to be a 2D OVSF code set, if

a) For any matrix $\mathbf{C}_{M,N}^{(i)} \in \boldsymbol{T}$ and an integer $l \in (0, N)$, the out-off phase values of the 2D even and odd autocorrelation of $\mathbf{C}_{M,N}^{(i)}$ are zeros,

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} c_{m,n}^{(i)} c_{m,n\oplus l}^{(i)} = 0$$

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1-l} c_{m,n}^{(i)} c_{m,n\oplus l}^{(i)} - \sum_{m=0}^{M-1}\sum_{n=N-l}^{N-1} c_{m,n}^{(i)} c_{m,n\oplus l}^{(i)} = 0$$

respectively, where"$\oplus$" denotes a modulo-N addition and $c_{m,n}^{(i)}$ is the entry in mth row and nth column of matrix $\mathbf{C}_{M,N}^{(i)}$.

b) For any two distinct matrices $\mathbf{C}_{M,N}^{(i)}$ and $\mathbf{C}_{M,N}^{(j)}$ within set $\boldsymbol{T}$ and an integer $l \in [0, N)$, the values of the 2D even and odd cross-correlation functions are zeros,

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} c_{m,n}^{(i)} c_{m,n\oplus l}^{(j)} = 0$$

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1-l} c_{m,n}^{(i)} c_{m,n\oplus l}^{(j)} - \sum_{m=0}^{M-1}\sum_{n=N-l}^{N-1} c_{m,n}^{(i)} c_{m,n\oplus l}^{(j)} = 0$$

respectively, where again $c_{m,n}^{(i)}$ is the entry of matrix $\mathbf{C}_{M,N}^{(i)}$ in mth row and nth column.

B. Complementary Sets

A set of M sequences $\{\mathbf{a}_0, \mathbf{a}_1, \cdots, \mathbf{a}_{M-1}\}$ is said to be a set of complementary sequences, if the sum of the aperiodic autocorrelation functions (ACF) of the M sequences vanishes for any integer shift $l \in (0, N)$,

$$\sum_{i=0}^{M-1} \theta_{\mathbf{a}_i,\mathbf{a}_i}(l) = \sum_{i=0}^{M-1}\sum_{n=0}^{N-1-l} a_{i,n} a_{i,n+l} = 0$$

where $\theta_{\mathbf{a}_i,\mathbf{a}_i}$ denotes the aperiodic ACF of sequences $\mathbf{a}_i$ with length N and $a_{i,n}$ denotes the nth element in the sequence $\mathbf{a}_i$. When $M = 2$, $\{\mathbf{a}_0, \mathbf{a}_1\}$ are called complementary pair. Particularly, we call $\{\mathbf{a}_0, \mathbf{a}_1\}$ binary complementary pair (a.k.a Golay pair) if $a_{i,n} \in \{+1, -1\}$ and ternary complementary pair (TCP) if $a_{i,n} \in \{+1, 0, -1\}$.

The complementary sequence set $\{\mathbf{b}_0, \mathbf{b}_1, \cdots, \mathbf{b}_{M-1}\}$ is a mate of complementary set $\{\mathbf{a}_0, \mathbf{a}_1, \cdots, \mathbf{a}_{M-1}\}$ if the length of $\mathbf{b}_i$ is equal to the length of $\mathbf{a}_i$, $0 \le i \le M-1$, and for integer shift $l \in [0, N)$,

$$\sum_{i=0}^{M-1} \theta_{\mathbf{a}_i,\mathbf{b}_i}(l) = \sum_{i=0}^{M-1}\sum_{n=0}^{N-1-l} a_{i,n} b_{i,n+l} = 0$$

Mutually orthogonal (MO) complementary set is a collection of complementary sets in which any two of them are mates to each other.

Theorem 1: MO complementary sets satisfy the definition of 2D OVSF codes.

MO complementary sets are mutually orthogonal in both periodic and aperiodic sense. Based on the relationship between periodic/aperiodic correlations and odd/even correlations [9], the MO complementary sets satisfy the odd and even correlation properties of 2D OVSF codes.

The following lemmas are used in the construction of MO complementary sets in next section. We list them here,

Lemma 1: Let $\{\mathbf{a}_1, \mathbf{b}_1\}$ be a complementary pair, then $\{\overleftarrow{\mathbf{b}_1}, -\overleftarrow{\mathbf{a}_1}\}$ is its mate, where $\overleftarrow{\mathbf{b}_1}$ denotes the reverse of the sequence $\mathbf{b}_1$ and $-\overleftarrow{\mathbf{a}_1}$ denotes the sequence whose ith element is the negation of the ith element in sequence $\overleftarrow{\mathbf{a}_1}$.

Lemma 2: Let $\{\mathbf{a}_1, \mathbf{a}_2\}$ be a complementary set, then $\{\mathbf{a}_1\mathbf{a}_1, \mathbf{a}_2\mathbf{a}_2,\ \mathbf{a}_1(-\mathbf{a}_1), \mathbf{a}_2(-\mathbf{a}_2)\}$ and $\{\mathbf{a}_1(-\mathbf{a}_1), \mathbf{a}_2(-\mathbf{a}_2), \mathbf{a}_1\mathbf{a}_1, \mathbf{a}_2\mathbf{a}_2\}$ are mates of each other, where $-\mathbf{a}_1$ denotes the sequence whose ith element is the negation of the ith element of $\mathbf{a}_1$ and $\mathbf{a}_1(-\mathbf{a}_1)$ denotes the concatenation of two sequences $\mathbf{a}_1$ and $-\mathbf{a}_1$.

Lemma 3: Let $\{\mathbf{a}_1, \mathbf{a}_2\}$ be a complementary set and $\{\mathbf{b}_1, \mathbf{b}_2\}$ be its mate, then $\{\mathbf{a}_1\mathbf{a}_1, \mathbf{a}_2\mathbf{a}_2, \mathbf{a}_1(-\mathbf{a}_1), \mathbf{a}_2(-\mathbf{a}_2)\}$ and $\{\mathbf{b}_1\mathbf{b}_1, \mathbf{b}_2\mathbf{b}_2,\ \mathbf{b}_1(-\mathbf{b}_1), \mathbf{b}_2(-\mathbf{b}_2)\}$ are also mates.

C. Design Algorithm

We illustrate the algorithm with the design of ternary 2D OVSF codes. The construction of binary 2D OVSF codes is straightforward by using binary seed in stead of ternary one.

Step 1: Start from any seed TCP $\{\mathbf{c}_1, \mathbf{c}_2\}$ with sequence length $N^{(1)}$ and present them in matrix form as $\mathbf{C}_{2,N^{(1)}}^{(1)}$,

$$\begin{aligned} \mathbf{C}_{2,N^{(1)}}^{(1)} &= \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix}_{2\times N^{(1)}} \\ &= \begin{bmatrix} c_{0,0} & c_{0,1} & \cdots & c_{0,N^{(1)}-1} \\ c_{1,0} & c_{1,1} & \cdots & c_{1,N^{(1)}-1} \end{bmatrix} \end{aligned} \quad (1)$$

From *Lemma* 1, we obtain its mate $\mathbf{C}_{2,N^{(1)}}^{(2)}$,

$$\begin{aligned} \mathbf{C}_{2,N^{(1)}}^{(2)} &= \begin{bmatrix} \overleftarrow{\mathbf{c}_2} \\ -\overleftarrow{\mathbf{c}_1} \end{bmatrix}_{2\times N^{(1)}} \\ &= \begin{bmatrix} c_{1,N^{(1)}-1} & \cdots & c_{1,1} & c_{1,0} \\ -c_{0,N^{(1)}-1} & \cdots & -c_{0,1} & -c_{0,0} \end{bmatrix} \end{aligned} \quad (2)$$

Then, $\mathbf{C}_{2,N^{(1)}}^{(1)}$ and $\mathbf{C}_{2,N^{(1)}}^{(2)}$ are the first layer 2D OVSF codes.

Step 2: From $\mathbf{C}_{2,N^{(1)}}^{(1)}$ and $\mathbf{C}_{2,N^{(1)}}^{(2)}$, the recursive procedure generates second layer 2D OVSF codes $\mathbf{C}_{2^2,N^{(2)}}^{(1)}$, $\mathbf{C}_{2^2,N^{(2)}}^{(2)}$, $\mathbf{C}_{2^2,N^{(2)}}^{(3)}$ and $\mathbf{C}_{2^2,N^{(2)}}^{(4)}$ with aid of two 2 by 2 orthogonal matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ below,

$$\mathbf{H}_1 = \begin{bmatrix} + & + \\ + & - \end{bmatrix}$$

and

$$\mathbf{H}_2 = \begin{bmatrix} + & - \\ + & + \end{bmatrix}$$

where "$+$" denotes $+1$ and "$-$" denotes -1. Then,

$$\mathbf{C}_{2^2,N^{(2)}}^{(1)} = \mathbf{H}_1 \otimes \mathbf{C}_{2,N^{(1)}}^{(1)} = \begin{bmatrix} \mathbf{C}_{2,N^{(1)}}^{(1)} & \mathbf{C}_{2,N^{(1)}}^{(1)} \\ \mathbf{C}_{2,N^{(1)}}^{(1)} & -\mathbf{C}_{2,N^{(1)}}^{(1)} \end{bmatrix} \quad (3)$$

$$\mathbf{C}^{(2)}_{2^2,N^{(2)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(1)}_{2,N^{(1)}} & -\mathbf{C}^{(1)}_{2,N^{(1)}} \\ \mathbf{C}^{(1)}_{2,N^{(1)}} & \mathbf{C}^{(1)}_{2,N^{(1)}} \end{bmatrix} \quad (4)$$

$$\mathbf{C}^{(3)}_{2^2,N^{(2)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(2)}_{2,N^{(1)}} & \mathbf{C}^{(2)}_{2,N^{(1)}} \\ \mathbf{C}^{(2)}_{2,N^{(1)}} & -\mathbf{C}^{(2)}_{2,N^{(1)}} \end{bmatrix} \quad (5)$$

$$\mathbf{C}^{(4)}_{2^2,N^{(2)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(2)}_{2,N^{(1)}} & -\mathbf{C}^{(2)}_{2,N^{(1)}} \\ \mathbf{C}^{(2)}_{2,N^{(1)}} & \mathbf{C}^{(2)}_{2,N^{(1)}} \end{bmatrix} \quad (6)$$

where $\otimes$ denotes Kronecker product of two matrices and the second layer 2D OVSF codes with dimension 2^2 by $N^{(2)}$ and $N^{(2)} = 2N^{(1)}$.

Step 3: We numerate 2^i 2D OVSF codes at ith layer by index k, thus $k \in [1, 2^i]$. Then, $(i+1)$th layer 2D OVSF codes can be constructed from ith layer 2D OVSF codes using the following general formula,

$$\begin{aligned} \mathbf{C}^{(2k-1)}_{2^{i+1},N^{(i+1)}} &= \mathbf{H}_1 \otimes \mathbf{C}^{(k)}_{2^i,N^{(i)}} \\ &= \begin{bmatrix} \mathbf{C}^{(k)}_{2^i,N^{(i)}} & \mathbf{C}^{(k)}_{2^i,N^{(i)}} \\ \mathbf{C}^{(k)}_{2^i,N^{(i)}} & -\mathbf{C}^{(k)}_{2^i,N^{(i)}} \end{bmatrix} \end{aligned} \quad (7)$$

and

$$\begin{aligned} \mathbf{C}^{(2k)}_{2^{i+1},N^{(i+1)}} &= \mathbf{H}_2 \otimes \mathbf{C}^{(k)}_{2^{i-1},N^{(i-1)}} \\ &= \begin{bmatrix} \mathbf{C}^{(k)}_{2^i,N^{(i)}} & -\mathbf{C}^{(k)}_{2^i,N^{(i)}} \\ \mathbf{C}^{(k)}_{2^i,N^{(i)}} & \mathbf{C}^{(k)}_{2^i,N^{(i)}} \end{bmatrix} \end{aligned} \quad (8)$$

where $N^{(i+1)} = 2N^{(i)} = 2^i N^{(1)}$.

Theorem 2: At any step i, the constructed matrices $\mathbf{C}^{(k)}_{2^i,N^{(i)}}$, $k \in [1, 2^i]$, are MO complementary set. Thus, they are 2D OVSF codes at ith layer of code tree.

Proof: $Lemma$ 2 and $Lemma$ 3 guarantee that constructed matrices $\mathbf{C}^{(k)}_{2^i,N^{(i)}}$, $k \in [1, 2^i]$ are MO complementary sets. Thus, they meet the autocorrelation and cross-correlation requirements of 2D OVSF codes.

Example 1: Based on *Lemma* 1, from seed TCP

$$\mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix} = \begin{bmatrix} + & + & - \\ + & 0 & + \end{bmatrix}_{2\times 3} \quad (9)$$

we obtain its mate $\mathbf{C}^{(2)}_{2,N^{(1)}}$,

$$\mathbf{C}^{(2)}_{2,N^{(1)}} = \begin{bmatrix} \overleftarrow{\mathbf{c}}_2 \\ -\overleftarrow{\mathbf{c}}_1 \end{bmatrix} = \begin{bmatrix} + & 0 & + \\ + & - & - \end{bmatrix}_{2\times 3} \quad (10)$$

Then, we use recursive procedure to generate second layer 2D OVSF codes $\mathbf{C}^{(1)}_{2^2,N^{(2)}}$, $\mathbf{C}^{(2)}_{2^2,N^{(2)}}$, $\mathbf{C}^{(3)}_{2^2,N^{(2)}}$ and $\mathbf{C}^{(4)}_{2^2,N^{(2)}}$,

$$\mathbf{C}^{(1)}_{2^2,2N^{(1)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} ++- & ++- \\ +0+ & +0+ \\ ++- & --+ \\ +0+ & -0- \end{bmatrix}_{4\times 6} \quad (11)$$

$$\mathbf{C}^{(2)}_{2^2,2N^{(1)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} ++- & --+ \\ +0+ & -0- \\ ++- & ++- \\ +0+ & +0+ \end{bmatrix}_{4\times 6} \quad (12)$$

$$\mathbf{C}^{(3)}_{2^2,2N^{(1)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} +0+ & +0+ \\ +-- & +-- \\ +0+ & -0- \\ +-- & -++ \end{bmatrix}_{4\times 6} \quad (13)$$

$$\mathbf{C}^{(4)}_{2^2,2N^{(1)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} +0+ & -0- \\ +-- & -++ \\ +0+ & +0+ \\ +-- & +-- \end{bmatrix}_{4\times 6} \quad (14)$$

We can go further to generate the 3rd layer 2D OVSF codes using general formula (7) and (8).

The tree structure of the first 3 layer 2D OVSF codes is illustrated in Figure 1. From Theorem 2, any two 2D OVSF codes in the same layer are orthogonal. Furthermore, any two codes of different layers are also orthogonal except for the case that one of the two codes is a mother code of the other. The mother codes are defined as all high layer codes linking this particular code to the first layer code [1]. For example, both $C^{(1)}_{2,N^{(1)}}$ and $C^{(2)}_{4,N^{(2)}}$ are mother codes of $C^{(3)}_{8,N^{(3)}}$.

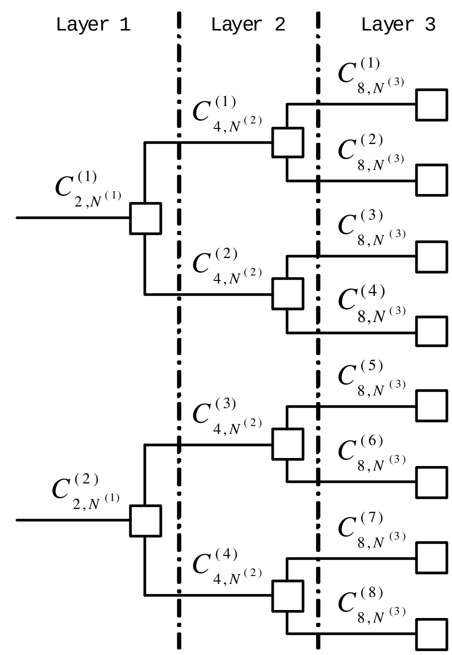

Fig. 1. The tree structure of 2D OVSF codes with first 3 layers

The parameters of proposed 2D OVSF codes are listed in Table 1. The most left column indicates the total number of layers in a multirate system. For example, if the total number of layers is 3, then the number of orthogonal channels used in the system is 16, as well as the maximum number of users that the system can support. If we assume the lowest data rate R_{min} is R, the highest data rate in the system could be $16R$. For total number of layers 3, the corresponding maximum spreading sequence length is $4N^{(1)}$. For binary 2D OVSF codes, $N^{(1)} = 2^\alpha 10^\beta 26^\gamma$, where α,β and γ are non-negative integers. For ternary 2D OVSF codes, $N^{(1)}$ can be any positive integers.

TABLE I

THE RELATIONSHIP OF PARAMETERS IN PROPOSED 2D OVSF CODES.

Total Layers	Number of Channels	Sequence Length	Maximum Users	$\frac{R_{max}}{R_{min}}$
1	2	$N^{(1)}$	2	1
2	4	$2N^{(1)}$	4	4
3	8	$4N^{(1)}$	8	16
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
i	2^i	$2^{i-1}N^{(1)}$	2^i	4^{i-1}

III. SYSTEM MODEL

We transmit the same information bit over M parallel channels. Each bit is modulated by the channel pulse train. The corresponding transmitted signal for user k is given by

$$s^{(k)}(t) = \sum_{r}\sum_{m=1}^{M} b_r^{(k)} P_m^{(k)}(t - rT_p) \tag{15}$$

where the pulse train for user k and code channel m is

$$P_m^{(k)}(t) = \sum_{n=0}^{N-1} c_{m,n}^{(k)} \psi_m(t - nT_c) \tag{16}$$

r is the index of the information symbols and N is the length of spreading sequences. The spreading sequence c_m is the mth row in the 2D OVSF code matrix $C_{M,N}^{(i)}$, thus the spreading sequence set $\{c_m\}_{m=1}^{M}$ for each user is a 2D OVSF code set. b_r are binary antipodal symbols transmitted over M parallel channels, T_c is the chip duration time and $T_p = NT_c$ is the symbol period. $\psi_m(t)$ is the unit energy signaling pulse chosen from the orthogonal set of pulses and assumed known to the receiver.

The impulse response of the UWB channel with L resolvable paths is

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \tag{17}$$

where α_l and τ_l denote the channel gain and the propagation delay of the l_{th} path, respectively.

When sufficient multipath resolution is available, small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients α_l can be modelled as independent real valued random variables whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [10].

For an asynchronous UWB system with K users, the corresponding received signal model is:

$$r(t) = \sum_{k=1}^{K}\sum_{l=0}^{L-1} \alpha_l s^{(k)}(t - \tau_l^{(k)}) + n(t) \tag{18}$$

where $\tau_l^{(k)}$ accounts for the propagation delay of l_{th} path of user k and lack of synchronism between transmitters, $n(t)$ is a white Gaussian noise with zero mean and two-sided power spectral density $\frac{N_0}{2}$.

IV. INTERFERENCE ANALYSIS

The receiver employs M correlators with corresponding orthogonal chip pulses to separate parallel channels. M outputs are then summed and help to make statistic decision.

Let us assume user 1 is the desired user and all the M orthogonal channels behave the same fading with channel coefficients $\{\alpha_l\}_{l=0}^{L-1}$. As we defined before, $\tau_l^{(k)}$ is the delay of lth path of user k, and T_c is the chip duration. Let $\lambda_l^{(k)} = \lfloor \tau_l^{(k)}/T_c \rfloor$, and $\delta_l^{(k)} = \tau_l^{(k)} - \lambda_l^{(k)} T_c$, then the MAI from user k is given by,

$$MAI_k = \frac{1}{T_c}\sum_{l=0}^{L-1}\sum_{m=1}^{M} \alpha_l A_{m,l}^{(k)}\left(b_r^{(k)}\right) \tag{19}$$

We express $A_{m,l}^{(k)}$ as

$$A_{m,l}^{(k)} = \begin{cases} b_r^{(k)}\widetilde{A}_{m,l}^{(k)} & if\ \ b_r^{(k)} = b_{r-1}^{(k)} \\ b_r^{(k)}\widehat{A}_{m,l}^{(k)} & if\ \ b_r^{(k)} = -b_{r-1}^{(k)} \end{cases} \tag{20}$$

where

$$\widetilde{A}_{m,l}^{(k)} = \widetilde{\theta}_m^{(k)}(\lambda_l^{(k)})\Phi_m'(\delta_l^{(k)}) + \widetilde{\theta}_m^{(k)}(\lambda_l^{(k)}+1)\Phi_m(\delta_l^{(k)}) \tag{21}$$

$$\widehat{A}_{m,l}^{(k)} = \widehat{\theta}_m^{(k)}(\lambda_l^{(k)})\Phi_m'(\delta_l^{(k)}) + \widehat{\theta}_m^{(k)}(\lambda_l^{(k)}+1)\Phi_m(\delta_l^{(k)}) \tag{22}$$

$\widetilde{\theta}_m^{(k)}$ and $\widehat{\theta}_m^{(k)}$ respectively denote the even and odd correlation of spreading sequences from mth channel of user 1 and user k. Φ_m' and Φ_m are partial autocorrelation functions of the chip waveform and given by,

$$\Phi_m'(\delta) = \int_{\delta}^{T_c} \psi_m(t)\psi_m(t-\delta)dt \tag{23}$$

$$\Phi_m(\delta) = \int_{0}^{\delta} \psi_m(t)\psi_m(t+T_c-\delta)dt \tag{24}$$

If $\Phi_m'(\delta)$ and $\Phi_m(\delta)$ are the same for all M pulses ψ_m over any delay value δ or we synchronize the system in chip, based on cross-correlation properties of 2D OVSF codes, the total $MAI = \sum_{k=2}^{K} MAI_k$ is zero.

The multipath interference from lth path denotes by $MPI(l)$. From autocorrelation properties of 2D OVSF codes, using the similar arguments as above, the total multipath interference is given by,

$$MPI = \sum_{k=1}^{\lfloor L/N \rfloor} MPI(kN+1) \tag{25}$$

If $N > L$, the system is multipath interference free.

If $N < L$, the BER performance is degraded due to the multipath interference caused by bit modulation. Also, if the M orthogonal channels suffer different fading (i.e. with different channel coefficients $\{\alpha_{m,l}\}$), the complementarity of 2D OVSF codes is lost and MAI is no longer zero. These scenarios are simulated and discussed in the next section.

V. Numerical Results

We study correlator and RAKE receiver performance of 2D OVSF codes in both single and multiple user system. We either assume the channel coefficients are identical over M channels, or they are independent M Nakagami fading channels. The mean power of multipath components are chosen to be equal to average value given in [11], which is based on the in-door line of sight (LOS) measurements performed in 23 homes. The sign of the reflected path coefficient is modelled as a uniformly distributed random variable [12]. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration T_c. We assume that each bin contains exactly one multipath component (emulating a dense multipath environment) and the delay spread was restricted to be 4 nanosecond. The effects of interchip interference has been assumed negligible.

In Fig. 2, we compare the BER performances of 2D OVSF codes with different sequence length N in a multipath single user scenario. $N = 6, 12$ and 24 correspond to 2D OVSF codes $C_{4,6}^{(1)}$,$C_{8,12}^{(1)}$, and $C_{16,24}^{(1)}$ constructed in Example 1. We assume M orthogonal channels either with identical channel coefficients or suffer independent Nakagami fading, and mark them as ”identical” or ”different” respectively. Note that the multipath number $L = 10$, so the multipath interference is free as long as $N > L$ in the identical parallel channel case. Thus $C_{8,12}^{(1)}$ and $C_{16,24}^{(1)}$ with MRC receiver have the identical performance which achieves the BER lower bound of antipodal signaling in AWGN channel. Due to the interference from path 6, $C_{4,6}^{(1)}$ with MRC receiver has around 2dB penalty at $BER = 10^{-3}$. With chip-matched filter which only extracts signal energy from the direct path, $C_{16,24}^{(1)}$ lost near 3dB gain compared with MRC receiver. Furthermore, when 16 orthogonal channels suffer independent Nakagami fading with shape factor $m = 2$, due to the loss of complementarity, $C_{16,24}^{(1)}$ has another 2dB lost at $BER = 10^{-3}$.

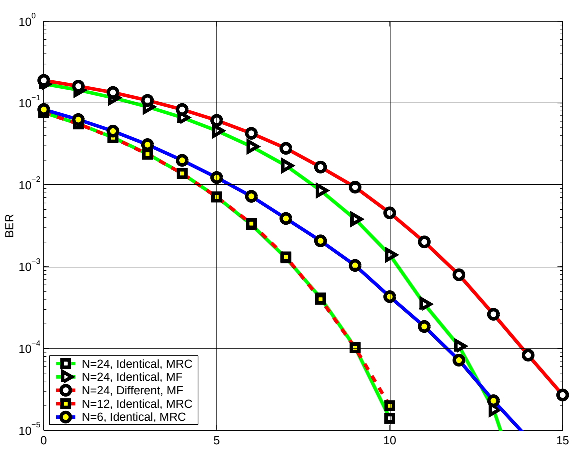

Fig. 2. Single user case, BER performance of 2D OVSF codes $C_{4,6}^{(1)}$,$C_{8,12}^{(1)}$, and $C_{16,24}^{(1)}$

Based on the tree-structure of 2D OVSF codes, we assign $C_{4,6}^{(2)}$, $C_{8,12}^{(1)}$, $C_{8,12}^{(2)}$ and $C_{16,24}^{(16)}$ to four users. Non of them is mother code of other codes, thus they are mutually orthogonal. Fig. 3 compares BER performances of $C_{16,24}^{(16)}$ in four users and single user scenarios. When 16 parallel channels are identical, there is no BER difference observed in two scenarios. When channels suffer independent Nakagami fading with shape factor $m = 1$ or 2, due to the loss of complimentary of 2D OVSF codes, the MAI is not longer zero. The smaller the m is, the more penalty upon the BER.

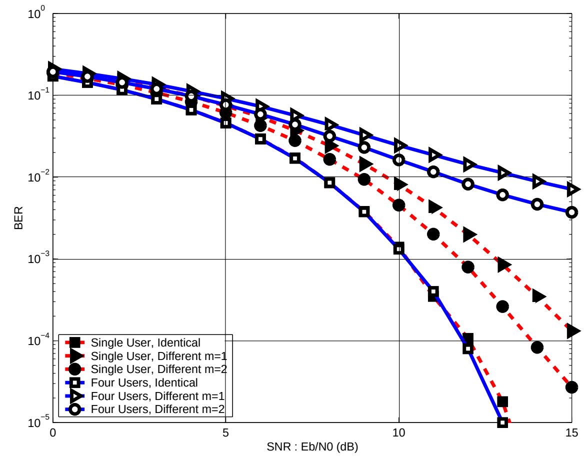

Fig. 3. Comparison on the BER performances of 2D OVSF code $C_{16,24}^{(16)}$ in single user and four users systems

References

[1] F. Adachi, M. Sawahashi, and K. Okawa, “Tree-structured generation of orthogonal spreading codes with differnt lengths for forward link of DS-CDMA mobile radio,” *Electronics Letters*, vol. 33, pp. 27 –28, Jan. 1997.

[2] S. M. Tseng and M. Bell, “Asynchronous multicarrier DS-CDMA using mutually orthogonal complementary sets of sequences,” *IEEE Transactions on Communications*, vol. 48, pp. 53 –59, Jan. 2000.

[3] D. Wu, P. Spasojević, and I. Seskar, “Ternary complementary sets for orthogonal pulse based UWB,” *Conference Record of the Thirty-Seventh Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1776–1780, Nov. 2003.

[4] C. M. Yang, P. H. Lin, G. C. Yang, and W. C. Kwong, “2D orthogonal spreading codes for multicarrier DS-CDMA systems,” *IEEE International Conference on Communications, ICC’03*, vol. 5, pp. 3277 – 3281, May 11-15 2003.

[5] N. Suehiro and M. Hatori, “N-shift cross-orthogonal sequences,” *IEEE Transactions on Information Theory*, vol. 34, pp. 143 –146, Jan. 1988.

[6] C. C. Tseng and C. Liu, “Complementary sets of sequences,” *IEEE Transactions on Information Theory*, vol. 18, pp. 644 –652, Sep. 1972.

[7] A. Gavish and A. Lempel, “On ternary complementary sequences,” *IEEE Transactions on Information Theory*, vol. 40, pp. 522 –526, Mar. 1994.

[8] M. Golay, “Complementary series,” *IEEE Transactions on Information Theory*, vol. 7, pp. 82 –87, Apr. 1961.

[9] D. V. Sarwate and M. B. Pursley, “Cross correlation properties of pseudo-random and related sequences,” *IEEE Proc.*, vol. 68, pp. 593–619, May 1980.

[10] T. S. Rappaport, *Wireless Communications Principles and Practice*. Prentice Hall, 1997.

[11] S. Ghassemzadeh, R. Jana, V. Tarokh, C. Rice, and W. Turin, “A statistical path loss model for in-home UWB channels,” *Ultra Wideband Systems and Technologies, 2002. Digest of Papers. 2002 IEEE Conference on*, pp. 59 –64, May 2002.

[12] G. Durisi and G. Romano, “Simulation analysis and performance evaluation of an UWB system in indoor multipath channel,” *IEEE Conference on Ultra Wideband Systems and Technologies, 2002. Digest of Papers*, pp. 255–258, May 2002.