332:545 **Communications Theory** **Spring 1999**

FINAL EXAMINATION

You have 3 hours to answer the following 4 questions. Read through the exam carefully before beginning work.

1. **Rutgera Univera and Fly By Night Coding Corporation:** *(50 points)* Rutgera Univera has been approached by Fly By Night Coding, Inc. (FBNC) who wishes to sell her a proprietary coder/decoder for use with her binary symmetric channel (crossover probability $0 < \varepsilon < 1/2$). Their claim is that the FBNC coder/decoder will allow error free decoding of the channel output for each and every channel use.

   Discuss their claim quantitatively and help Rutgera decide whether to buy the FBNC apparatus, or call security and have them removed from the premises.

2. **What if $M > 2$?:** *(50 points)* Consider a communications system with three arbitrary equiprobable $1-$dimensional points in signal space s_1, s_2 and s_3. You may assume none of the points are coincident. The receiver obtains $r = s_i + n$ with n a zero mean Gaussian random variable with variance N_0 (i.e., $n \sim N(0, N_0)$)

   Let D_i be the decision region for which we will declare signal s_i was sent.

   (a) *(10 points)* Given D_1 and D_2, what is D_3?

   (b) *(10 points)* Write down an expression for the probability of error in terms of the decision regions $\{D_i\}$.

   (c) *(10 points)* Find a minimum P_e decision rule for this system.

   (d) *(20 points)* Generalize your results to K dimensions (assume the noise along each dimension is I.I.D. $\sim N(0, N_0)$).

3. **Mystery Channel:** *(50 points)*

   Two spies, Alice and Bob, are communicating behind enemy lines. Alice sends binary messages to Bob by tying either a red or white string to the leg of a carrier pigeon. Bob watches the skies for the carrier pigeon and if he sees one, notes the color of the string.

   Assume that each day Alice looses a carrier pigeon and later in the day Bob sees it as it continues its journey to some distant place from which it never returns.

   (a) *(10 points)* Sketch a channel diagram. What is the capacity of this channel in bits per day?

   (b) *(20 points)* Suppose each carrier pigeon independently has probability ε of being shot down by enemy agents and not seen by Bob. Sketch a channel diagram. What is the capacity of the channel in bits per day?

   (c) *(20 points)* Suppose that Alice and Bob change their method and Alice either ties a red or white string as before, but also may decide not to send a pigeon at all. As before, a given pigeon has probability ε of not being seen by Bob. Sketch the channel diagram. What is the capacity of the channel in bits per day?

4. **The Grand Finale:** *(50 points)* This question is carefully worded and has no typos.

   (a) *(25 points)* A given additive Gaussian noise channel has a bandwidth of 1 Hz and the noise is of spectral height 1 Watt/Hz in this band (white over the band). What is the maximum entropy rate (in bits per second) of a source which can be reliably sent over this channel? Carefully and concisely explain your result from signal constellation to information theory arguments.

   (b) *(25 points)* Once again assume an additive Gaussian noise channel. Let the noise have spectral height 1 Watt/Hz (into a 1 ohm load) assumed white over the 1 Hz bandwidth channel. Assume also that the information signal is generated using a source which has a ± 10 volt range (also into 1 ohm load). Find the capacity of this channel (or a good upper bound) in bits/second. Carefully and concisely explain your result from signal constellation to information theory arguments.