Obtaining Alternating Current Through a Solar Panel: Mechanical Alternatives to the Electrical Inverter

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Abstract

The current Photovoltaic (PV) systems collect DC voltage and then use an electrical inverter to convert this to AC voltage. Inverters, like any electronic equipment, dissipate heat and loose some of the power collected by the solar panels. In this study, we attempt to use a mechanical system that periodically occludes the solar panels to simulate an AC wave. We establish 3 designs that, if produced on a micro-electromechanical scale, can prove to be cheaper and more energy efficient than the current stationary solar panel systems.

Motivation

The increasing energy demand in the near future creates a need to seek renewable energy sources (1). Solar energy is a natural solution to this problem but current PV systems are not optimal in the way they capture this energy. They are stationary and collect DC voltage from the sun. Then this DC voltage has to be sent through an inverter to produce AC voltage (2). Inverters, like other electronic equipment, dissipate heat and loose some of the power collected by the solar panels. Current commercial electric inverters have an efficiency of about 90% (3) but in some cases approximately 30%-40% (4) of the cost of a solar electric system is spent converting DC to AC.

The inverter is also responsible for the majority of failures in PV systems and accurate forecasting of inverter reliability requires a reasonably large amount of failure data that reflects the components, subsystem designs, and operating conditions typically found in PV inverters (5). Other problems that inverters face are: the most effective and realistic decomposition will be based on inverter topology (5), the inverter's reliance on capacitors whose functionality degrades at the end of the capacitor's service life (6), and the potential need for multiple inverters to maintain high energy yields for large PV arrays (7).

Moreover, on such a small scale, solar energy conversion is considered the most viable energy harvesting technique because of the high power density of solar energy compared to other environmental energy harvesting techniques (8). For this reason, several researchers have tried integrating micro-solar power systems in wireless embedded systems (8), (9), (10). However, in such cases, energy consumption by controlling devices (e.g. charging controller, regulator) takes a large fraction of the energy budget and there is substantial interaction among the solar-powered system components resulting in a lower efficiency (11). Thus, there is a need for a non-electrical solution that can create AC without having to suffer from all the deficiencies that inverters face.

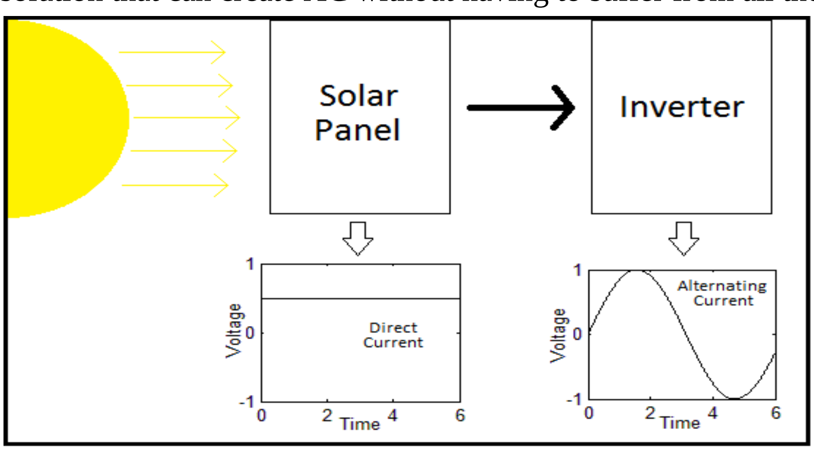
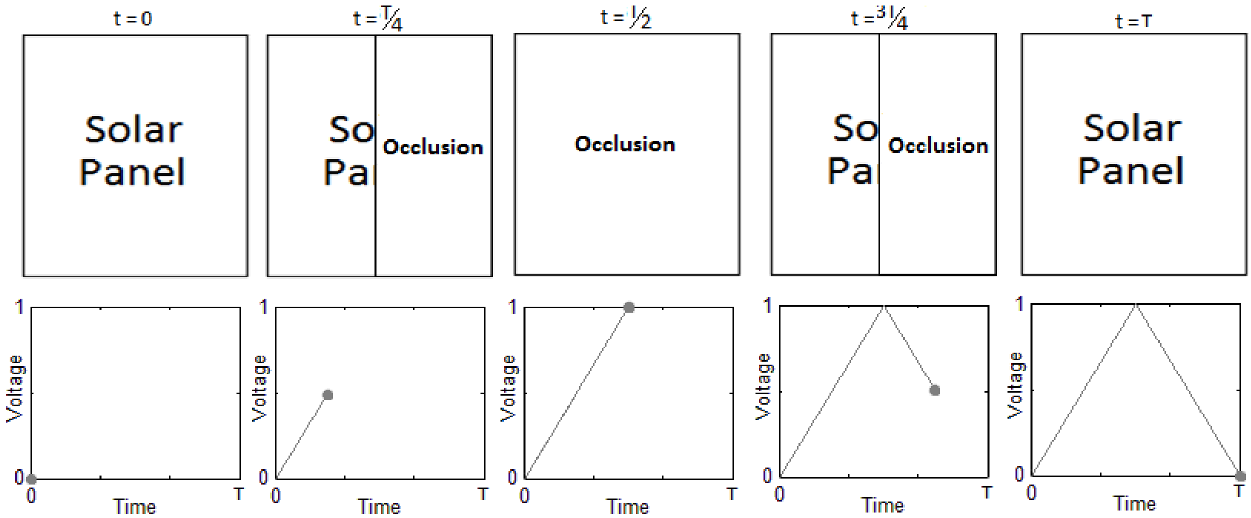


Fig. 1 The Current System of Solar Energy Conversion into Electrical Energy.

A Mechanical Solution

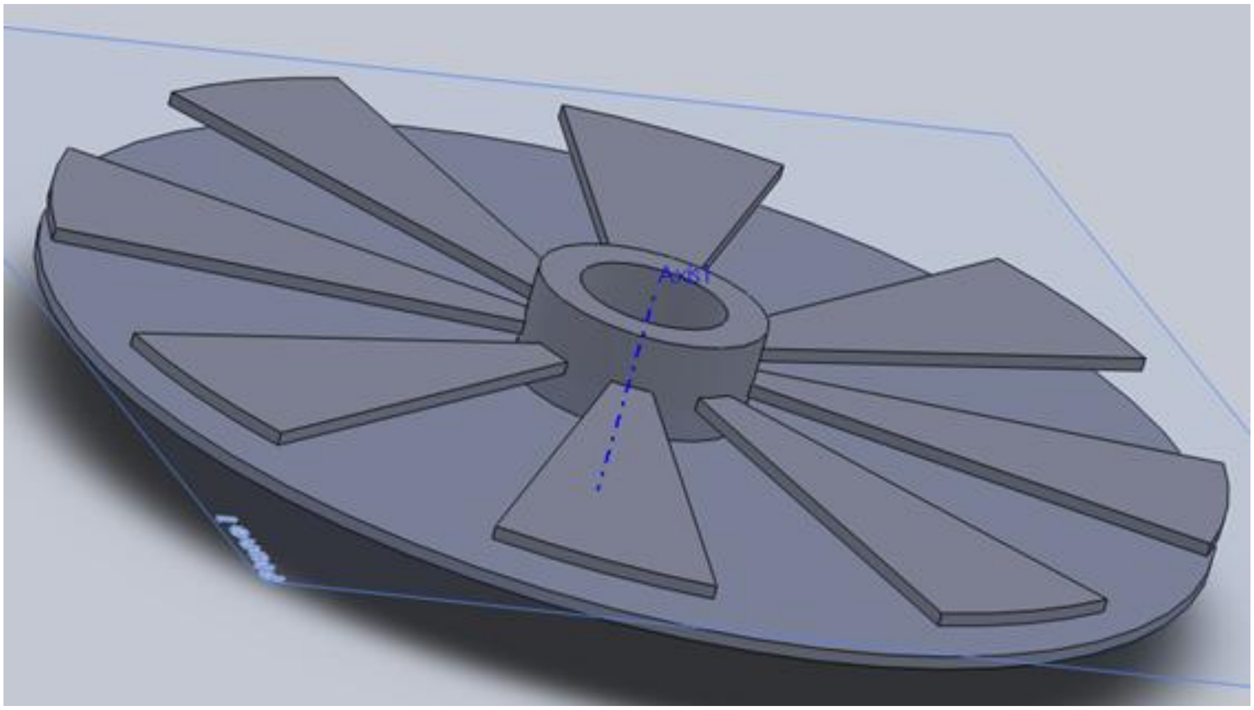
In recent years, high efficiency electric motors have been able to realize conversion rates of electrical to mechanical energy higher than 96% (12). Since the energy collected by a solar panel is proportional to the area of the panel exposed to the sun, by periodically occluding parts of the panel, an AC wave can be created. If the energy required by a motor is less than the energy loss incurred by an inverter, then the mechanical solution will be more optimal than an electrical inverter. It is also important to note that commercial electric motors are cheaper than inverters (13). The simulation of an AC wave can be summarized in the following picture:



Common Problems

When parts of a solar panel are occluded, the sunlight that does not reach the solar panel is lost. In all the designs we propose, we try to occlude part of the solar panel and still conserve the energy that falls on the occluded portion. The most important problem with a solar panel is that the energy collected is proportional to the angle of the incident light. It collects the most energy when the incident light is normal and collects 0% when the incident light is parallel to the surface of the solar panel. If the incident light is θ with respect to the normal, the fraction of energy that is collected is cos(θ) (11). Thus most designs that include a occuluder that reflects light onto the solar panel at large angle will not be optimal and will lose most of the energy. Moreover, any apparatus that involves light traveling through a medium will also lose a lot of the energy. For example, any mechanical system that is covered in a transparent vacuum chamber, or one that uses a reflecting glass will have a lower efficiency. The transmittance of a material is the fraction of incident light that passes through a medium, or the ratio of the intensity of the output to the input light. If x is the path length of the medium, the transmittance of a material is proportional to exp{-cx} where c is the medium's constant (14). As x, increases, the intensity of the light that is passed though exponentially decreases. Finally, the power that is collected is a function of r^2 where r is the radius of the mechanical system. But the power that the mechanical system has to put in to keep the system in steady state is a function of I, where I is the moment of inertia. I is proportional r^4 and thus as the radius increases, the power required by the motor increases faster than the power collected by the panels. Thus, it is most optimal to create micro-electromechanical systems with a very small radius.

Design #1



We can put the system in a vacuum and this will decrease the energy losses of the system in the following capacities: the energy required by the motor, the energy lost by the reflection of the material in the wedges and the energy lost due to the reflected light having an incident angle when hitting the solar cells. Overall, efficiency can be computed from 3 independent variables: the radius, r, and the rotational speed, ω. The overall efficiency can be computed as:

$$E(r,\omega) = \frac{P_{collected} - P_{motor}}{P_{total}} \quad (1)$$

where, E is the efficiency of total system, $P_{collected}$ is the AC power collected by panels, P_{motor} is the power required by motor to keep system running, and P_{total} is the total possible energy collected. $P_{collected}$ can be computed as:

$$P_{collected} = \alpha P_{total} = \alpha\beta\pi r^2 \quad (2)$$

where α is the percent of incident energy converted to AC power, and β is the current conversion rate of incident light energy to DC power of current solar panels. The total energy required by the motor is a function of the load of the motor equated as:

$$P_{motor} = \frac{load}{\eta_{motor}} \quad (3)$$

where load is the power required by the motor to keep the system in steady state and η_{motor} is the efficiency of motor. η_{motor} can be as high as even 99.9% in some very efficient motors but these motors require a very high load of usually > 300W. In low power systems, of ≈ 1W, its efficiency is around 80%.

Half the energy collected at each period is stored by the top panel as DC energy (and will need to be send through an inverter resulting in an efficiency of .9 (3)) and the other half will be automatically collected as AC. Thus $P_{collected}$ will be equal to ½(.9+1) = .95 of the total possible energy collected P_{total}.

During each period, the motor has to keep producing this load power to keep the system in steady state. This load is proportional to the rotational energy of the system times some internal frictional energy loss per second, Fr. This includes the frictional energy required by the motor to keep the system in stady state as well as the frictional energy caused by the electric brushes that collect DC energy.

The mass, $m(r,\theta)$, of all the wedges is also a function of r and θ, that is computed as the overall surface area of the wedges times a certain density, ρ.

$$load = \frac{1}{2}F_r I\omega^2 \quad (5)$$

$$= \frac{1}{2}F_r(m(r,\theta)r^2h)\omega^2 \quad (6)$$

$$= \frac{1}{2}F_r\frac{1}{2}\rho\pi r^2 h\omega^2 \quad (7)$$

$$= \frac{1}{4}F_r\rho\pi r^4 h\omega^2 \quad (8)$$

So, combing these equations together:

$$E(r,\omega) = \frac{P_{collected} - P_{motor}}{P_{total}} = \frac{1}{P_{total}}(P_{collected} - P_{motor}) \quad (9)$$

$$= \frac{1}{P_{total}}\left((\frac{19}{20})P_{total} - \frac{load}{\eta_{motor}}\right) \quad (10)$$

$$= \frac{1}{P_{total}}\left((\frac{19}{20})P_{total} - \frac{\frac{1}{4}F_r\rho\pi r^4 h\omega^2}{\eta_{motor}}\right) \quad (11)$$

$$= \frac{19}{20} - \frac{\frac{1}{4}F_r\rho\pi r^4 h\omega^2}{P_{total}\eta_{motor}} \quad (12)$$

$$= \frac{19}{20} - \frac{\frac{1}{4}F_r\rho r^2 h\omega^2}{\beta\eta_{motor}} \quad (13)$$

We can now formally establish the formulation of Fr. Fr is ratio of the power lost by the system in a second to the rotational energy that it already has. The power lost is simply the torque T, multiplied by the rotational frequency, ω:

$$T\omega = \mu Nr\omega = \mu g\omega * I/r \quad (14)$$

$$Fr = \frac{T\omega}{\frac{1}{2}Iw^2} = \frac{2\mu g}{r\omega} \quad (15)$$

This leads to a final Efficiency of

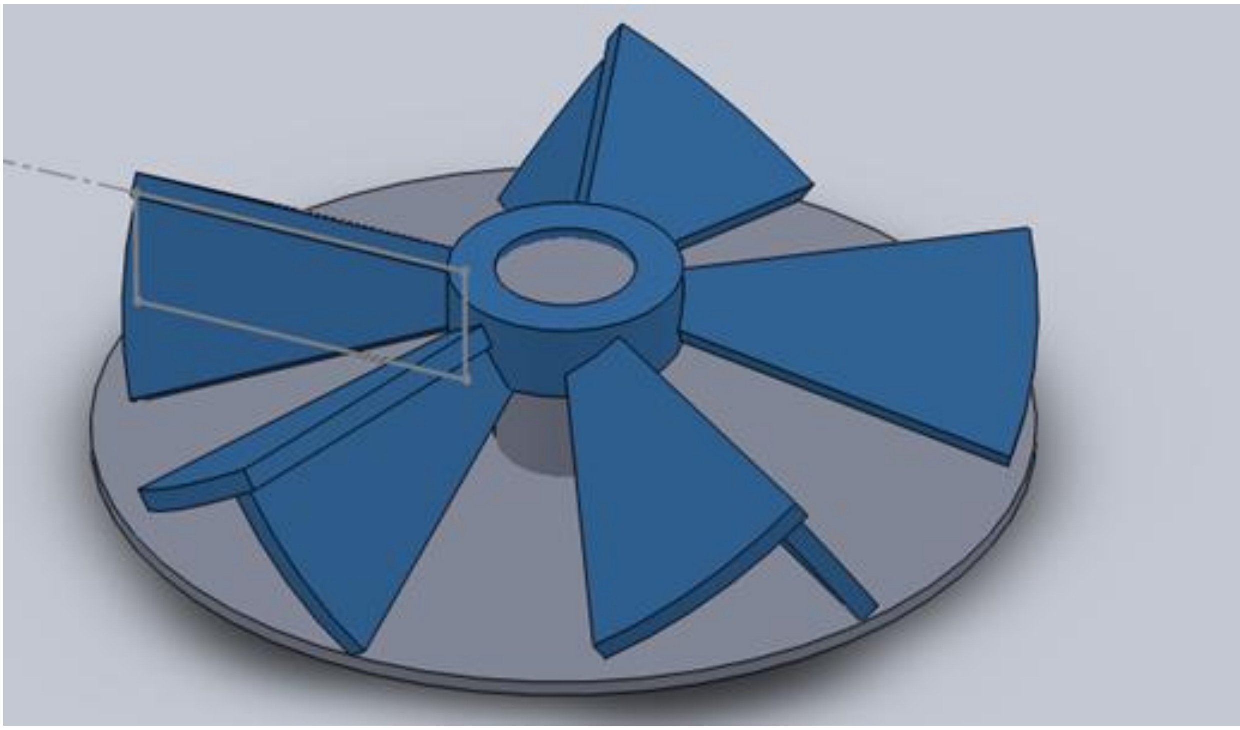
$$E(r,\omega) = \frac{19}{20} - \frac{\mu g \rho r h \omega}{2\beta\eta_{motor}} \quad (16)$$

Theoretically, η_{motor} is a function of load and thus a function of θ, r, and ω but its range is only .8<, η_{motor}< .999. Compared to the other variables this change is very small such that

$$\frac{\partial\eta_{motor}}{\partial\theta} \approx \frac{\partial\eta_{motor}}{\partial r} \approx \frac{\partial\eta_{motor}}{\partial\omega} \quad (17)$$

From equation 13, we can see that r and ω only decrease the value of E. Thus, a smaller radius of the system will result in a better efficiency. Moreover, the more wedges or subsections there are in the system, the smaller the rotational frequency of the system will need to be. This also results automatically into a higher efficiency.

Design #2



We can put the system in a vacuum and this will decrease the energy losses of the system in the following capacities: the energy required by the motor, the energy lost by the reflection of the material in the wedges and the energy lost due to the reflected light having an incident angle when hitting the solar cells. Overall, efficiency can be computed from 3 independent variables: the radius, r, the rotational speed, ω, and the angle of incidence of the light which is also the angle that describes the height of the wedge, θ . The overall efficiency can be computed as:

$$E(\theta, r, \omega) = \frac{P_{collected} - P_{motor}}{P_{total}} \quad (1)$$

where, E is the efficiency of total system, $P_{collected}$ is the AC power collected by panels, P_{motor} is the power required by motor to keep system running, and P_{total} is the total possible energy collected. $P_{collected}$ can be computed as:

$$P_{collected} = \alpha P_{total} = \alpha\beta\pi r^2 \quad (2)$$

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$$P_{motor} = \frac{load}{\eta_{motor}} \quad (3)$$

where load is the power required by the motor to keep the system in steady state and η_{motor} is the efficiency of motor. η_{motor} can be as high as even 99.9% in some very efficient motors but these motors require a very high load of usually > 300W. In low power systems, of $\approx$ 1W, its efficiency is around 80%. α is the efficiency of light that lands on the solar panels + efficiency that is reflected onto solar panels = .5 + .5(loss due to angular deflection) (loss due to imperfect reflectivity of reflective material):

$$\alpha(\theta) = .5 + .5\sin(\theta)\, R(\theta, \beta) \quad (4)$$

R(θ) , the reflectivity, varies widely for a lot of materials across (θ), the angle of incidence and the λ, wavelength of light that falls on the material. This value is also sensitive and degrades over time if left to corrode outside. This makes putting the system in vacuum more crucial. Aluminum is cheap and works well in the visible spectrum, so we can use that. Because, the Reflectivity also varies across the wavelength of the wave being reflected, R also depends on β. For this analysis, we are assuming R will be constant.

During each period, the motor has to keep producing this load power to keep the system in steady state. This load is proportional to the rotational energy of the system times some internal frictional energy loss per second, F_r. The mass, $m(r,\theta)$, of all the wedges is also a function of r and θ, that is computed as the overall surface area of the wedges times a certain density, ρ.

$$load = \frac{1}{2} F_r I \omega^2 \quad (5)$$

$$= \frac{1}{2} F_r (m(r,\theta) r^2 h)\omega^2 \quad (6)$$

$$= \frac{1}{2} F_r \left(\frac{\frac{1}{2}\rho\pi r^2 h}{\cos(\theta)} \right) \omega^2 \quad (7)$$

$$= \frac{1}{4} F_r \rho\pi \sec(\theta)\, r^4 h \omega^2 \quad (8)$$

So, combing these equations together:

$$E(\theta, r, \omega) = \frac{P_{collected} - P_{motor}}{P_{total}} = \frac{1}{P_{total}}(P_{collected} - P_{motor}) \quad (9)$$

$$= \frac{1}{P_{total}} \left(\alpha(\theta) P_{total} - \frac{load}{\eta_{motor}} \right) \quad (10)$$

$$= \frac{1}{P_{total}} \left(\alpha(\theta) P_{total} - \frac{\frac{1}{4} F_r \rho\pi \sec(\theta)\, r^4 h \omega^2}{\eta_{motor}} \right) \quad (11)$$

$$= \alpha(\theta) - \frac{\frac{1}{4} F_r \rho\pi \sec(\theta)\, r^4 h \omega^2}{P_{total}\eta_{motor}} \quad (12)$$

$$= \alpha(\theta) - \frac{F_r \rho \sec(\theta)\, r^2 h \omega^2}{4\beta\eta_{motor}} \quad (13)$$

We can now formally establish the formulation of Fr. Fr is ratio of the power lost by the system in a second to the rotational energy that it already has. The power lost is simply the torque T, multiplied by the rotational frequency, ω:

$$T\omega = \mu N r\omega = \mu g\omega * I/r \quad (14)$$

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This leads to a final Efficiency of

$$E(r,\omega) = \alpha(\theta) - \frac{\mu g\rho \sec(\theta)\, rh\omega}{2\beta\eta_{motor}} \quad (16)$$

Theoretically, η_{motor} is a function of load and thus a function of θ, r, and ω but its range is only .8<, η_{motor}< .999. Compared to the other variables this change is very small such that

$$\frac{\partial\eta_{motor}}{\partial\theta} \approx \frac{\partial\eta_{motor}}{\partial R} \approx \frac{\partial\eta_{motor}}{\partial\omega} \quad (17)$$

From equation 13, we can see that r and ω only decrease the value of E. However, as θ increases, $\alpha(\theta)$ increases but so does tan(θ). This means that the system is most efficient when it is as small and as slow as possible. Only θ can be varied from 45 < $\theta < 90$ to maximize E.

$$\frac{\partial E(\theta,r,\omega)}{\partial\theta} = 0 = \frac{1}{\partial\theta}\partial\left(\alpha(\theta) - \frac{\mu g\rho \sec(\theta)\, rh\omega}{2\beta\eta_{motor}}\right) \quad (18)$$

which is equivalent to:

$$\frac{\partial\alpha(\theta)}{\partial\theta} = \frac{1}{\partial\theta}\partial\left(\frac{\mu g\rho \sec(\theta)\, rh\omega}{2\beta\eta_{motor}}\right) \quad (19)$$

$$\frac{1}{\partial\theta}\partial(.5 + .5\sin(\theta)\, R(\theta)) = \frac{1}{\partial\theta}\partial\left(\frac{\mu g\rho \sec(\theta) rh\omega}{2\beta\eta_{motor}}\right) \quad (20)$$

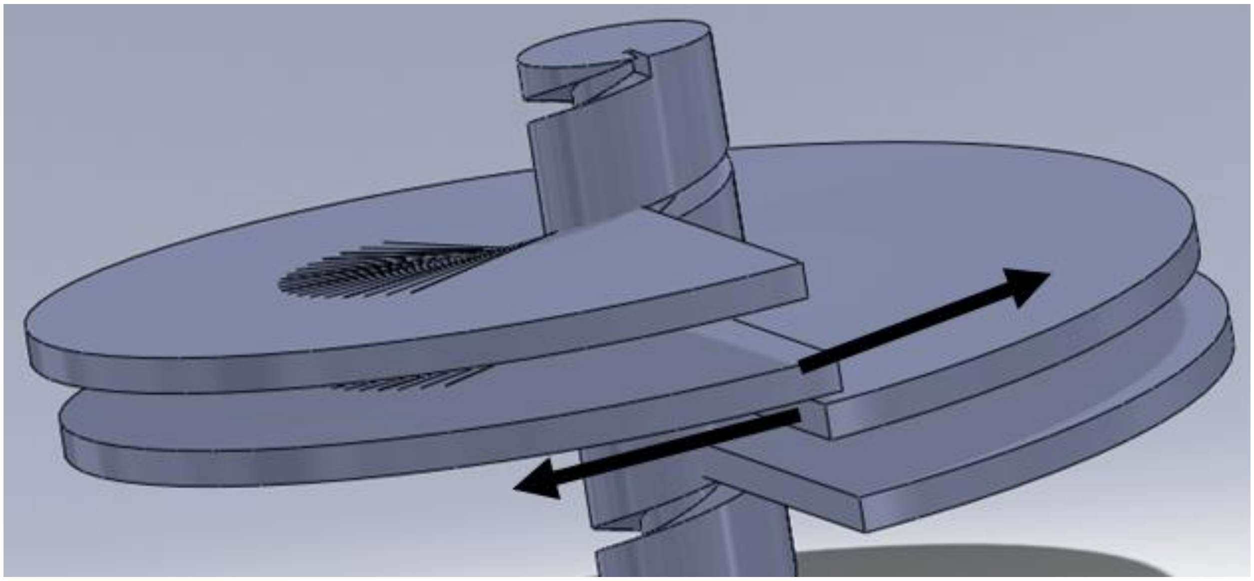
$$\frac{R\big(d\sin(\theta)\big)}{d\theta} = \frac{\mu g\rho rh\omega}{2\beta\eta_{motor}}\frac{d\sec(\theta)}{d\theta} \quad (21)$$

Reflectivity is a function of θ but for Aluminum, it is usually very high, especially in the visible and infared spectrum. So R is given a value of .92 here. This solution to this ODE is equivalent to the following answer:

$$\cos(\theta)^3 \csc(\theta) = \frac{\mu g\rho rh\omega}{2R\beta\eta_{motor}} \quad (22)$$

Given this optimal θ, we still see a similar pattern from Design #1 that a smaller radius and a smaller rotational frequency, drastically help the efficiency of the system.

Design #3



We first determined how much energy is lost in free vibration of a torsion spring. We then set the energy loss of the unideal system to the energy required by the motor to maintain a desired steady state response. Our solar panel design requires every oscillation to maintain a rotation from an angle of π to $-\pi$. With the equations of motion, and the energy loss, we can calculate the efficiency of the system.

To find the equation of motion, we took the sum of moments about the spring's central axis.

$$\sum M = T(t) - c\dot{\theta} - k\theta = I\ddot{\theta} \qquad (1)$$

Where T is the forcing torque induced by a motor, θ is the displaced angle of the torsion spring, c is the viscous damping property of the spring, k is the stiffness of the spring, and I is the inertia of the body. For free vibration, the forcing torque was set to zero.

$$\ddot{\theta} + \frac{c}{I}\dot{\theta} + \frac{k}{I}\theta = 0 \qquad (2)$$

This second order differential equation can be rewritten as the following.

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + {\omega_n}^2\theta = 0 \qquad (3)$$

For

$$\omega_n = \sqrt{k/I} \qquad (4)$$

$$\zeta = \frac{c}{2\omega_n} = \frac{c}{2\sqrt{k/I}} \qquad (5)$$

Where ω_n is the natural frequency of the spring and mass, and ζ is the dimensionless viscous damping factor. The solution to this differential equation can be assumed to take the form of an exponential function (15). Taking the derivative and plugging in yields the following.

$$(A\lambda^2 e^{\lambda t}) + 2\zeta\omega_n(A\lambda e^{\lambda t}) + {\omega_n}^2 A e^{\lambda t} = 0 \qquad (6)$$

Which reduces to

$$\lambda^2 + 2\zeta\omega_n\lambda + {\omega_n}^2 = 0 \qquad (7)$$

Solving for lambda leads to.

$$\lambda = -\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2} \qquad (8)$$

Or

$$\lambda = -\zeta\omega_n \pm \omega_d \qquad (9)$$

Where $\omega_d = \sqrt{\zeta^2 - 1}$, for $\zeta < 1$, and is termed the damped natural frequency.
Since ζ will be less than unity for a spring, this reduces our solution can be written in the form.

$$\theta(t) = e^{-\zeta\omega_n t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) \qquad (10)$$

Where B_1 and B_2 are coefficients to be determined by initial conditions.

Our initial displacement was set to θ_0 and since the displacement is maximum at time zero, we assumed that the initial angular velocity was zero. This leads to the following values for B_1 and B_2 respectively.

$$B_1 = \theta_0 \qquad (11)$$

$$B_2 = \frac{\zeta\omega_n\theta_0}{\omega_d} \qquad (12)$$

Now we have the particular solution to the equation of motion of a torsion spring, with a mass and damping, in free vibration.

$$\theta(t) = \theta_0 e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta\omega_n}{\omega_d}\sin\omega_d t\right) \qquad (13)$$

In general, the magnitude of the angular displacement will be the following

$$\theta\left(\frac{nT}{2}\right) = \theta\left(\frac{n\pi}{\omega_d}\right) = \theta_0 e^{-\frac{\zeta n\pi}{\sqrt{1-\zeta^2}}} \qquad (14)$$

Where n is the number of half periods. It is clear that if a forcing torque were applied, applying a forcing torque more frequently, ideally continuously, would result in less diminishing angular displacement due to the exponential nature of the effects of damping.

Equation 12 quickly leads to the expression of the efficiency of the spring for n number of half periods:

$$\eta_{spring} = \frac{E_{out}}{E_{in}} = \frac{\frac{1}{2}k\left[\theta\left(\frac{n\pi}{\omega_d}\right)\right]^2}{\frac{1}{2}k[\theta(0)]^2} = e^{-2\frac{\zeta n\pi}{\sqrt{1-\zeta^2}}} \qquad (15)$$

If a forcing torque is applied to maintain the magnitude of the initial angular displacement during every half period (n = 1), then, for 90% efficiency, it is then required that $\zeta < 0.016766$ for the energy loss in the spring alone to be considered against efficiencies of electrical inverters.

Interestingly, as the inertia increases, the damping factor decreases and thus efficiency actually increases. But as the moment of inertia increases a frequency of 60Hz will be more difficult to achieve (inertia is the resistance of an object to a change in its state of motion).

If the forcing torque were continuous and sinusoidal with amplitude A, namely $A/m\ \cos(\omega_d t)$, the response would be of the form

$$\theta_{steady}(t) = D\cos(\omega_d t - \phi) \qquad (16)$$

where

$$D = \frac{A}{k}\delta \qquad (17)$$

$$\delta = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2}} = \frac{1}{2\zeta} \qquad (18)$$

$$\phi = \tan^{-1}\left[\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right] = \pi/2 \qquad (19)$$

Letting $\omega = \omega_d$ our steady state solution simplifies to

$$\theta_{steady}(t) = \frac{A}{2\zeta k}\cos(\omega_d t - \pi/2) \qquad (20)$$

This continuous forcing torque would eliminate the decreasing of the angular displacement amplitude at the cost of energy to maintain the motor to run.

Lastly, the efficiency of the whole solar panel system is:

$$\eta_{system} = \frac{P_{collected} - P_{motor}}{P_{total}} \qquad (21)$$

where $P_{collected}$ is the AC power collected by panels, P_{motor} is the power required by motor to keep system running, and P_{total} is the total possible energy collected.

$$P_{motor} = \frac{load}{\eta_{motor}} \qquad (22)$$

where $load$ is the power required by the motor to keep the system in steady state and η_{motor} is the efficiency of motor.

$$P_{collected} = \alpha P_{total} = \alpha\beta\pi r^2 \qquad (23)$$

where we can assume α to be 1 if the solar panels are arranged and angled in such a way that all of the incoming solar rays are normal to the panels. The panels may be arranged in such a way that they are a stepwise array, and thus perpendicular to the incoming solar rays. The β is the amount of solar energy being converted to electrical energy.

The power that has to be supplied by the motor to keep the system in steady state was estimated by the total energy loss of the system per half period

$$load = \left(\frac{1}{2}k[\theta(0)]^2 - \frac{1}{2}k\left[\theta\left(\frac{\pi}{\omega_d}\right)\right]^2\right)\frac{1}{T/2} = \frac{\frac{1}{2}k[\theta(0)]^2\left(1-\eta_{spring}\right)}{T} \qquad (24)$$

The overall efficiency of the system can be described as follows:

$$E = 1 - \frac{\pi\rho r^2\omega^3 h}{8\beta\eta_{motor}}\left(1 - e^{\left(\frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}\right)}\right) \qquad (25)$$

Figures

Figure 1. Free vibration response with no forcing function and initial angular displacement of π for ten seconds.

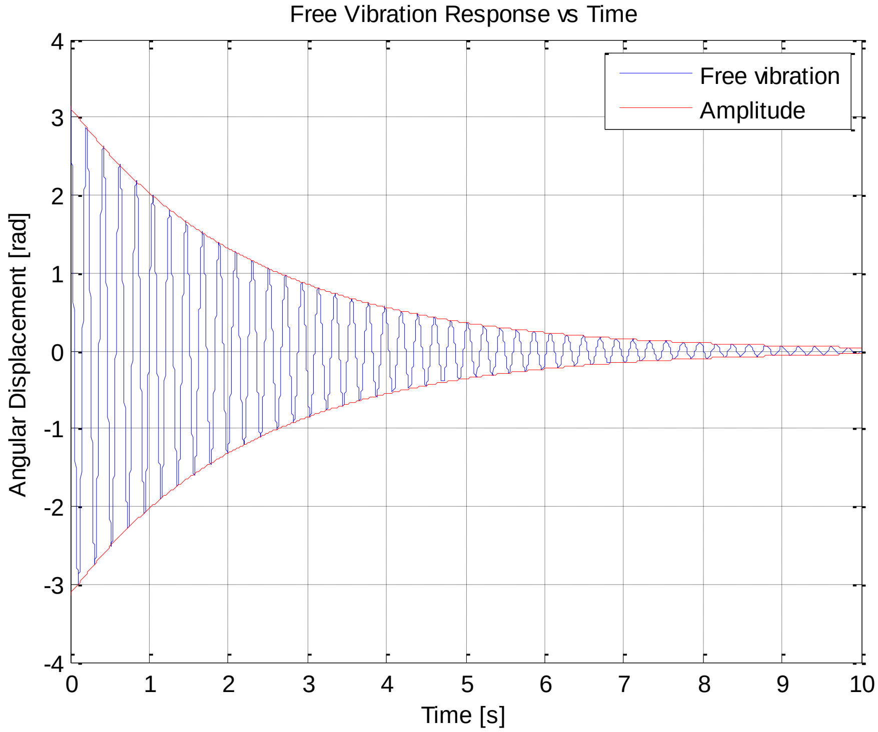
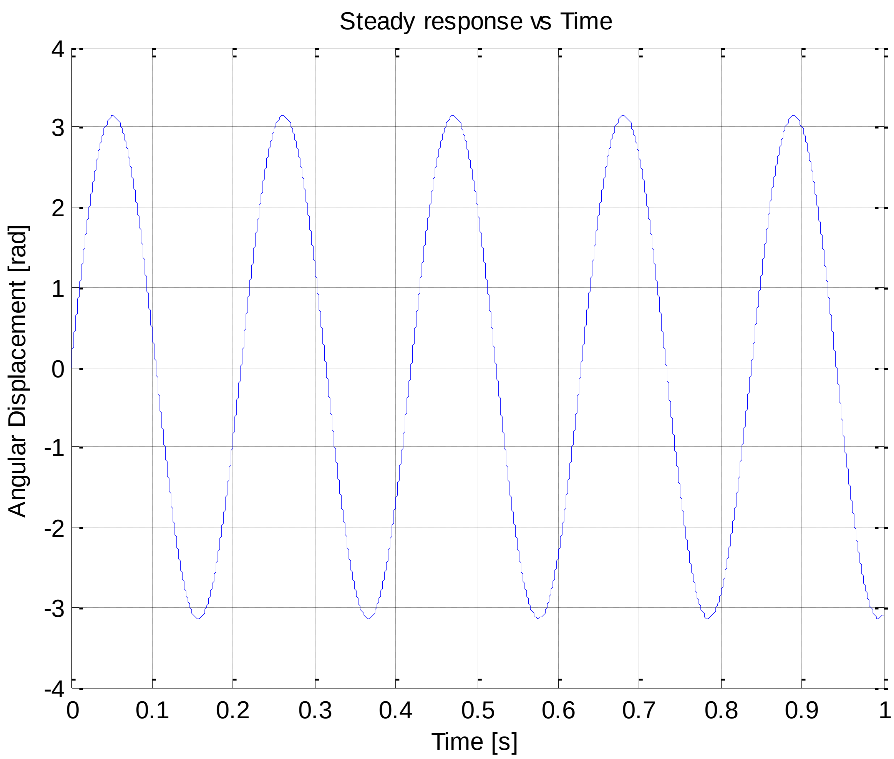


Figure 2. Steady state response for one second.



Matlab Code used.

```
%Symbols
syms lbs Newtons_per_meter

%Variables
a=0;
b=10;                          % Set b = 1 for figure 2, b = 10 for figure 3
t=linspace(a,b,10000);         % [s]-----10 seconds of time viewed

% Given Values
m=1;                          % [kg]
m_english=2.204622*m*lbs      % [lbs]
r=1/3;                         % [m]
I=1/2*m*r^r;                  % [kg m^2]--Inertia
%c=0.3;                       % []    ------Damping coefficient
Xi=1*10^-5;                   % []----------For Steel only
wn=30;                        % [rad/s]-----Natural Frequency
wd=30;                        % [rad/s]-----Damped Frequency
w=wd;                         % [rad/s]-----Forcing torque frequency
X0=pi;                        % [rad] ------Initial Angular displacement
V0=0;                         % [rad/s]-----Initial angular velocity

% Calculated Values
k=wn^2*I;                   % []    ------Spring constant
k_=k*Newtons_per_meter

%wn=sqrt(k/I);             % [rad/s]
Xi=c./(2*I.*wn);            % []
wd=wn.*sqrt(1-Xi.^2);       % [rad/s]

P=2.*Xi.*w./wn;
Q=1-(w./wn).^2;

phi=atan(P./Q);
Beta=1./sqrt(Q.^2+P.^2);

A=pi*k/Beta;                          % [N m] ------Amplitude of forcing torque

D=A.*Beta./k;                         % Assumes input function of A/I cos(wt)

B1=X0;
B2=B1.*Xi.*wn/wd;

Theta_free=exp(-Xi.*wn.*t).*(B1.*cos(wd.*t)+B2.*sin(wd.*t));
Theta_steady=D.*cos(w.*t-phi);
Theta_full=Theta_free+Theta_steady;


ARF=X0.*exp(-Xi.*wn.*t);

% figure(1)
% plot(t,Theta_full,'b')
% grid on
```

```
% ylabel('Angular Displacement [rad]')
% xlabel('Time [s]')
% title('Full response vs Time')

figure(2)
plot(t,Theta_steady,'b')
grid on
ylabel('Angular Displacement [rad]')
xlabel('Time [s]')
title('Steady response vs Time')
legend('Steady state response')

figure(3)
plot(t,Theta_free,'b')
hold on
plot(t,ARF,'r')
hold on
plot(t,-ARF,'r')
grid on
ylabel('Angular Displacement [rad]')
xlabel('Time [s]')
title('Free Vibration Response vs Time')
legend('Free vibration','Amplitude')
```

```
Output:

Figure 1 (b=10)
Figure 2 (b=1)

m_english = 2.2 lbs
k_ = 312 Newtons_per_meter
```

Such high spring constant values exist (16).

Deflection of Disks and Wedges

The dimensions a given sector of the cover plays an important role in the efficiency of the solar panel system. The smaller the sector, the less rotation is required for the solar panels to change between covered and uncovered states, therefore decreasing the amount of energy needed to rotate the system. The following analysis was conducted to observe limitations in the thinness of these sectors to increase efficiency.

A circular sector has area

$$A = \frac{1}{2} r^2 \theta$$

A uniform load, due to the weight of the material, is applied to the disk, which is proportional to the disks volume.

$$w = \rho g h \frac{1}{2} r^2 \theta \qquad [1]$$

The center of mass of a circular sector is given by

$$\bar{r} = \frac{4r}{3\pi} \qquad [2]$$

Therefore, the moment at the center of the circular sector is calculated as the total load times the distance from the center of the circular disk to the point where the load is applied.

$$M_c = \left(\rho g h \frac{1}{2} r^2 \theta\right)\left(\frac{4r}{3\pi}\right) \qquad [3]$$

- Note how the moment is a function of the angle theta.

The relationship between moment and deflection is as follows.

$$M_c = EI \frac{d^2 v}{dr^2} \qquad [4]$$

Where E is young's Modulus (we use a value of E = $6*10^6$ Pa (17)), I is the polar moment of inertia about the 'z' axis. Since we are interested in small angles of theta, the polar moment of inertia was approximated as a rectangular beam with the following relationship.

$$I = \frac{(r\theta)h^3}{12} \qquad [5]$$

Combining equations 1 through 5 leads to the following expression:

$$\frac{d^2 v}{dr^2} = 8 \frac{\rho g}{\pi E h^2} r^2 \qquad [6]$$

Integrating twice, yields the following.

$$v = 8 \frac{\rho g}{E h^2 \pi} \frac{r^4}{12} + c_1 r + c_2 \qquad [7]$$

Since the disk is symmetric about the origin, the slope is zero, and since the disk is supported at the origin, the deflection is zero, leading to both constants of integration canceling out, yielding.

$$v = \frac{2}{3} \frac{\rho g}{\pi E h^2} r^4 \qquad [8]$$

Analysis of the Three Designs

In the below table, let S denote the cost of solar panels for a certain constant area, I as the cost of an inverter, M as the cost of a motor, L = .1 (3) as the efficiency loss of a inverter, α as the reflectance of a material used, n the number of "wedges" in Design's #1 and #2.

	Current Systems	Design #1	Design #2	Design #3
Cost	S + I	1.5S+I+M	S+M	2S+M
Maximum Efficiency	1-L	1-1/2L	(1+R)/2	1
Required ω	-	60/n	60/n	60
Optimal r	-	0	0	0
Required r to meet efficiency of Current Systems	-	897 microns	374 microns	2462 microns
Deflection (at "r" measured above)	-	7.5 microns	1.3 microns	57 microns

The "Maximum Efficiency" values were generated using very ideal parameters, setting the power required by the motor to be 0. The values used to calculated the "Required r to meet efficiency of Current Systems" were based on values found in literature, specifically ω = 60hz (18), β = .2 (19), η_{motor} = .8 (12), R = .9 (20), ρ = 2700 kg/m3 (21), h = .01r (22), n=4 (22), and ζ = .1 (15). The only value left to calculate is θ, which happens to be a function of the radius. We choose instead a conservative value of θ=80 to perform all our calculations.

A smaller radius for any design implies that less the power is required by the motor to keep the system in steady state. This smaller radius automatically results in a higher efficiency. Thus, if any design can meet the efficiency of the inverter, even at a large radius, it means that that design is the most energy efficient. We see that Design #3 achieves the same efficiency of Design #1 and #2 even at a radius 3 times and 8 times bigger. These results indicate that, of the 3 designs proposed, Design #3 is the most viable and energy efficient mechanical alternative to the electrical inverter.

We further check these results to make sure the deflection from the edge of the disk to its axle is not too large. If the deflection is too large, then the angle of the incident light will not be normal to the panels and this might result in large energy losses. We can see a heavier and larger disk is going to result in a higher deflection. Yet, we can see that none of the deflections are larger than .01*r. This indicates that the efficiency loss from the deflection is about .01%, which is negligible to 90%, the efficiency that the designs obtain.

Thus, we can establish that mechanical alternatives to the electric inverter can produce AC with a higher efficiency. It is further important to reiterate, that the efficiency values above will increase when the radius becomes smaller.

Concluding Remarks

We now use this space to list the limitations of current technology for building such a micro-electronic design.

First, electrostatic forces appear to be the most viable option to create motion at such a small level (23). However, friction plays a dominant role in the dynamic behavior of such systems (24). In the design, we stated that the optimal radius would be "0" meters, however as the radius gets smaller to a few hundred microns, the frictional forces tend to become very high and the system will start to become very inefficient again.

Second, solar panels are not usually manufactured at such a small scale. The closest ones that are manufactured at that level are as large as a few millimeters wide (25), (26), (27). This relative large size makes it impossible to obtain a high efficiency, especially for Design #1 and #1. For Design #3, the height or the thickness of the current solar panels becomes too large. Design #3 requires that the height of the disk be much smaller than the length. If this constraint is not met, then the angular separation of the disks will have to be much larger. This angular separation will force the incident light to hit the panels at a very poor angle. In addition, the added thickness and weight of these panels will further increase the bending deflection in the panels and create an even worse incident angle.

These two facts indicate that, using current technology, the manufacturing of the 3 designs discussed in this paper is impossible. New technology needs to arise that helps 1) alleviate the friction caused in electro-statically driven micro-motor systems and 2) create smaller, yet efficient, micro-scale solar panels.

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