THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring**

Problem Set 5

Reading: Haykin 3.1–3.4

1. **Nyquist 101:** Specify the Nqyusit rate and Nyquist interval for each of the following signals. Note that $sinc(x) \equiv \frac{\sin(\pi x)}{\pi x}$.

   (a) $g(t) = sinc(200t)$

   SOLUTION: *This sinc pulse corresponds to a bandwidth of $W = 100$ Hz. Hence, the Nyquist rate is 200 Hz, and the Nyquist interval is $1/200$ seconds.*

   (b) $g(t) = sinc^2(200t)$

   SOLUTION: *This signal may be viewed as the product of the sinc pulse $sinc(200t)$ with itself. Since multiplication in time domain corresponds to convolution in frequency domain, we find that the signal $g(t)$ has a bandwidth equal to twice that of the sinc pulse $sinc(200t)$, that is 200 Hz. The Nyquist rate of $g(t)$ is therefore 400 Hz, and the Nyquist interval is $1/400$ seconds.*

   (c) $g(t) = sinc(200t) + sinc^2(200t)$

   SOLUTION: *The bandwidth of $g(t)$ is determined by the highest frequency content of either $sinc(200t)$ or $sinc^2(200t)$. From earlier parts, we know that $sinc^2(200t)$ has the higher bandwidth equal to 200 Hz. Correspondingly, the Nyquist rate is 400 Hz and the Nyquist interval is $1/400$ seconds.*

2. **Nyquist 102:** Suppose we have samples of a signal $a_k = g(k\Delta)$ where Δ is shorter than the Nyquist interval for the bandlimited function $g(t)$. Derive an explicit time-domain expression for how we recover the function $g(t)$ from the samples $\{a_k\}$.

   SOLUTION: *Sampled signal can be expressed as*

$$g_\delta(t) = \sum_{k=-\infty}^{\infty} g(k\Delta)\delta(t - k\Delta)$$

   The Fourier transform of $g_\delta(t)$ is Now, we apply a perfect low pass filter of bandwidth W to obtain the original signal $g(t)$. The inverse Fourier transform of such a filter which is 1 on $-W \le f \le W$ is

$$h(t) = \frac{\sin 2\pi W t}{\pi t}$$

   Convolution of $g_\delta(t)$ with $h(t)$ yields

$$g(t) = \sum_{k=-\infty}^{\infty} g(k\Delta)\frac{\sin 2\pi W(t - k\Delta)}{\pi(t - k\Delta)}$$

so the "sinc" function is the interpolation function *for Nyquist sampling.*

3. **Nyquist Grad School:** Does the Nyquist Sampling Theorem apply to strictly time limited signals? If not why not? If so, why? This problem is a bit subtle so think carefully and analytically (and justify any assumptions).

   SOLUTION: *The Nyquist Sampling Theorem can't be directly applied to strictly limited signals, because these signals are not band-limited. The reason is pretty simple. You can think of an time-limited signal $s(t)$ as an unlimited time signal $q(t)$ (that has limited bandwidth) multiplied by a window function $w(t)$ (a unit pulse of a certain duration which is zero everywhere else). So, $s(t) = q(t)w(t)$. Multiplication in time domain implies convolution in frequency domain and since the fourier transform of the window function is a sinc and has infinite extent, when it's convolved with the finite extent (because ($q(t)$ is bandlimited) fourier transform of $q(t)$, the result will be of infinite extent too. So time-limited means band-unlimited. And by duality, band-limited means time-unlimited.*

   In practice, we may take the following two measures:

   (a) *Prior to sampling, a low-pass filter is used to limit the sinal bandwidth in which high-frequency components are not essential to the information being conveyed by the sinal.*
   (b) *The filtered signal is sampled at a rate slightly higher than the Nyquist rate just to be sure that nothing really significant slips through the filter.*

   But not matter how you slice it, it's just an approximation to the theory (or the theory is an approximation to the reality).

4. **Pulse Modulation**

   (a) What is Pulse Amplitude Modulation? Provide a pictorial example.

   SOLUTION: *The ampliude of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal, as illustrated in figure 3.5 in text.*

   (b) What is Pulse Position Modulation? Provide a pictorial example.

   SOLUTION: *The position of a pulse relative to its unmodulated time of occurrence is varied in accordance with the message signal, as illustrated in figure 3.8(d) in text.*

   (c) What is Pulse Frequency Modulation? Provide a pictorial example.

   SOLUTION: *The pulse repetition rate is varied in accordance with the modulating signal.*

   (d) What is Pulse Width Modulation? Provide a pictorial example.

   SOLUTION: *Samples of the message signal are used to vary the duration of the individual pulses in the carrier, as illustrated in figure 3.8(c) in text.*

   (e) Consider a full wave rectified AM signal $r(t) = m(t)\cos 2\pi f_c t$ where we assume $m(t) \geq 0\ \forall t$. Assuming the highest frequency content of $m(t)$ is much less than f_c, can $r(t)$ be considered the approximate result of a pulse modulation method applied to $m(t)$? If so, which one?

   SOLUTION: *Yes, $r(t)$ can be considered the approximation result of PAM, in which the pulse is not a rectangle, but $\cos 2\pi f_c t$ ($0 \leq t \leq T/2$), where $T = \frac{1}{f_c}$ instead.*

5. **Problem 3.5 in Haykin**

SOLUTION: *We can determine the spectrum of the resulting PAM signal according to*

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f)$$

Where f_s *is the sampling rate and* $H(f)$ *is the spectrum of the pulse.*

$$\begin{aligned} H(f) &= Tsinc(fT)e^{-j\pi fT} \\ &= 10^{-4}sinc(10^{-4}f)e^{-j\pi 10^{-4}f} \end{aligned}$$

where T is the pulse duration, which is 0.1*ms. So,*

$$S(f) = \frac{1}{10} \sum_{k=-\infty}^{\infty} M(f - 10^3 k)sinc(10^{-4}f)e^{-j\pi 10^{-4}f}$$