

College of Engineering
Department of Electrical and Computer Engineering

332:322

Principles of Communications Systems
Linear Systems Assessment Examination

Spring 2006

There are three questions. You have the class period to answer them. Place all answers on the test sheet. You may use whatever materials you'd like – including text books – but you're better off just trying to figure out everything on your own using first principles.

1. The Fourier transform of $x(t)$ is $X(f)$. Prove that the Fourier transform of $x(t - t_0)$ is $e^{-j2\pi ft_0}X(f)$.
2. Derive the convolution integral from first principles (as outlined in class) given by $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau$, where $h(t)$ is the impulse response of a LTI system, $x(t)$ the input and $y(t)$ the output.
3. The energy in a signal $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Prove that if $X(f)$ is the Fourier transform of $x(t)$, then we must also have

$$E = \int_{-\infty}^{\infty} |X(f)|^2 df$$

You may use the fact that

$$\int_{-\infty}^{\infty} e^{j2\pi vt} dt = \delta(v)$$

4. Show that the overall system function $H(s)$ for the feedback system in FIGURE 1 is given by $H(s) = \frac{F(s)}{1-F(s)G(s)}$.
5. A signal $x(t)$ of finite energy is applied to a square-law device whose output is defined by $y(t) = x^2(t)$. The spectrum of $x(t)$ is limited to the frequency interval $-W \leq f \leq W$. Hence show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq 2W$.

Figure 1: Feedback system for problem 4.

