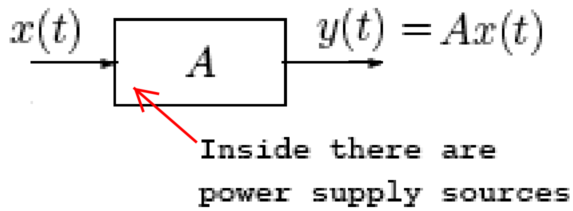
Operational Amplifiers and their use

What is an Operational Amplifier? or even What is an Amplifier?

An Amplifier is a device which enhances or increases the magnitude level of its input. If its signal input is $x(t)$, its signal output $y(t) = Ax(t)$ where A is the amplifier gain which is normally much greater than 1.



Any amplifier is an active circuit. It has certain other DC inputs to make it work. Such inputs are called bias voltages, they are not related at all to signal inputs and signal outputs. An analogy is your cell phone, to make the cell phone work, it requires a battery that needs to be charged now and then. The signal input is your talk or voice signal. Its output is some other signal that can be radiated into space.

What is an Operational Amplifier?

Operational Amplifier, or for short *op-amp*, is a very high gain amplifier. The gain is at least in the order of 1,000 or 10,000 or even more.

But, why the adjective ‘operational’?

To answer this, let us look at all the *basic operations* involved in mathematics and hence in its applications:

- Addition
- Subtraction
- Multiplication
- Division
- Differentiation
- Integration.

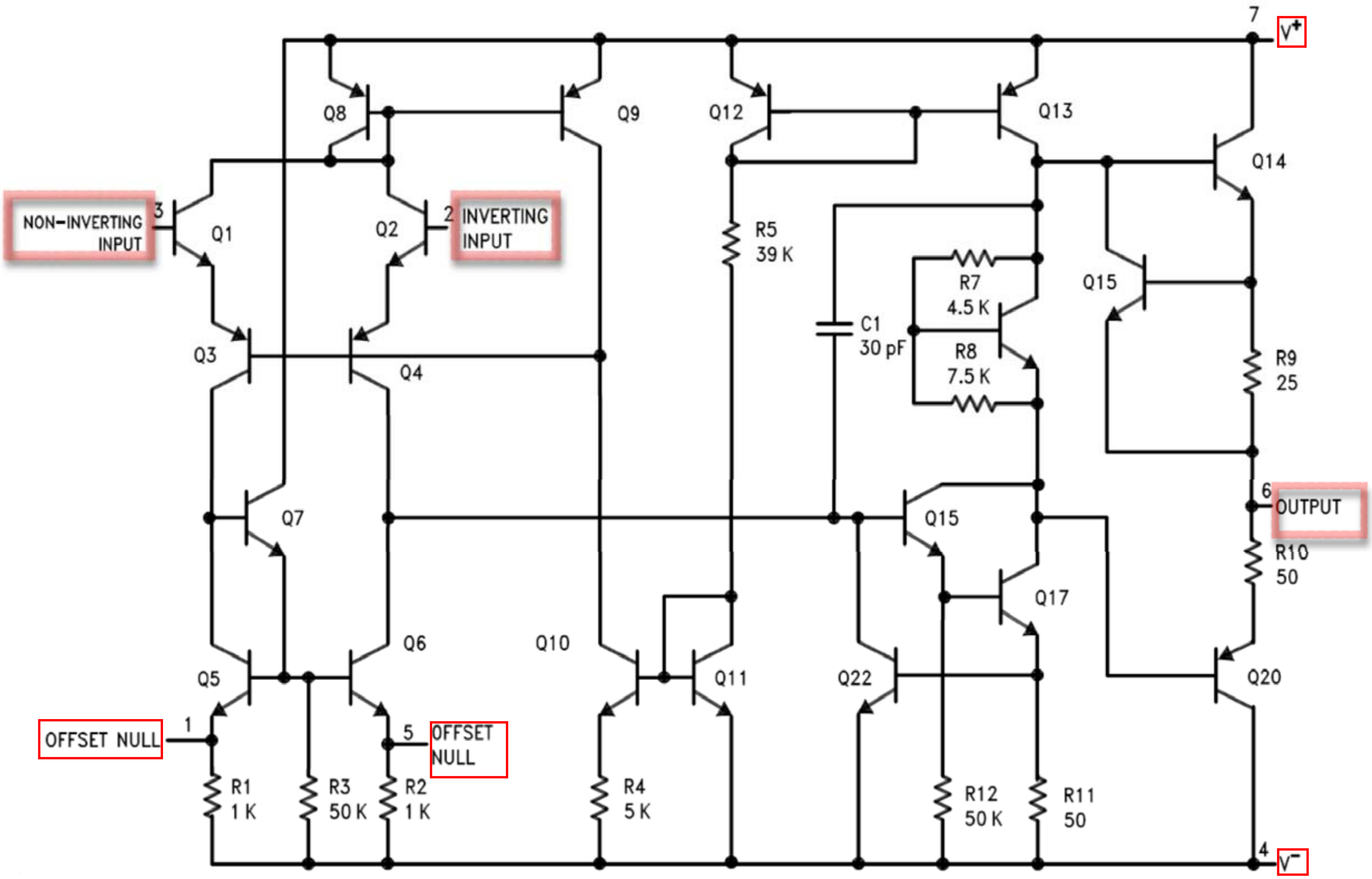
A very high gain amplifier can be used to construct circuits which are capable of performing each of these operations except possibly multiplication and division. Hence a high gain amplifier is termed as *operational amplifier*. We will build all such circuits.

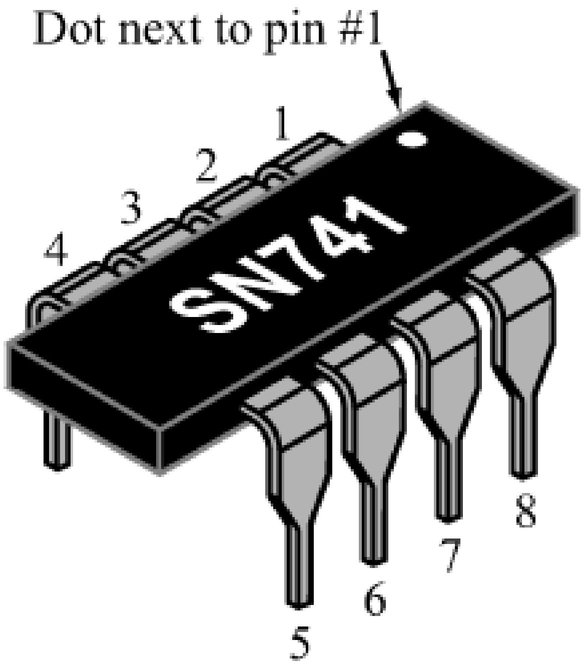
Op-Amp is a chip having many transistors (devices).

Inside The Op-Amp (741)

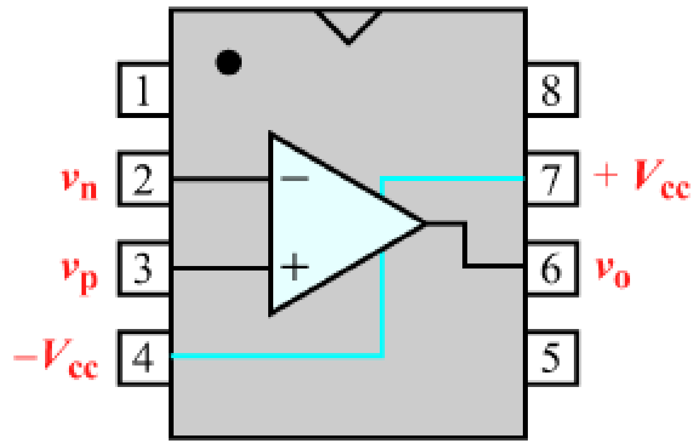
Seven terminals come out of the circuit.

Schematic Diagram

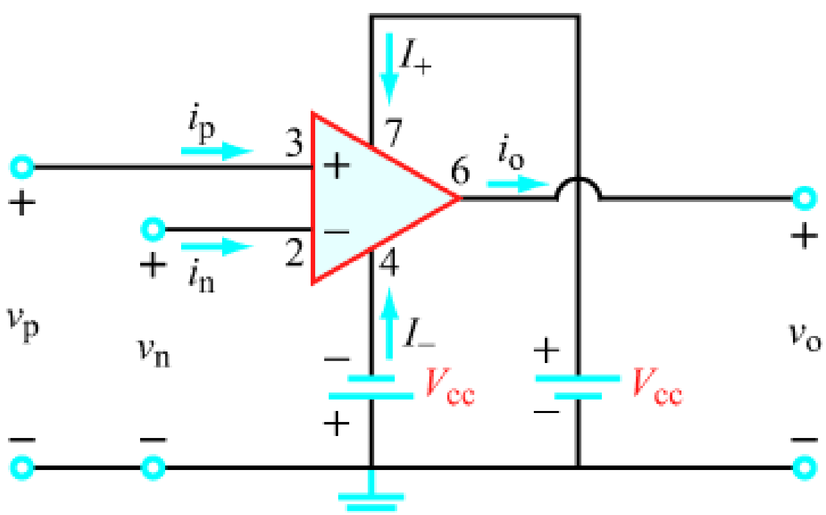


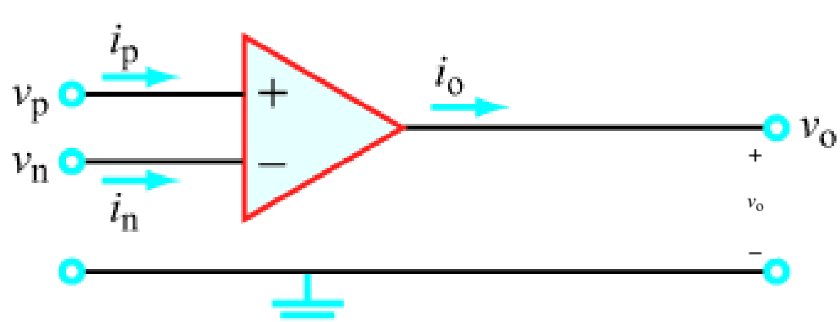


Typical op-amp package



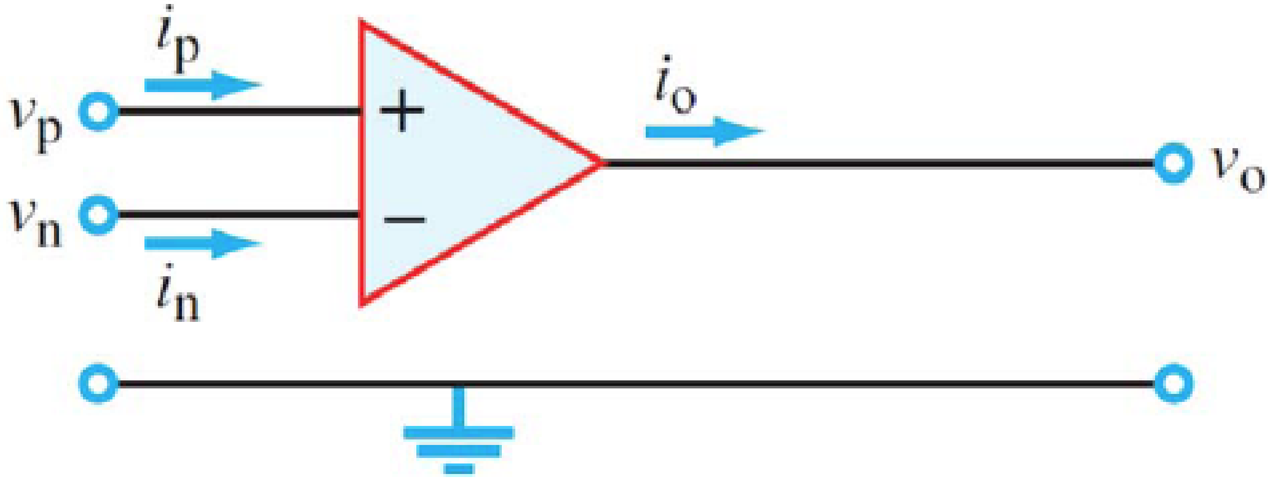
(b) Pin diagram





$$v_o = A(v_p - v_n)$$

(d) Op amp diagram without showing V_{cc} sources explicitly



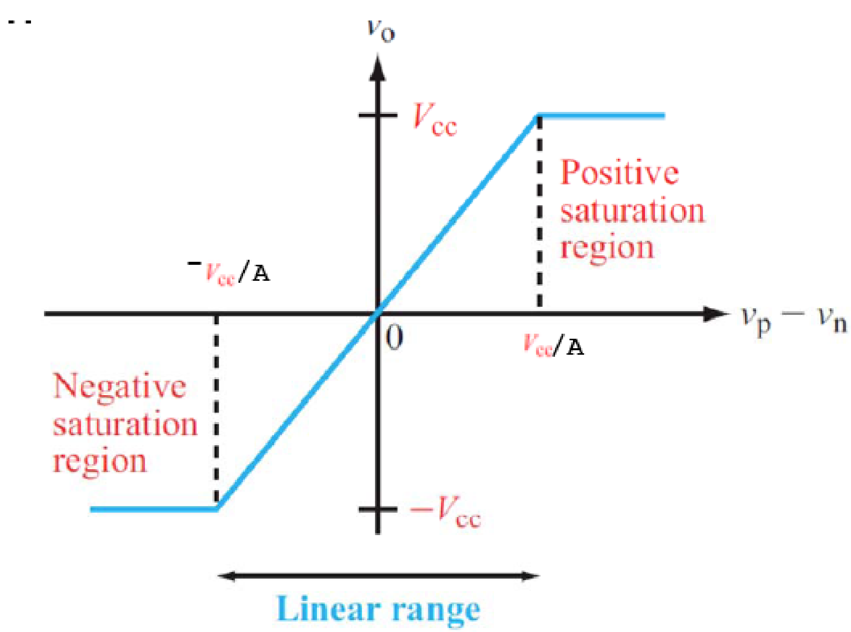
important aspect of op amp:

high voltage gain

$$v_o = A(v_p - v_n)$$

A is op-amp gain

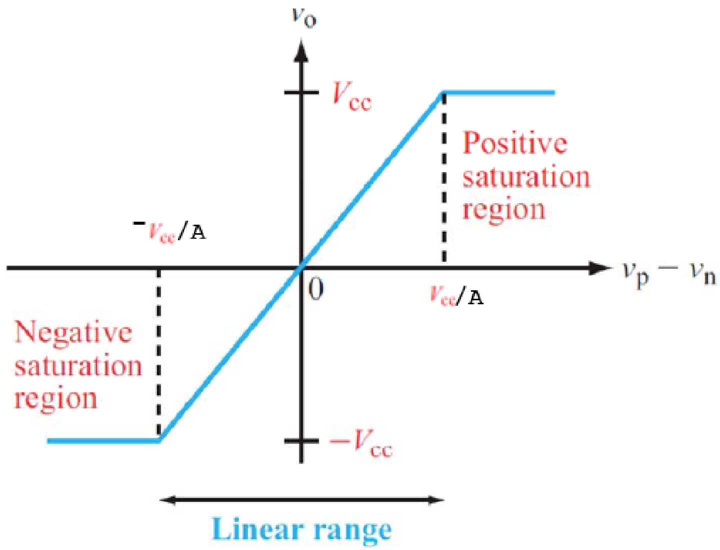
A is very large and is in excess of 10,000.

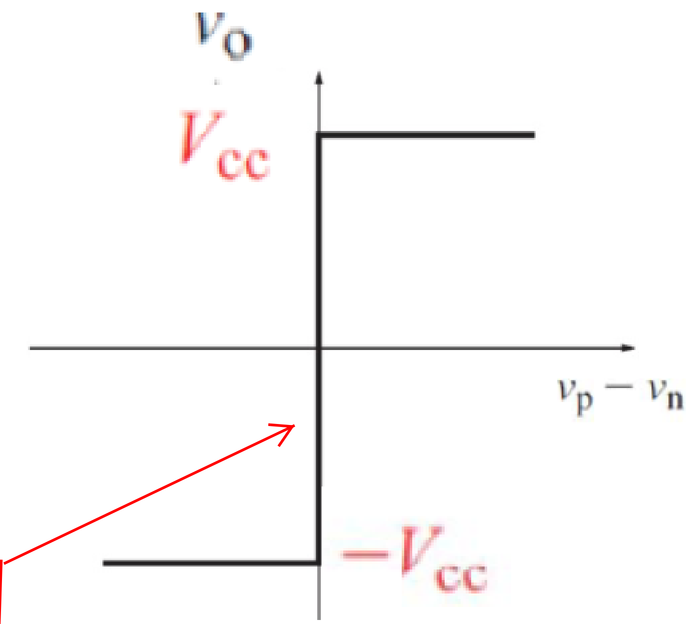


V_{cc} is in the order of 15 V.

Thus the linear region is very small.

V_{cc}/A is in the order of milli-volts.





The gain A is assumed to be high but finite.

The gain A is assumed to be infinite.

$$v_p = v_n$$

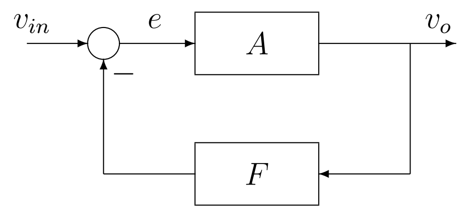
Note that the linear region is along the vertical axis.

How do we know where exactly the output is on the vertical axis?

We find other ways of determining the output.

Principles of Electrical Engineering I

A Block Diagram with a High-gain Amplifier



The gain of Op-Amp although high is not known precisely. Because of this, an Op-Amp is often embedded in a circuit in which there exists negative feedback from output to the input side. The negative feedback renders the over-all gain of the feedback circuit reliable. Let us illustrate the concept of negative feedback by considering a block-diagram in which A is the main amplifier gain and F is the feedback amplifier gain. We will first calculate the output signal v_0 in terms of the input signal v_{in}. To do so, we can easily write the equations of the given block-diagram,
the output of the main amplifier $= Ae$,
the output of the feedback amplifier $= Fv_0$,
the output of the summing device $= e = v_{in} - Fv_0$.

....Then came the morning of Tuesday, August 2, 1927, when the concept of negative feedback amplifier came to me in a flash while I was crossing the Hudson river on the Lacakawana Ferry, on my way to work.....

Harold S. Black

Thus, $v_0 = Ae = A(v_{in} - Fv_0)$. This yields,

$$\begin{aligned} v_0 &= \frac{A}{1+AF} v_{in} = \frac{1}{\frac{1}{A} + F} v_{in}, \quad \text{and} \\ e &= \frac{v_0}{A} = \frac{1}{1+AF} v_{in}. \end{aligned}$$

Next, we would like to find the limit of v_0 and e as the amplifier gain A tends to infinity. We can easily see that

- $v_0 \to \frac{v_{in}}{F}$ as $A \to \infty$,
- $e \to 0$ as $A \to \infty$.

We could have **easily obtained the above limiting behavior directly from the block-diagram by setting** $e = 0$, i.e. by setting $e = v_{in} - Fv_0 = 0$. This simplifies to $v_0 = \frac{v_{in}}{F}$.

Motivation to use negative feedback: You would like to build a device, say an amplifier. Components of the device especially when they are mass produced are never perfect. There always exists certain tolerance and hence certain unreliability. The unreliability increases as the number of components in the device increases. This begs the question: *What should be done to make the device reliable?* One obvious answer is to make the components very reliable; but this is expensive. How do we solve the riddle? In the case of designing amplifiers and the related devices, the concept of *high gain feedback* solves the riddle in a meaningful way. The idea here is simple. Suppose we can design a high-gain amplifier cheaply by utilizing the mass produced components having a large variance in their values. In such a case, although we cannot guarantee the exact value of gain, we can perhaps guarantee that it is greater than a fixed number, say greater than 10^4. As shown above, by using the feedback concept and using only a few reliable components, we can easily design a reliable amplifier. In the above feedback amplifier circuit, F is reliable and A is unreliable, however it can be guaranteed to have a high value, say greater than 10^4. Then the overall feedback amplifier circuit has a gain close to $\frac{1}{F}$, and as A tends to infinity, the overall gain tends to $\frac{1}{F}$.

Principles of Electrical Engineering I

A Realistic Circuit Model of an Op-Amp and an Ideal Op-Amp

A realistic Circuit Model of an Amplifier is shown in Figure 1 where
$A =$ Amplifier gain,
$R_{in} =$ Amplifier Input Resistance,
$R_o =$ Amplifier Output Resistance,
$v_1 = v_P - v_N =$ Amplifier Input,
$v_o =$ Amplifier Output Voltage,
$i_P = -i_N =$ Amplifier Input Current.
Here all the node voltages marked in circles are with respect to G.

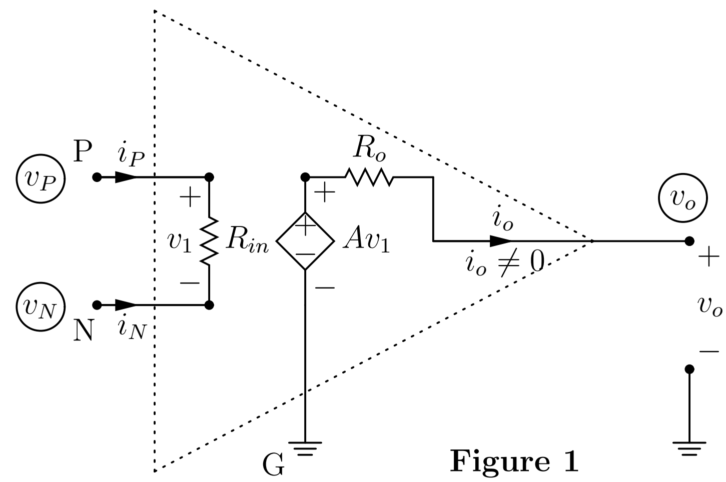


Figure 1

Any **Op-Amp circuit** (that is, a circuit in which Op-Amps are imbedded) can be analyzed by using the above circuit model for each Op-Amp. Such an analysis usually by **Node Voltage Method** is straightforward. However, we ask ourselves whether such an analysis **can be simplified**? If so under what conditions? It turns out that if the Op-Amp is close to what is called an ideal Op-Amp, we can simplify the analysis greatly.

An Op-Amp is said to be **good or ideal** if the following conditions hold:

Good Op-Amp:

- Op-Amp gain A is very high (in the order of 10^6),
- Op-Amp input resistance R_{in} is very high (in the order of $10^6\,\Omega$),
- Op-Amp output resistance R_o is very small (in the order of $1\,\Omega$).

Ideal Op-Amp:

- Op-Amp gain A is infinity,
- Op-Amp input resistance R_{in} is infinity,
- Op-Amp output resistance R_o is zero.

Let us assume that the Op-Amp is operating in its linear region. If an Op-Amp is **good or ideal** and if there exists **negative** feedback (that is, if there exists a branch connecting the output node of Op-Amp to the inverting input node of Op-Amp), the following conditions prevail:

- **Virtual Short:** Op-Amp gain A is high implies that the input voltage to the Op-Amp $v_P - v_N$ is negligibly small and can be assumed to be zero. In other words, the **input terminals of Op-Amp behave like being shorted although in reality they are not shorted**. Whenever there exists feedback, the output voltage can be computed by other means than by using $v_o = A(v_P - v_N)$. Note that v_o can still be non-zero when $A = \infty$ and when $v_P - v_N = 0$. That is, zero times infinity is not necessarily zero.

- **Virtual Open:** The input resistance R_{in} is high implies that the input currents to the Op-Amp i_P and i_N are negligibly small and can be assumed to be zero. In other words, the **input terminals of Op-Amp behave like being opened although in reality they are not open**.

The concepts of Virtual Short and Virtual Open are very useful in analyzing Op-Amp circuits as will be illustrated soon. Most of the Op-Amps commercially available can be assumed to be more or less ideal at least for the initial analysis of an Op-Amp circuit. For more accurate results one can utilize a more realistic circuit model for the Op-Amp as given in Figure 1. For even more precision work, there exist other Op-Amp circuit models in the literature.

Important note about i_o: The output current of Op-Amp is **NOT** zero. It depends on the load connected to Op-Amp. In view of this, it is always better to analyze circuits by writing **node equations at the input terminals N and P of Op-Amp**.

If Op-Amp is not saturated, we can analyze any circuit having an Op-Amp by replacing it by its equivalent circuit. Are there any simpler methods?

Often we need to determine vo. If Op-Amp is saturated, not much analysis needs to be done. We know in the saturated region, vo is either Vcc or -Vcc. All we need then is to determine whether vo is positive or negative.

In a linear region, vp-vn is at the most Vcc/A which is very small.
If there is a way to determine vo other than using vo=A(vp-vn),
we can neglect vp-vn. That is, we can assume v_p=vn.
This simplifies the analysis drastically as we shall see.

A feel for numbers – What is small? – What is large?

We are going to consider what can be called ideal Op-amps. To understand some of the assumptions we make in this regard, you need to have a feel for numbers, in particular what is small and what is large when compared to something else. Without comparison between numbers, nothing can be said to be small and nothing can be said to be large.
Let us first understand the following:

- The highest possible output of an Op-amp is V_{cc} which is of the order of 4 to 24 volts as prescribed by the manufacturer.
- All signals in an Op-amp circuit are at the most in the order of V_{cc}, some of the signals are much smaller than V_{cc} and thus are negligible in comparison with V_{cc}.
- When Op-amp gain is **large**, the linear region of its input-output characteristic is **small**.

To get a feel for numbers, let us consider an equation where A is large and is in the order of 10^7 and x, y and z are at the most 10,

$$Ax + y = z \quad \Rightarrow \quad x = \frac{z-y}{A}.$$

We note that x tends to zero as A tends to infinity. Another way of saying the same is, x is negligible in comparison with $z-y$, or $z-y$ is large in comparison with x.

Let us look at one of the equations of 'non-inverting' Op-amp circuit,

$$\frac{v_o - v_n}{R_1} + \frac{v_o - A(v_p - v_n)}{R_o} = 0.$$

This can be re-written as

$$A(v_p - v_n) = \frac{R_o}{R_1}(v_o - v_n) + v_0 \;\Rightarrow\; v_p - v_n = \frac{\frac{R_o}{R_1}(v_o - v_n) + v_0}{A}.$$

The right hand side of the right equation tends to zero as A tends to infinity. This implies that for a large A, $v_p - v_n$ is small in comparison with other signals. For $A = \infty$, we have $v_p - v_n = 0$, that is $v_p = v_n$. The input current $i_p = \frac{v_p - v_n}{R_i}$ is also small when $v_p - v_n$ is small.

The above discussion implies that for the 'non-inverting' Op-amp circuit we have considered, the input to the Op-amp, namely $v_p - v_n$, is small or negligible. Also, input current i_p to the Op-amp is small or negligible as well. In general, whenever the output of Op-amp is fed back to its input side, we have

- input voltage to Op-amp is negligible **(virtual open)**.
- input current to Op-amp is negligible **(virtual short)**.

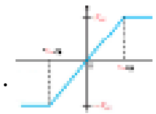
Summary

- When does life begin?
  your choice: $\rightarrow$ *at conception, at birth, when you get driver license*

- When does EE life begin?
  there is only one choice: $\rightarrow$ *when you learn circuit analysis and logic design*

- Op-Amp is a device having a very large gain A from input to output.

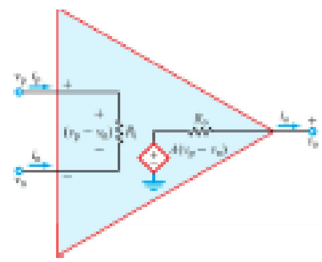
- A typical input-output characteristic of an Op-Amp is as shown where the x-axis is $v_p - v_n$ and the y-axis is v_o.



saturation level is Vcc

- Op-Amp circuits (those circuits in which Op-Amps are imbedded) are popular as they can do several operations such as: addition, subtraction, differentiation, and integration of signals.

- Op-Amp can be modeled at its input side by a resistance R_i, and on its output side by a dependent source $A(v_p - v_n)$ in series with a resistance R_o.



- Op-Amp gain A is normally very high (10^4 to 10^8) but unreliable, input resistance R_i is very high as well (10^{10} to $10^{13}\Omega$), while the output resistance R_o is small (1 to 100Ω).

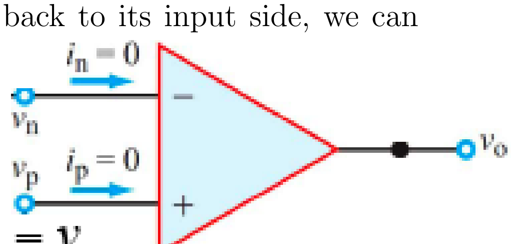
- One method of analyzing Op-Amp circuits is by replacing an Op-Amp with its equivalent model. Node Voltage Method is best suited for Op-Amp circuit analysis. Such an analysis of an Op-Amp circuit using equivalent model for an Op-Amp yields whatever is needed. However, it presents a lot of algebra, and a *quick grasp of what a given circuit does is not feasible.*

- In order to have a *quick grasp*, one can simplify the analysis by making some assumptions or by neglecting certain small numbers, **just for doing analysis**.

- In an Op-Amp circuit, if Op-Amp is saturated, the output of it is either the supply voltage V_{cc} or $-V_{cc}$. So, the analysis gets simplified. If the Op-Amp is in its linear region, the input to Op-Amp which is $v_p - v_n$ is in the range $-\frac{V_{cc}}{A}$ to $\frac{V_{cc}}{A}$. Note that this range is very small for a very large A. If A is in the range 10^4 to 10^8, $v_p - v_n$ is in the range 2 milli volts to 2 micro volts. **As such, whenever there is feedback from output side to the input side via a circuit component, one can neglect $v_p - v_n$ for analysis purpose. That is, for analysis purpose, v_p and v_n can be assumed to have the same values. Also, $i_p = -i_n$ is then negligible because $i_p = \frac{v_p - v_n}{R_i}$; note that $v_p - v_n$ is negligible and moreover R_i is large and thus i_p is even more negligible.**

- **Note that even when $v_p - v_n$ is negligible, the output of an Op-Amp $v_o = A(v_p - v_n)$ is not negligible; A number (very small)$\times$(very large) is not negligible. Also, the output current i_o of an Op-Amp is not negligible either.**

- In general, whenever the output of an Op-amp is fed back to its input side, we can assume the following, **just for doing analysis**.
  - input voltage to Op-amp is zero **(virtual open)**.
  - input current to Op-amp is zero **(virtual short)**.



$v_n = v_p$

We recognize that vp-vn is negligibly small, and often can be neglected in analyzing a circuit in which Op-Amps are embedded (such circuits are called Op-Amp circuits). Once vp-vn is neglected, that is once we assume vp=vn, we get ip=in=0. When we make these approximations, the Op-Amp is said to be ideal. Ideal analysis of an Op-Amp circuit is simple and straightforward as shown in the following pages.

Characteristics of the ideal op-amp model

Ideal Op Amp	
• Current constraint	$i_p = i_n = 0$
• Voltage constraint	$v_p = v_n$
• $A = \infty$ $\quad R_i = \infty$	$R_o = 0$

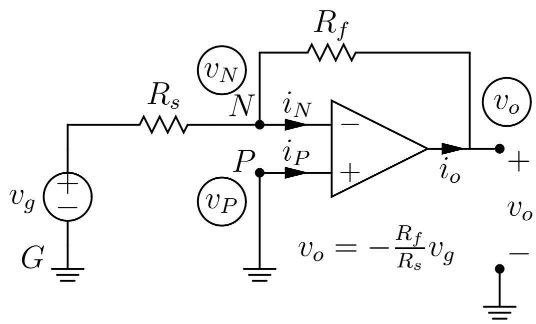
Virtual Open

ONLY AT THE INPUT SIDE

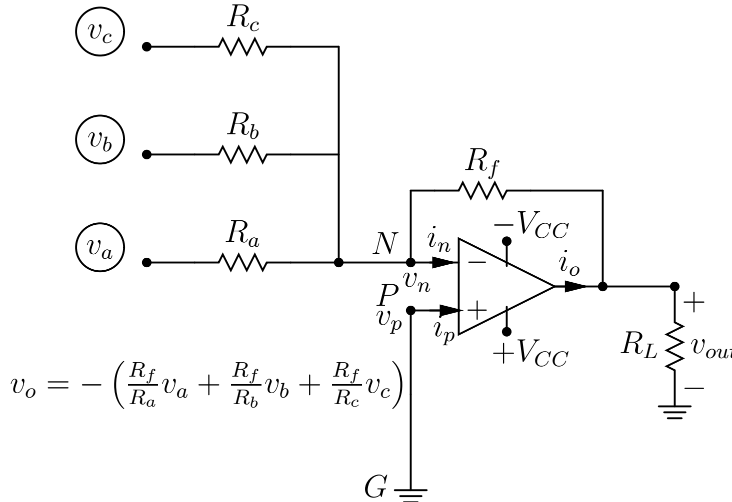
Virtual Short

THE OUTPUT SIDE OF OP-AMP:
The output current of OP-Amp is non-zero.as is output voltage.

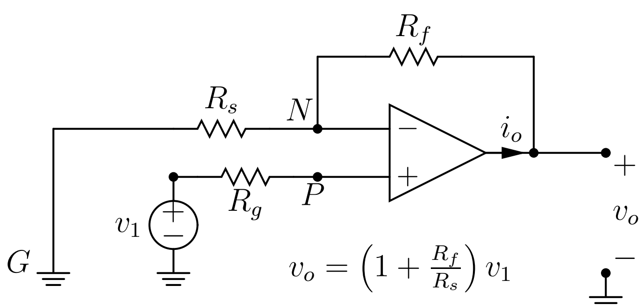
Principles of Electrical Engineering I
Certain Standard Op-Amp Circuits



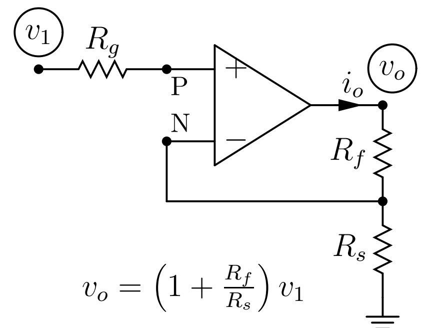
An inverting amplifier circuit



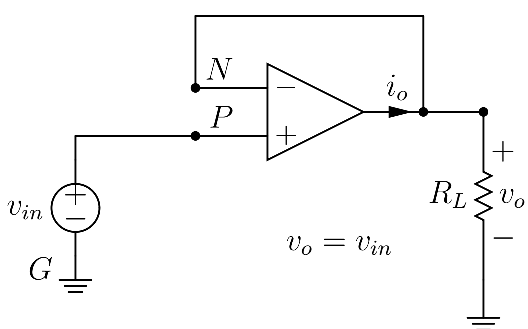
A summing amplifier circuit



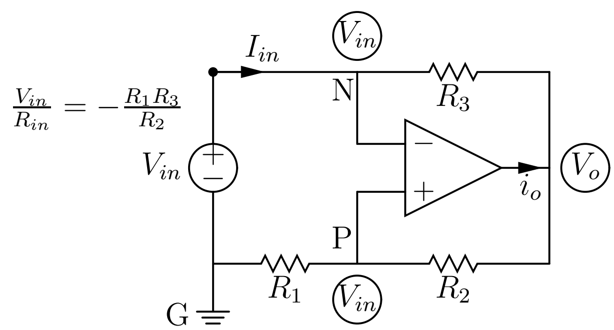
A non-inverting amplifier circuit



Non-inverting amplifier (re-drawn)



A voltage follower (buffer) circuit

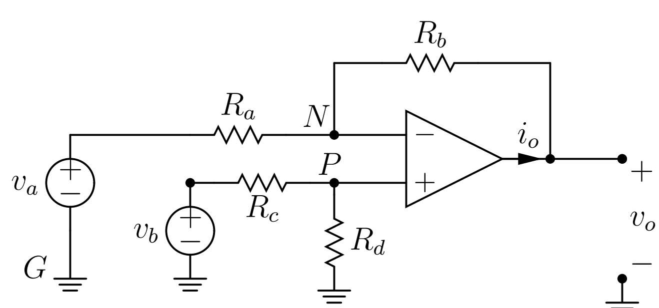


Negative resistance converter

$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$$

If $\frac{R_a}{R_b} = \frac{R_c}{R_d}$, then

$$v_o = \frac{R_b}{R_a}\left(v_b - v_a\right).$$



A Differential amplifier circuit

Principles of Electrical Engineering I

Analysis of Non-ideal Inverting Op-Amp Circuit

Consider the inverting Op-Amp circuit shown in Figure 1. Let $R_f = 200\,K\Omega$ and $R_s = 100\,K\Omega$. Let the input signal v_g be 1 V. Determine the output signal v_o. At first, analyze the circuit assuming that the **Op-Amp is ideal**, that is assume that the input terminals of Op-Amp behave **as virtually shorted as well as virtually open**. This implies that $v_P = v_N$ and $i_P = i_N = 0$.

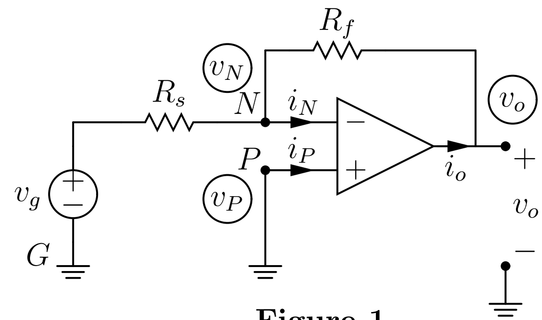


Figure 1

Solution: From the given circuit configuration, we note that $v_p = 0$ and thus from the concept of virtual short, $v_N = 0$. Next, from the concept of virtual open, we note that $i_N = 0$. Then, the node equation at N yields

$$\frac{v_o}{200} + \frac{v_g}{100} = 0 \quad \Rightarrow \quad v_o = -2v_g = -2.0\,V.$$

Let us next analyze the circuit of Figure 1 assuming that the **Op-Amp is non-ideal**. Let the Op-Amp has the following worst possible parameters:
The gain of Op-Amp is 1000,
The input resistance R_{in} is $100\,K\Omega$, and
The output resistance R_o is $10\,K\Omega$.

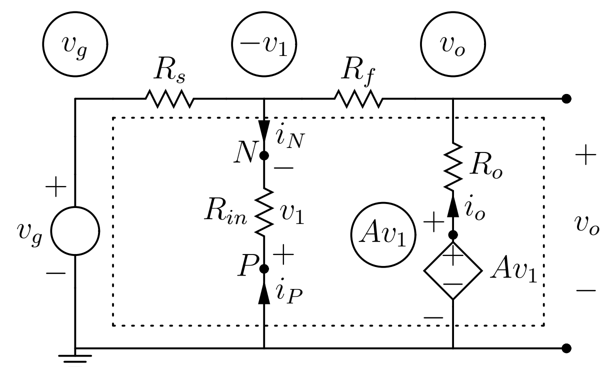


Figure 1a

Solution: The equivalent circuit of the given Op-Amp circuit is drawn in Figure 1a. The dotted box shows the model of Op-Amp. Here all the node voltages marked in circles are with respect to G. We note that $v_P = 0$ and $v_N = -v_1$. Then the pertinent equations are

$$\begin{aligned}
\frac{v_o + v_1}{200} + \frac{v_o - 10^3 v_1}{10} = 0 \quad &\Rightarrow \quad v_1 \approx (1.05)10^{-3} v_o, \\
\frac{1 + v_1}{100} + \frac{v_1}{100} + \frac{v_o + v_1}{200} = 0, \quad &\Rightarrow \quad v_o \approx -1.98955\,V, \\
v_1 \approx (1.05)10^{-3} v_o \quad &\Rightarrow \quad v_1 \approx -(2.08903) \times 10^{-3}\,V. \\
i_p = \frac{v_1}{100K} \quad &\Rightarrow \quad i_p \approx -(2.08903) \times 10^{-8}\,A.
\end{aligned}$$

(In the above, the second equation is solved for v_o after substituting for $v_1 \approx (1.05)10^{-3} v_o$.) The above algebra indicates that even for the worst possible values of $R_{in} = 100\,K\Omega$, $R_o = 10\,K\Omega$, and $A = 1000$, the Op-Amp behaves almost like an ideal one. **Determine i_o as Home-Work by both ideal and non-ideal analysis**.

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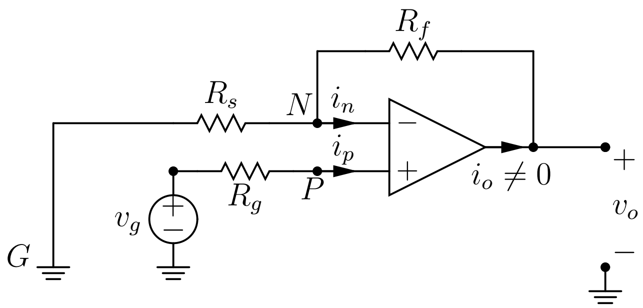
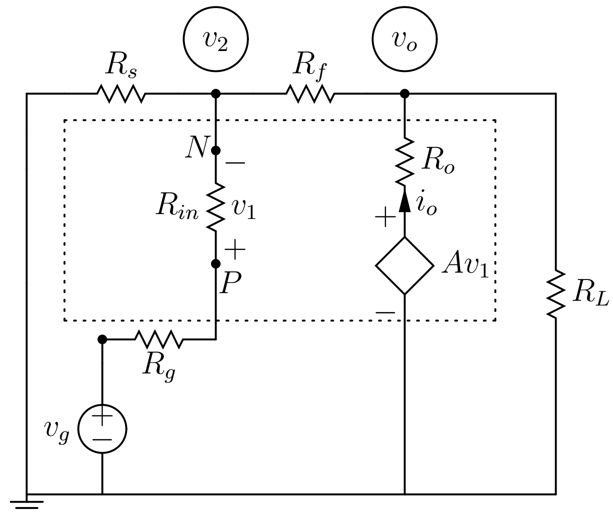
Non-inverting Op-Amp



Consider the non-ideal non-inverting Op-Amp circuit. Let $R_f = 160K\Omega$, $R_s = 80K\Omega$, $R_g = 200K\Omega$, and $R_L = 8K\Omega$. The Op-Amp has an input resistance of $800K\Omega$, and an output resistance of $10K\Omega$, while its gain is mere $20,000$. Determine, v_o, v_n, and v_p when $v_g = 1$V. The voltages v_n and v_p must be calculated to an accuracy of at least 10^{-4}V.



Node voltages are as marked. We note that

$$v_1 = R_{in}\frac{v_g - v_2}{R_{in} + R_g} = 0.8(v_g - v_2).$$

We can write the KCL equations at the nodes marked by v_0 and v_2. These are respectively given by (simplifications are made after substituting for v_1),

$$\frac{v_0 - Av_1}{R_o} + \frac{v_o - v_2}{R_f} + \frac{v_o}{R_L} = 0 \quad \Rightarrow \quad (256000 - 1)v_2 + 37v_o = 256000v_g,$$

$$\frac{v_2 - v_g}{R_{in} + R_g} + \frac{v_2 - v_o}{R_f} + \frac{v_2}{R_s} = 0 \quad \Rightarrow \quad 19.75v_2 - 6.25v_o = v_g.$$

The solution of the above equations yields,

$$v_o = 2.9986v_g, \quad v_2 = 0.999571v_g.$$

We note that

$$v_n = v_2 = 0.999571v_g, \quad v_1 = v_p - v_n = 0.8(v_g - v_2) = (0.3432)10^{-3}v_g, \quad v_p = v_n + v_1 = 0.999914v_g.$$

We also note that

$$i_p = -i_n = \frac{v_1}{R_{in}} = (0.429)10^{-9}v_g.$$

The above analysis confirms that $v_p \approx v_n$ and $i_p = -i_n \approx 0$.

Determine i_o by both ideal and non-ideal analysis as Home-Work.

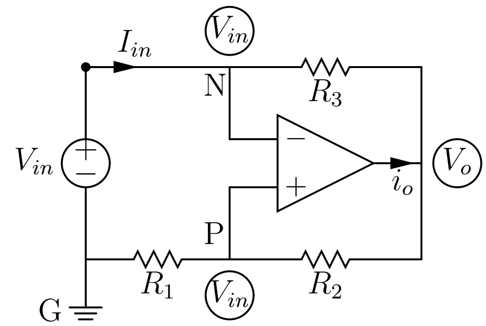
Ideal Analysis: Virtual open implies that $i_p = 0$. This in turn implies that $v_p = v_g$. Also, virtual short implies that $v_n = v_p = v_g$. Now, by writing the node equation at N, we get

$$\frac{v_g}{R_s} + \frac{v_g - v_o}{R_f} = 0 \text{ Note that } i_n = 0 \text{ because of Virtual open.}$$

By simplifying the above equation, we get $v_o = \left(1 + \frac{R_f}{R_s}\right)v_g = 3v_g$.

Negative Resistance Converter Circuit

Determine the input resistance R_{in} of the circuit shown.
Obviously, in the absence of Op-Amp, $R_{in} = R_1 + R_2 + R_3$. However, because of the Op-Amp, the node voltage with respect to the ground at P is forced to equal the input voltage V_{in}, while the other node voltage at the output of Op-Amp is different from V_{in}. This changes the input resistance drastically. In fact, the input resistance turns out to be negative. In other words, the output of Op-Amp becomes higher than the input signal and thus Op-Amp pumps current into the input signal via the resistance R_3 as discussed below.



Assuming that the Op-Amp is ideal, the node voltages are marked as shown. There is one unknown node voltage V_o (outputs of Op-Amp). It is clear that the appropriate node where we can easily write the node equation (without introducing additional unknowns) is node P as shown. The node equation at P is given by

$$\frac{V_{in} - V_0}{R_2} + \frac{V_{in}}{R_1} = 0 \quad \Rightarrow \quad V_o = V_{in}\left[1 + \frac{R_2}{R_1}\right].$$

The above equation defines V_o in terms of V_{in}. Note that $V_o > V_{in}$.
The node equation at N gives us

$$I_{in} = \frac{V_{in} - V_o}{R_3}.$$

Substituting for V_o and simplifying, we get

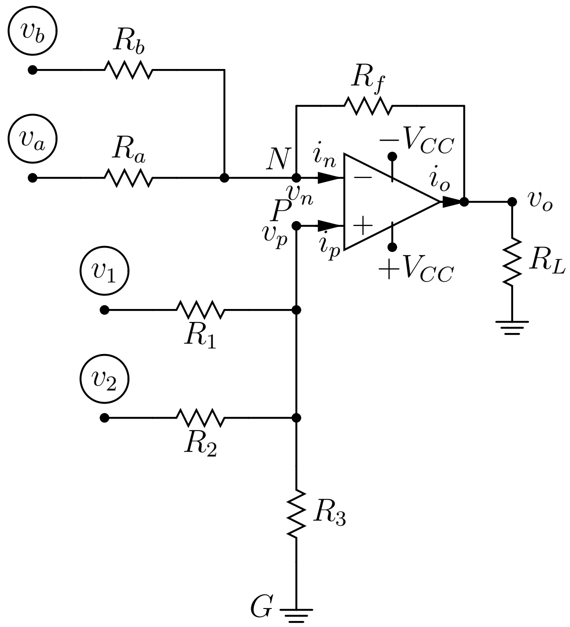
$$I_{in} = \frac{V_{in} - V_o}{R_3} = -V_{in}\frac{R_2}{R_1 R_3}.$$

Note that I_{in} is negative, and thus

$$R_{in} = \frac{V_{in}}{I_{in}} = -\frac{R_1 R_3}{R_2}.$$

Determine i_o as Home-Work.

332:221 Principles of Electrical Engineering I
An Op-Amp Circuit



We present here an op-amp circuit which is a generalization of inverting amplifier, non-inverting amplifier, adder, and differential amplifier circuits.

The fundamental equations of the **ideal Op-Amp** are

$$v_p = v_n \quad \textbf{(Virtual Short)} \quad \textbf{and}$$

$$i_p = -i_n = 0 \quad \textbf{(Virtual Open)}.$$

The node equations at N and P respectively are

$$\frac{v_n - v_a}{R_a} + \frac{v_n - v_b}{R_b} + \frac{v_n - v_o}{R_f} = 0, \qquad (1)$$

$$\frac{v_p - v_1}{R_1} + \frac{v_p - v_2}{R_2} + \frac{v_p}{R_3} = 0. \qquad (2)$$

For the case when $R_a = R_b = R_f$ and $R_1 = R_2 = R_3$, the above equations simplify to $3v_n = v_a + v_b + v_o$ and $3v_p = v_1 + v_2$. These equations together imply that

$$v_o = v_1 + v_2 - (v_a + v_b).$$

For the general case, simplifying (2) and solving for v_p we get

$$v_p \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] = \frac{v_1}{R_1} + \frac{v_2}{R_2} \quad \Rightarrow \quad v_p = \frac{R_3(R_2 v_1 + R_1 v_2)}{R_1R_2 + R_2R_3 + R_3R_1}.$$

Similarly, simplifying (1) and solving for v_o, we get

$$v_o = R_f \left[\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_f}\right] v_n - R_f \left[\frac{v_a}{R_a} + \frac{v_b}{R_b}\right]$$

This simplifies to $$v_o = \frac{R_aR_b + R_bR_f + R_fR_a}{R_aR_b} v_n - \frac{R_f(R_b v_a + R_a v_b)}{R_aR_b}.$$

Substituting for $v_n = v_p$, and simplifying, we get

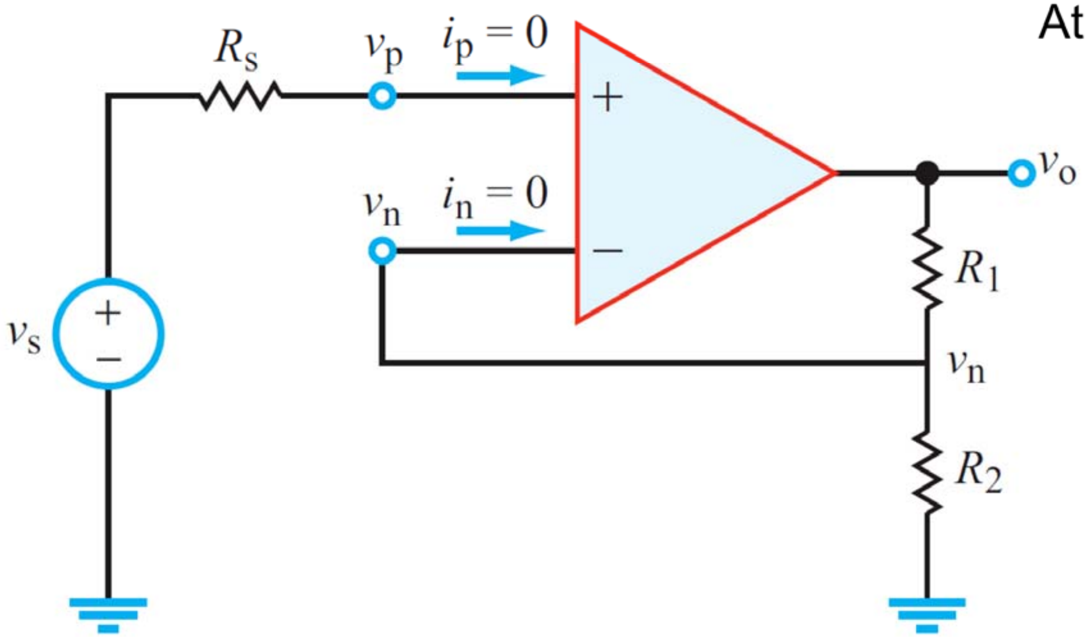
$$v_o = \frac{R_3(R_aR_b + R_bR_f + R_fR_a)(R_2v_1 + R_1v_2)}{R_aR_b(R_1R_2 + R_2R_3 + R_3R_1)} - \frac{R_f(R_b v_a + R_a v_b)}{R_aR_b}.$$

The above result is true whenever Op-Amp is working in its linear region. That is, whenever

$$-V_{CC} < v_o < V_{CC}.$$

Home-Work: Assume that $R_a = R_b = R_f$ and $R_1 = R_2 = R_3$, and determine the output current i_o.

Noninverting Amplifier

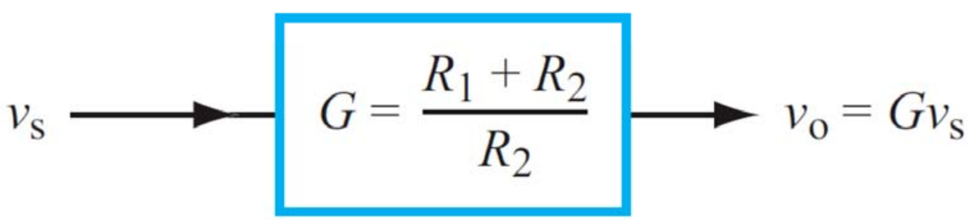


At node v_n

$$\frac{v_n - v_o}{R_1} + \frac{v_n}{R_2} = 0$$

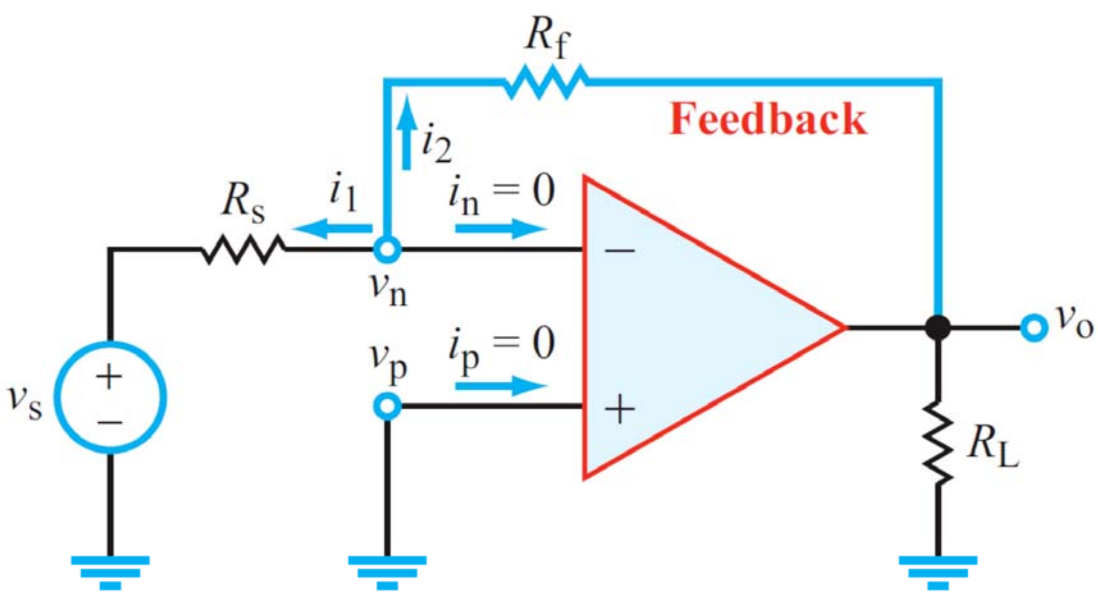
$$v_n = v_p = v_s$$

$$\boxed{v_o = \frac{R_1 + R_2}{R_2} v_s}$$



$$\boxed{v_o\,(\mathrm{max}) = V_{cc}}$$

Inverting Amplifier

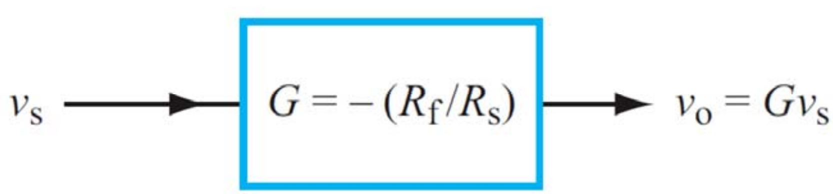


At node v_n

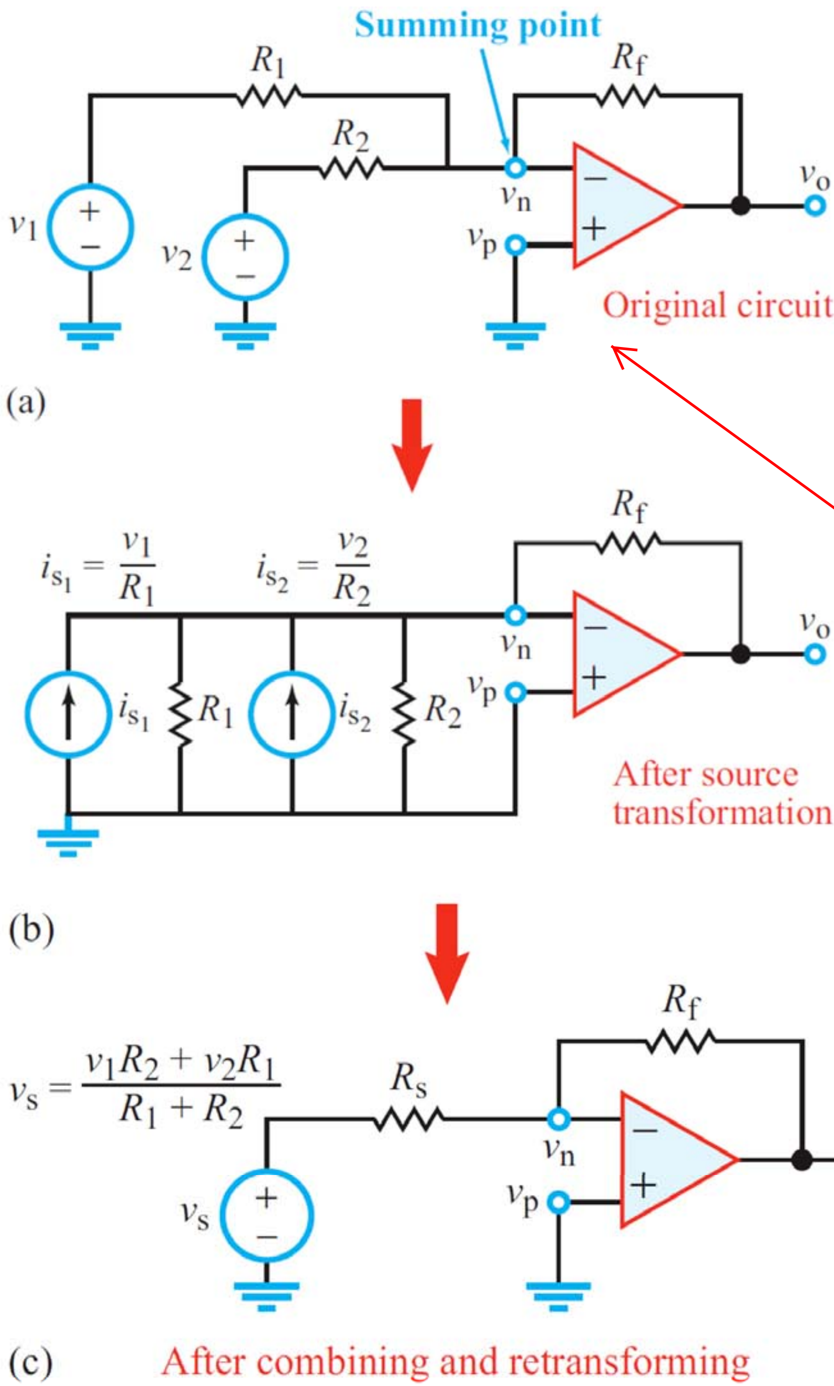
$$\frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} + i_n = 0.$$

$$v_n = v_p = 0$$

$$G = \frac{v_o}{v_s} = -\left(\frac{R_f}{R_s}\right).$$



Summing



We first observe that

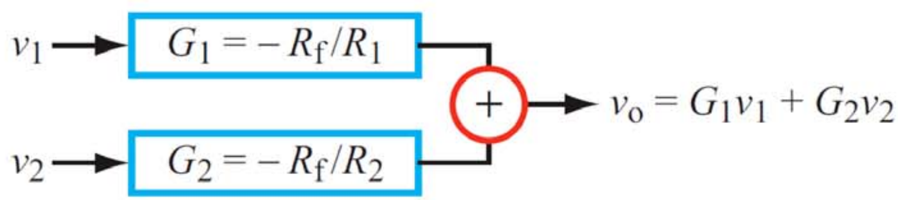
$v_n = v_p = 0$ for an ideal OP-Amp.

We can write a node equation at v_n,

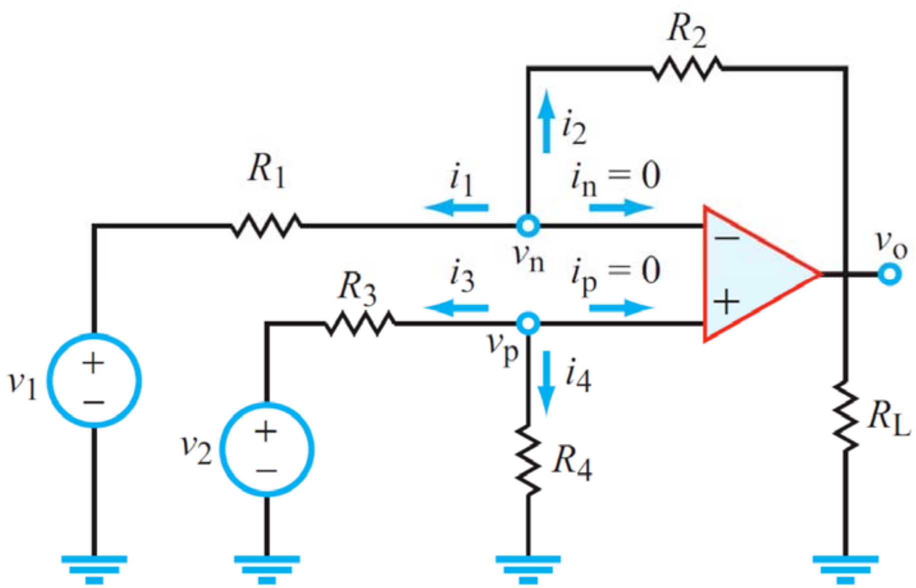
$$\frac{0 - v_1}{R_1} + \frac{0 - v_2}{R_2} + \frac{0 - v_o}{R_f} = 0.$$

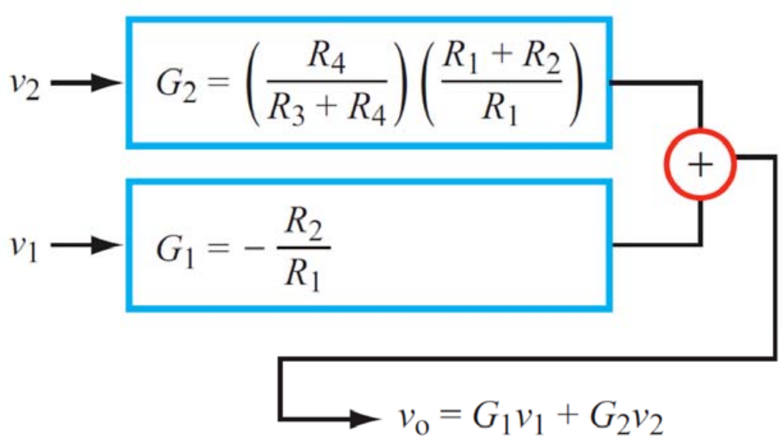
The above equation simplifies to

$$v_o = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2.$$



Difference Amplifier





At node v_n

$$\frac{v_n - v_1}{R_1} + \frac{v_n - v_o}{R_2} + i_n = 0. \tag{4.38}$$

At v_p, $i_3 + i_4 + i_p = 0$, or

$$\frac{v_p - v_2}{R_3} + \frac{v_p}{R_4} + i_p = 0. \tag{4.39}$$

Upon imposing the ideal op-amp constraints $i_p = i_n = 0$ and $v_p = v_n$, we end up with

$$v_o = \left[\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)\right] v_2 - \left(\frac{R_2}{R_1}\right) v_1, \tag{4.40}$$

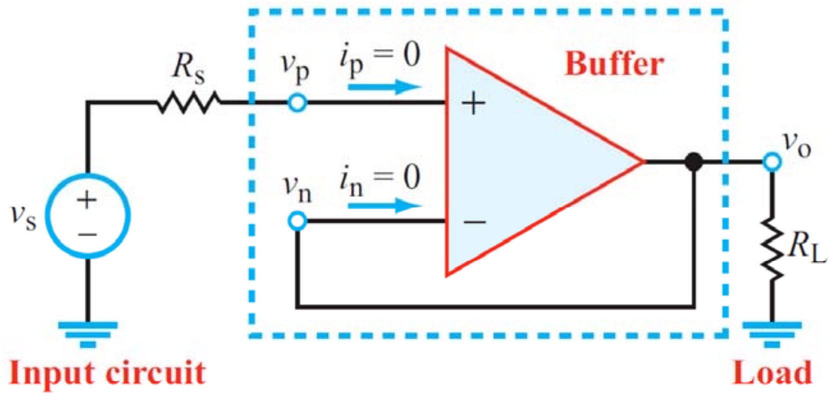
Note negative gain of channel 1

Voltage Follower

"Buffers" Sections of Circuit

$$v_o = \frac{v_s R_L}{R_s + R_L} \qquad \text{(without voltage follower)}$$

v_o depends on both input and load resistors



$$v_o = v_p = v_s \qquad \text{(with voltage follower)}$$

v_o is immune to input and load resist

What is the op amp doing?

Principles of Electrical Engineering I
Quiz Fall 1999

Student's name in capital letters:
Last four digits of SSN:

Operational Amplifier Circuit

Assume that the Op-Amps are ideal in the circuit shown on the right. Determine the output voltage v_o if the input voltage $v_g = 1$ V.

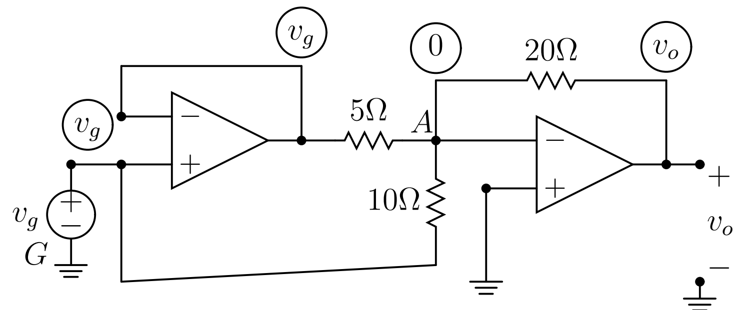


Figure1

Mark the node voltages with respect to G as shown. Write the node equation at the point A as

$$\frac{0 - v_g}{5} + \frac{0 - v_g}{10} + \frac{0 - v_o}{20} = 0.$$

By solving the above equation, we get

$$v_o = -6v_g = -6\,V.$$

Principles of Electrical Engineering I
Instrumentation Amplifier Circuit

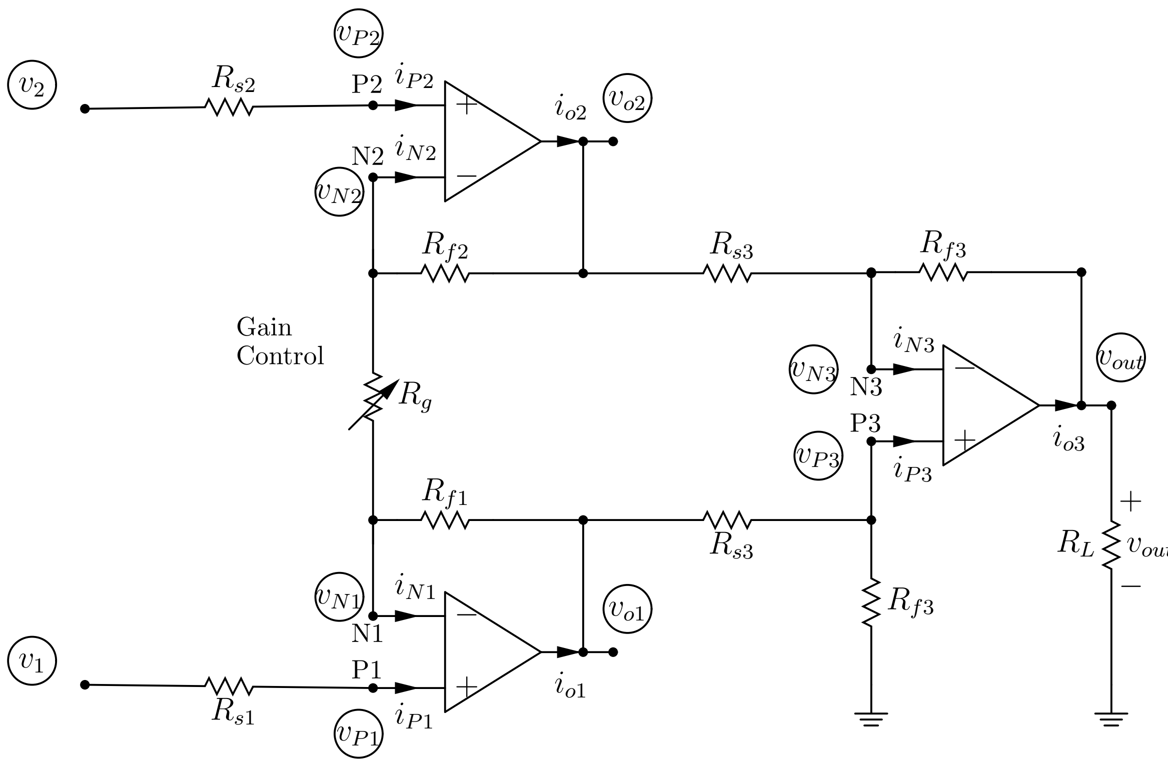
A differential amplifier circuit which is commonly used in instrumentation circuits is shown below. For simplicity, assume that

$$R_{s1} = R_{s2} = R_{s3} = R_{f1} = R_{f2} = R_{f3} = R.$$

Also, assume that all the three Op-Amps are ideal. Let v_1 and v_2 be the input signals as marked. Show that the output voltage v_{out} is given by

$$v_{out} = K(v_1 - v_2),$$

where K is a constant. Determine the value of K in terms of R and R_g.



All appropriate node voltages are marked in circles next to each node. We need to write the necessary equations to determine them. See next page which guides writing systematically the necessary equations.

Step 1: Since all Op-Amps are ideal, we can easily answer the following questions: What are the values of i_{P1}, i_{N1}, i_{P2}, i_{N2}, i_{P3}, and i_{N3} ?

$$i_{P1} = i_{N1} = 0, \quad i_{P2} = i_{N2} = 0, \quad i_{P3} = i_{N3} = 0.$$

How are the pair v_{P1} and v_{N1} related? Similarly, how are the pair v_{P2} and v_{N2} as well as the pair v_{P3} and v_{N3} related?

$$v_{P1} = v_{N1}, \quad v_{P2} = v_{N2}, \quad v_{P3} = v_{N3}.$$

Knowing the value of i_{P1}, how is v_{P1} related to v_1 ?

$$v_{P1} = v_1.$$

Similarly, knowing the value of i_{P2}, how is v_{P2} related to v_2 ?

$$v_{P2} = v_2.$$

Step 2: Write a node equation at $N1$.

$$\frac{v_1 - v_{o1}}{R} + \frac{v_1 - v_2}{R_g} = 0 \quad \Rightarrow \quad v_{o1} = \left(1 + \frac{R}{R_g}\right) v_1 - \frac{R}{R_g} v_2.$$

Step 3: Write a node equation at $N2$.

$$\frac{v_2 - v_{o2}}{R} + \frac{v_2 - v_1}{R_g} = 0 \quad \Rightarrow \quad v_{o2} = \left(1 + \frac{R}{R_g}\right) v_2 - \frac{R}{R_g} v_1.$$

Step 4: Write a node equation at $N3$ and another at $P3$.

$$\frac{v_{P3} - v_{o1}}{R} + \frac{v_{P3}}{R} = 0 \quad \Rightarrow \quad v_{P3} = 0.5 v_{o1}.$$

$$\frac{v_{N3} - v_{o2}}{R} + \frac{v_{N3} - v_{out}}{R} = 0 \quad \Rightarrow \quad v_{out} = 2v_{N3} - v_{o2}.$$

Step 5: Solve the above equations to determine v_{out} in terms of v_1 and v_2.
We note that

$$\begin{aligned} v_{out} &= 2v_{N3} - v_{o2} = 2v_{P3} - v_{o2} = v_{o1} - v_{o2} \\ &= \left(1 + \frac{R}{R_g}\right) v_1 - \frac{R}{R_g} v_2 - \left(1 + \frac{R}{R_g}\right) v_2 + \frac{R}{R_g} v_1 \\ &= \left(1 + \frac{2R}{R_g}\right) (v_1 - v_2). \end{aligned}$$

HW from Nilsson and Riedel 8th and 9th editions

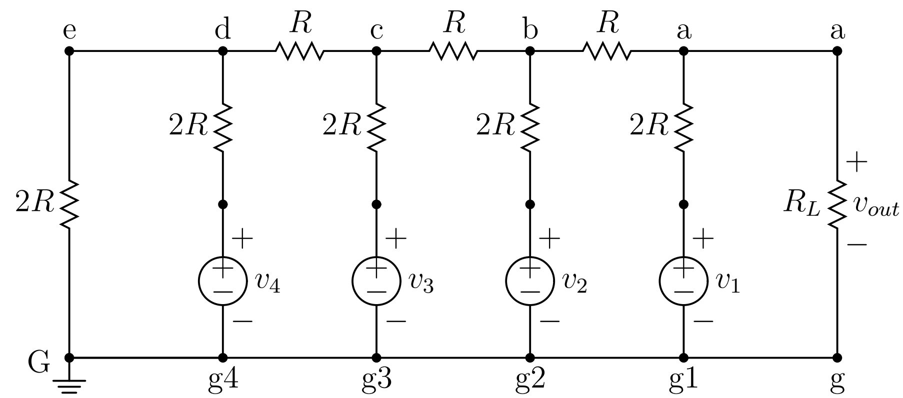
Nilsson and Riedel 8th edition:
5.2, 5.5, 5.18, 5.25

Nilsson and Riedel 9th edition:
5.3, 5.6, 5.18, 5.28

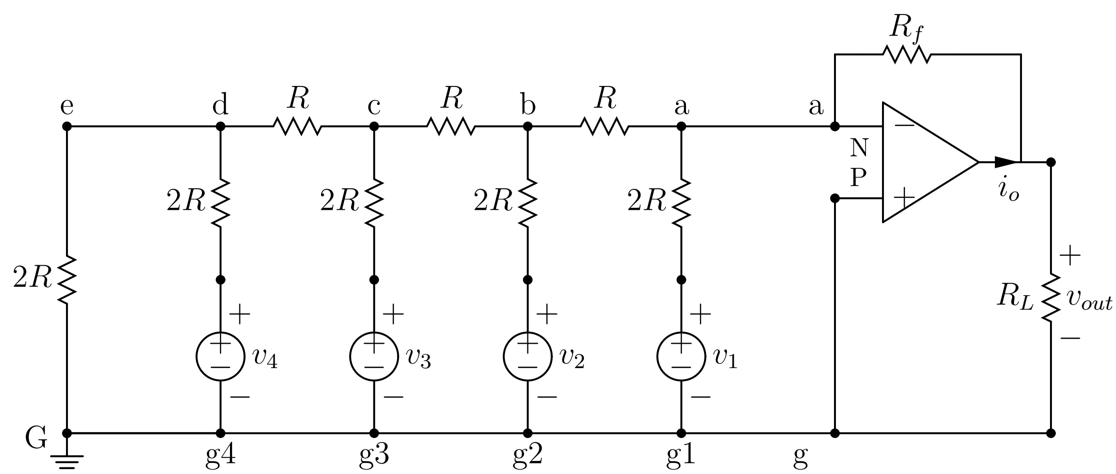
Name in CAPITAL LETTERS: **HW: DAC with Op-Amp**
LAST FOUR DIGITS OF ID NUMBER:

332:221 Principles of Electrical Engineering I

Theme Example – DAC– loading effect removed by adding an op-amp circuit: HW, collected and graded



The above DAC circuit with load has a problem. The output voltage v_{out} depends on the load R_L. To reduce the effect of load, we can add an inverting op-amp circuit as shown below.



Assuming that the op-amp is ideal, determine the feedback resistance R_f in terms of R such that the output voltage v_{out} is given by

$$v_{out} = -8v_1 - 4v_2 - 2v_3 - v_4.$$

Hint: The Thevenin equivalent of the DAC circuit to the left of terminals 'a' and 'g' is given by

$$v_{Th} = \tfrac{1}{2}v_1 + \tfrac{1}{4}v_2 + \tfrac{1}{8}v_3 + \tfrac{1}{16}v_4,$$

and

$$R_{Th} = R.$$

Name in CAPITAL LETTERS:
LAST FOUR DIGITS OF ID NUMBER:

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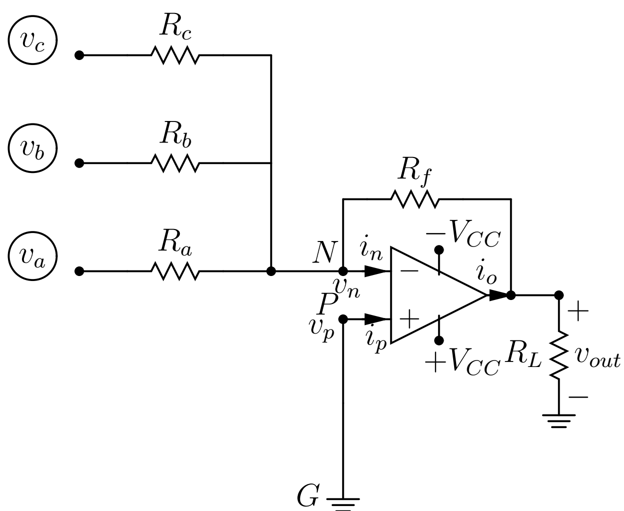
Summing Circuits

This design problem is a home-work which will be collected and graded.



Consider the **active summing or adder circuit** shown on the left. Design the values for resistances R_a, R_b, R_c, and R_f so that the output voltage v_{out} is the average of v_a, v_b, and v_c except for the sign inversion. That is, we need

$$v_{out} = -\frac{1}{3}(v_a + v_b + v_c).$$

Assume that the Op-Amp is ideal. To get the design equation, write a node equation at the negative input node N of Op-Amp.

Note: In Op-Amp circuits, one should choose resistors large enough not to load the outputs significantly but small enough that stray capacitances do not cause problems. A rule of thumb is to choose resistor values in the range, 500 Ω to 50 KΩ.

Name in CAPITAL LETTERS: **HW: Temperature Gauge**
LAST FOUR DIGITS OF ID NUMBER:

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Design of a Temperature Gauge

This design problem is a home-work which will be collected and graded.

We need to design a Temperature Gauge that displays the temperature of radiator coolant of an automobile. Obviously, such a gauge requires a temperature sensor and a display unit. The lead project engineer selected a temperature sensor which generates a voltage v_T depending upon the temperature of coolant. Its characteristic is given by

$$v_T = -0.01T + 2.0 \text{ Volts},$$

where v_T is the output voltage of the sensor and T is the temperature of coolant in degrees centigrade. Also, the lead project engineer selected a 5 V volt-meter as a display unit. Note that by the definition of a 5 V volt-meter, it has full scale deflection when the voltage applied to it is 5 V, and zero deflection when there exists no voltage across it.

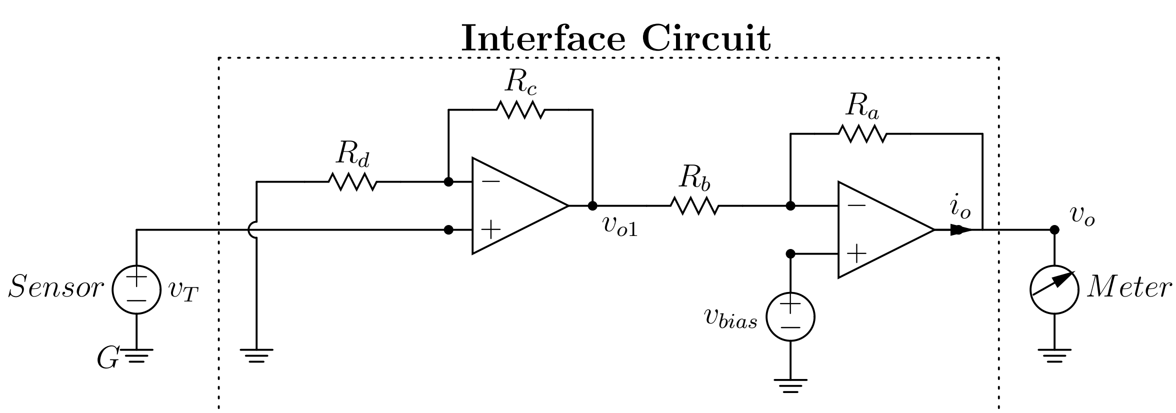
The lead project engineer assigned you to design an interface circuit which gets its input from the temperature sensor and feeds its output to the 5 V volt-meter. The specifications are that the voltmeter display zero deflection and full deflection respectively when the temperature of coolant is -20 and 120 degrees centigrade.

Your experience as a circuit engineer prompts you to select the following differential amplifier circuit as an interface circuit. Determine the values of resistances R_a, R_b, R_c, and R_d appropriately to meet the specifications. It turns out that the ratios $k_1 = \frac{R_a}{R_b}$ and $k_2 = \frac{R_c}{R_d}$ play important roles. Note that there are only two constraints, namely output voltage v_o be zero and 5 V respectively when the temperature of coolant is -20 and 120 degrees centigrade. As such k_1 and k_2 can be determined from the two constraints. Since only the ratios k_1 and k_2 are important, one of the resistances in each ratio can be arbitrarily chosen. It is advisable to have all resistance values between $20K\Omega$ and $200K\Omega$. Select v_{bias} as a proper fraction of the battery voltage 12 V. Assume that the Op-Amps are ideal.

This design problem has a number of solutions depending upon v_{bias} and the arbitrary values you selected for the two resistances. Do not worry that your design is different from that of your friends.

For your final design, obtain the expression that relates the output voltage v_o to the temperature T. Check that $v_o = 0$ when $T = -20$ degrees and $v_o = 5$ when $T = 120$ degrees.

Determine i_o when $T = 120$ degrees, assume that the meter has a resistance of $200K\Omega$.



Guide lines:

1. **Circuit Analysis:** Write a node equation at the negative terminal of each Op-Amp. Solve them to get the following equation,

$$v_o = \left(1 + \frac{R_a}{R_b}\right) v_{bias} - \frac{R_a}{R_b}\left(1 + \frac{R_c}{R_d}\right) v_T.$$

   Denote $k_1 = \frac{R_a}{R_b}$ and $k_2 = \frac{R_c}{R_d}$, they play important roles.

2. Substitute for v_T in terms of the temperature T to get a relationship between the output voltage v_o and the temperature T with v_{bias} still unknown.

3. By utilizing the above equation and the design specifications, derive two constraint equations in terms of k_1 and k_2 with v_{bias} still unknown.

4. Select v_{bias} as a ~~proper~~ fraction of the battery voltage 12 V and substitute it in the constraint equations.

5. Solve the resultant constraint equations to get values for k_1 and k_2.

   If the values for k_1 and k_2 are not positive, select a different value for v_{bias}, and solve for k_1 and k_2. Repeat this step until both the values for k_1 and k_2 are positive.

6. Now that we have values for $k_1 = \frac{R_a}{R_b}$ and $k_2 = \frac{R_c}{R_d}$, select appropriate resistance values. Note that all resistance must be positive. It is advisable to have all resistance values between $20K\Omega$ and $200K\Omega$.

7. For your final design, obtain the expression that relates the output voltage v_o to the temperature T. Check that $v_o = 0$ when $T = -20$ degrees and $v_o = 5$ when $T = 120$ degrees.