Transfer Function of a Circuit

Let us first emphasize the concept of impedance in Laplace domain and in Phasor domain:

All electrical engineering signals exist in time domain where time t is the independent variable. One can **transform a time-domain signal to phasor domain for sinusoidal signals**.

For general signals not necessarily sinusoidal, one can **transform a time domain signal into a Laplace domain signal**.

$$\text{The impedance of an element in Laplace domain} = \frac{\text{Laplace Transform of its voltage}}{\text{Laplace Transform of its current}}.$$

$$\text{The impedance of an element in phasor domain} = \frac{\text{Phasor of its voltage}}{\text{Phasor of its current}}.$$

The impedances of elements, R, L, and C are given by

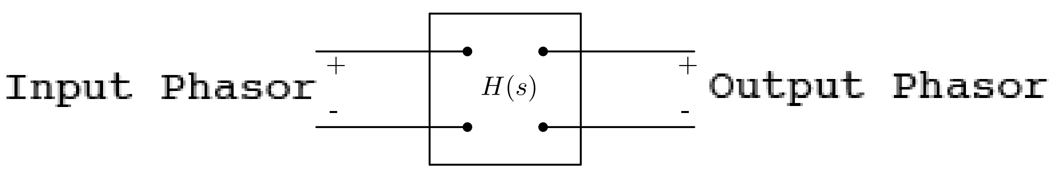
Element :	Resistance **R**	Inductance **L**	Capacitance **C**
Impedance in Laplace domain :	**R**	**sL**	$\frac{\mathbf{1}}{\mathbf{sC}}$
Impedance in Phasor domain :	**R**	**j**$\boldsymbol{\omega}$**L**	$\frac{\mathbf{1}}{\mathbf{j\boldsymbol{\omega} C}}$

$$s = j\omega$$

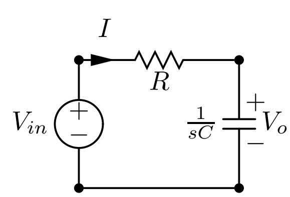
For Phasor domain, the Laplace variable $s = j\omega$ where ω is the radian frequency of the sinusoidal signal.

The transfer function $H(s)$ **of a circuit is defined as**:

$$H(s) = \text{The transfer function of a circuit} = \frac{\text{Transform of the output}}{\text{Transform of the input}} = \frac{\text{Phasor of the output}}{\text{Phasor of the input}}.$$



Example: As a simple example, consider a RC circuit as shown on the right. By voltage division rule, it is easy to determine its transfer function as

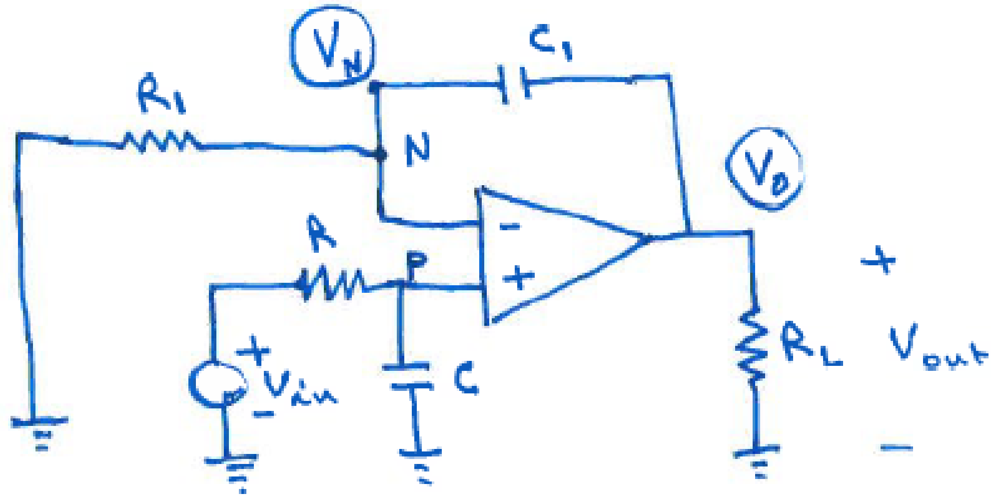


$$H(s) = \frac{V_o}{V_{in}} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC} = \frac{1}{RC}\,\frac{1}{s + \frac{1}{RC}} = \frac{\alpha}{s + \alpha}$$

where $\alpha = \frac{1}{RC}$.

Transfer function is normally expressed in a form where the coefficient of highest power in the denominator is unity (one).

Example: Determine the transfer function of the circuit shown. Assume that the Op-Amp is ideal.



The solution is simple. In what follows we show all steps clearly showing all the mathematical manipulations.

By voltage division rule,

$$V_N = V_P = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in} = \frac{1}{1 + sRC} V_{in} = \frac{1}{RC} \frac{1}{s + \frac{1}{RC}} V_{in} = \frac{\alpha}{s + \alpha} V_{in}$$

where $\alpha = \frac{1}{RC}$.

We can write the node equation at N as

$$(V_N - V_0) s C_1 + \frac{V_N}{R_1} = 0.$$

We can simplify the above equation as

$$V_N - V_0 + \frac{V_N}{sC_1R_1} = 0 \;\Rightarrow\; V_0 = V_N + \frac{V_N}{sC_1R_1} = V_N \left[1 + \frac{1}{sC_1R_1}\right] = V_N \frac{1 + sC_1R_1}{sC_1R_1}.$$

Thus

$$V_0 = V_N \frac{1 + sC_1R_1}{sC_1R_1} = V_N \frac{s + \frac{1}{C_1R_1}}{s} = V_N \frac{s + \beta}{s}$$

where $\beta = \frac{1}{C_1R_1}$.

We get

$$\text{The transfer function } = H(s) = \frac{V_0}{V_{in}} = \frac{V_0}{V_N} \frac{V_N}{V_{in}} = \frac{s + \beta}{s} \; \frac{\alpha}{s + \alpha} = \frac{\alpha(s + \beta)}{s(s + \alpha)}.$$

This is often used in deriving filter circuits.