Network Coding Aware Power Control in Wireless Networks

Kai Su, Dan Zhang, Narayan B. Mandayam
WINLAB, Rutgers University
671 Route 1 South, North Brunswick, NJ 08902
Email: {kais, bacholic, narayan}@winlab.rutgers.edu

***Abstract*—In the past decade, remarkable progress has been made in the area of network coding in terms of theory, code design, and applications. However, from a cross-layer perspective, there have been fewer efforts on understanding the impact of lower layers on coding of packets at network layer to improve the network throughput. In this paper, we study and design power control that can enhance the performance of random network coding in wireless networks. Specifically, we utilize a differential equation based framework to analyze random network coding throughput, and design dynamic power control algorithm to achieve higher multicast throughput.**

I. Introduction

Network coding has attracted a large amount of attention within the networking research community since its inception [1], [2], and the focus has been placed on establishing the performance bounds that are usually superior to traditional copy-and-forward routing [2], and on discovering a variety of applications, such as content delivery [3], [4] and distributed storage [5]. While network coding is considered as a network layer operation, one interesting and important question is how the lower layers would impact its performance. Taking an interference limited wireless network for example, increasing the transmit power at any transceiver has a positive effect in enhancing the signal-to-noise-and-interference-ratio for any receiver that intends to detect that signal. Yet at the same time it has a negative impact by raising interference for all the other transmissions. Such a situation can be regulated by an effective set of power control policies, as done for cellular networks [6]. However, when an information flow is to be designed across the wireless network using random network coding, the problem becomes more difficult if throughput is the primary concern. As a result, we want to design a power control policy that interacts closely with random network coding. There is a limited amount of existing work on this topic, mainly because the analytic tools used previously are more appropriate for uncoded networks. For example, in deriving the capacity region with an interference limited wireless network, [7] gives up full wireless multicast advantage in order to convert the wireless network to a conceptual wired network. The same approach was adopted in [8] with further restrictions on the form of interference. Without this approach, [9] , [10] studied the problem on cellular and linear networks that have a simplistic topology. It would be desirable to have a framework by which the coupling between network coding and power control can be studied for any wireless network in general.

In this paper, we adopt the recently proposed differential equation framework (DE) in [11], which elegantly models random network coding dynamics. We begin with a numerical example to motivate the necessity of a network coding aware power control policy. After that, we leverage the DE framework and develop an algorithm to dynamically adapt transmit power at each node. The time scale of power adjustment implicitly assumed here is of the order of packet durations at the network layer. The overall objective is to maximize the minimum throughput at the destination nodes in a multicast session. The simulation results demonstrate the efficacy of the network coding aware power control. The contribution of our work is three-fold. First, we derive an algorithm which, unlike traditional power control algorithms, explicitly supports random network coding performance. Second, the simplicity and adaptivity of this algorithm make it more amenable to be incorporated in practical design and implementation. Third, the derivation of this policy serves to showcase how the differential equation framework can be applied to yield refreshing insights and design perspectives.

II. System Model

A wireless network is naturally modeled with a directed hypergraph $G = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, 2, \ldots, N\}$ denotes N network nodes and $\mathcal{E} = \{(i, \mathcal{K}) | i \in \mathcal{N}, \mathcal{K} \subset \mathcal{N}\}$ denotes the hyperarcs in the hypergraph. The details of hypergraph model can be found in [12]. We assume that every node is able to receive and decode the signal transmitted by all other nodes. Therefore, as each transmission in wireless environment is in essence a broadcast, a hyperarc $(i, \mathcal{K})$ captures the fact that a transmitted packet from node i can be received by a set of receivers $\mathcal{K}'$ without error, where $\mathcal{K}' \subset \mathcal{K}$. This model is illustrated in Figure 1 where an arrowed line denotes a possible direction of packet flow.

Moreover, consider that the network is performing random network coding with a source s transmitting original packets and a set of destinations $\mathcal{D} \subset \mathcal{N}$. We further assume there exists an underlying MAC protocol such that each node i is transmitting at λ_i packets per second on average and the probability that a packet sent from node i is successfully

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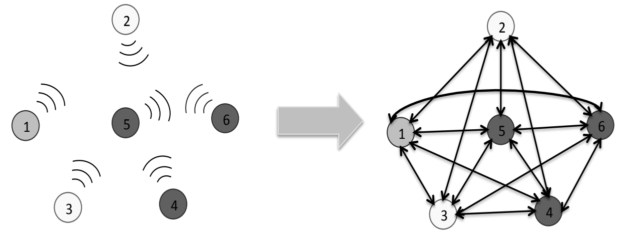

Fig. 1. Hypergraph model of a wireless network of six nodes with $s = 1$ and $\mathcal{D} = \{4, 5, 6\}$.

received by at least one node in $\mathcal{K}$ is $P_{i,\mathcal{K}}$. We can then define the transmission rate for the hyperarc $(i, \mathcal{K})$ as

$$z_{i,\mathcal{K}} = \lambda_i P_{i,\mathcal{K}}. \tag{1}$$

In addition, no routing operations are performed in the network; all the nodes merely receive packets from the network and broadcast the coded packets.

A. Random Network Coding

We now briefly describe the random network coding scheme. Consider a multicast session in which node 1 is the source trying to deliver m packets to every node in the destination set $\mathcal{D}$. Each packet $\underline{w}$ is a row vector of length L from $\mathbb{F}_q^L$ where $\mathbb{F}_q$ is a given finite field of size q. During the multicast session, each node keeps a reservoir of packets. In the initialization stage, node 1 puts all the source packets into its reservoir. Whenever a node receives a packet, it adds the packet to its reservoir. Suppose at time t, the reservoir of an arbitrary node i is $\text{Rsv}(i, t) = \{\underline{w}_{i,1}, \underline{w}_{i,2}, \cdots, \underline{w}_{i,n}\}$. Then whenever node i is to transmit a packet, a coded packet $\underline{v}$ will be formed such that $\underline{v} = a_1\underline{w}_{i,1} + a_2\underline{w}_{i,2} + \cdots + a_n\underline{w}_{i,n}$, where $[a_1, a_2, \cdots, a_n] \in \mathbb{F}^n$ is randomly generated. Since the coding operation is entirely linear, we get $\underline{v} = b_{i,1}\underline{w}_1 + b_{i,2}\underline{w}_2 + \cdots + b_{i,m}\underline{w}_m$ where $\underline{w}_1, \underline{w}_2, \cdots, \underline{w}_m$ are the m source packets and $[b_{i,1}, b_{i,1}, \cdots b_{i,m}] \in \mathbb{F}^m$ is called the global coefficient vector associated with $\underline{v}$. The global coefficient vector will be sent along with corresponding coded packets.

Let S_i denote the vector space spanned by the global coefficient vectors received by node i. We call $V_i = \dim S_i$ the rank of node i. Note V_i is time-varying and when $V_i = m$, node i can decode and recover the m original packets. Additionally, we extend the definition of rank to a subset $\mathcal{K}$ of $\mathcal{N}$, i.e. $V_{\mathcal{K}} = \dim\left(\sum_{i\in\mathcal{K}} S_i\right)$.

B. The DE Framework

In a hypergraph, a cut for $(\mathcal{S}, \mathcal{K})$, where $\mathcal{S}, \mathcal{K} \subset \mathcal{N}$ and $\mathcal{S} \cap \mathcal{K} = \emptyset$ is defined as $\mathcal{T}$ such that $\mathcal{K} \subset \mathcal{T} \subset \mathcal{S}^c$. The capacity of a cut is $c(\mathcal{T}) = \sum_{i\in\mathcal{T}^c} z_{i,\mathcal{T}}$ and the min cut for $(\mathcal{S}, \mathcal{K})$ is the cut with the minimum size. Then the throughput of a multicast session from s to destinations $\mathcal{D}$ is determined by the min cut between $\{s\}$ and $\mathcal{D}$, as is shown in [1].

In [11], it has been established that under the fluid approximation, we have $E[V_{\mathcal{K}}(t)] \doteq V_{\mathcal{K}}(t)$, and $V_{\mathcal{K}}$ satisfies a system of differential equations:

$$\dot{V}_{\mathcal{K}} = \sum_{i\notin\mathcal{K}} z_{i,\mathcal{K}} I(V_{i\cup\mathcal{K}} - V_{\mathcal{K}}),\ \forall\mathcal{K} \subset \mathcal{N}. \tag{2}$$

where

$$I(x) = \begin{cases} 1, & x > 0, \\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

A detailed treatment of the derivation of equations above can be found in [11]. Since $V_i(t)$ is the number of received innovative packets, we define $\dot{V}_i(t)$ as the rate that node i is receiving innovative packets, i.e., throughput.

C. Interference Model

We assume a path loss model for every point-to-point link in the network G with additive noise and interference. Let $P_{\text{Tx},i}$ denote the transmit power at node i, $i \in \mathcal{N}$. When i transmits a signal, the received signal power at node j is $P_{\text{Tx},i}h_{ji}$ where h_{ji} is the link gain. The noise power is denoted as σ^2 and the interference power is given by $\sum_{k\neq i,j} P_{\text{Tx},k}h_{jk}$. We further assume each node i implements certain processing gain g_i in its modulation, such that when j attempts to receive the signal from i, the observed interference power is given as

$$J_{ji} = \sum_{k\neq j,i} (P_{\text{Tx},k} \cdot h_{jk}/g_k). \tag{4}$$

Consequently, the point-to-point signal-to-noise-and-interference ratio (SINR) for the sender-receiver pair (i, j) is given as

$$\mathit{SINR}_{(i,j)} = \frac{P_{\text{Tx},i} \cdot h_{ji}}{J_{ji} + \sigma^2}. \tag{5}$$

In practice, given a modulation and coding scheme, the packet error rate $p_{i,j}$ for the pair (i, j) is a function of SINR, i.e., $p_{i,j} = p_{i,j}(\mathit{SINR}_{(i,j)})$. In this paper, we only need to assume that $p_{i,j}(\mathit{SINR}_{(i,j)})$ is a differentiable function. Without loss of generality, we assume each packet contains L bits. Then the probability that node j can receive a packet sent by node i without error, $P_{i,j}$, is

$$P_{i,j} = (1 - p_{i,j})^L. \tag{6}$$

While the work presented in this paper can be easily extended to the case of cooperative reception, for the sake of simplicity, we only consider independent receptions here. Therefore, the reception probability $P_{i,\mathcal{K}}$ is given as

$$P_{i,\mathcal{K}} = 1 - \prod_{j\in\mathcal{K}} (1 - P_{i,j}). \tag{7}$$

III. Effect of Interference on Network Coding Throughput

We now present an example to show that by manipulating transmit powers at each node in a wireless network, the rate of rank evolution at the destination nodes can be altered. Consider a wireless network with 6 nodes which are depicted in Figure 1. We assume the received signal is interfered by all the other nodes which are also transmitting and the packet loss is only due to bit error based on the interference model, i.e. there is no loss because of congestion or buffer overflow. We further assume that no routing operations are performed in this network, and each node merely receives packets and sends out linear combination of received packets. Consider a multicast session starting from $t = 0$ms in this network with source node 1 and destination nodes $\{4, 5, 6\}$. The source has 1000 packets to be delivered to the destinations and every node is transmitting at the rate of 1 packet per millisecond. At $t = 0$ms, the transmit power at each node is set to 13dBm. Subsequently, at $t = 500\text{ms}, t = 1000\text{ms}, t = 1500\text{ms}$, and $t = 2000\text{ms}$, $P_{\text{Tx},1}, P_{\text{Tx},3}, P_{\text{Tx},4}$ and $P_{\text{Tx},5}$ are increased to 14dBm, respectively. We utilize the DE framework to model this process. Figure 2 and Figure 3 show the rank evolution and throughput of the destination nodes respectively.

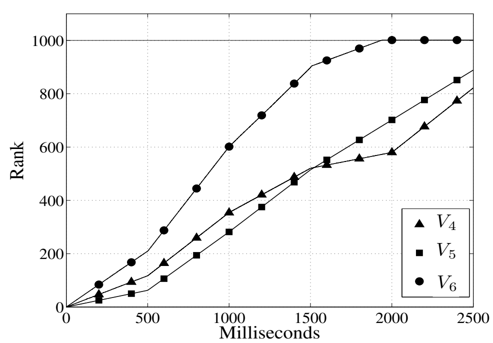

Fig. 2. Impact of interference on rank evolution

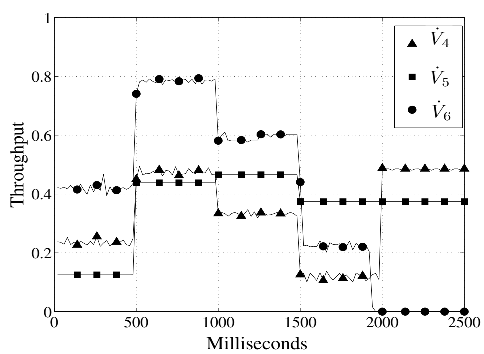

Fig. 3. Impact of interference on throughput

As can be seen from Figure 3, increasing transmit power at one node does not necessarily improve the throughput of all the destination nodes. In fact, if the increase in interference incurred by the power improvement compromises the throughput increase, incrementing transmit power would not be the optimal choice. This triggers our speculation that there is a way to adjust power so that network throughput can be maximized with the powers subject to a certain budget. In light of this, we present the network coding aware power control algorithm in the next section.

IV. Power Control Algorithm for Wireless Network Coding

A. Problem Formulation

We first algebraically formulate the power control problem in network coding based on the system model introduced in section II. We consider a wireless network $G = (\mathcal{N}, \mathcal{E})$ which is performing random network coding and running a multicast session with sender $s \in \mathcal{N}$ and destinations $\mathcal{D} = \{d_1, d_2, ..., d_r\}$, where $d_i \in \mathcal{N}, i = 1, ..., r$. Our goal is to adapt transmit powers at each node such that the destinations' throughput is improved. Note that the multicast throughput is bounded by the minimum unicast throughput [1]; therefore by increasing the minimum unicast throughput, the overall performance of network coding will be improved. Recall from section II-B that we use $\dot{V}_i$ to denote the throughput of node i. The problem of power control to maximize throughput can then be stated as:

$$
\begin{aligned}
\text{maximize} \quad & \min_{1\le j\le r} \dot{V}_{d_j} \\
\text{subject to} \quad & \dot{V}_{\mathcal{K}} = \sum_{i\notin\mathcal{K}} z_{i,\mathcal{K}}(\mathbf{P}_{\text{Tx}}) \cdot I(V_{i\cup\mathcal{K}} - V_{\mathcal{K}}),\ \forall\mathcal{K}\subset\mathcal{N}. \\
& z_{i,\mathcal{K}} = \lambda_i P_{i,\mathcal{K}} = \lambda_i \left(1 - \prod_{j\in\mathcal{K}} (1 - P_{i,j})\right). \\
& P_{i,j} = P_{i,j}\left(\mathit{SINR}_{(i,j)}\right). \\
& 0 \preceq \mathbf{P}_{\text{Tx}} \preceq \mathbf{P}_{\text{Tx}}^{\max}. \\
\text{variables} \quad & \mathbf{P}_{\text{Tx}}.
\end{aligned}
\tag{8}
$$

where $\mathbf{P}_{\text{Tx}} \in \mathbb{R}^N$ is the vector of transmit powers at each node.

Since random network coding is performed, the first three constraints hold due to the DE framework and our network model. The last constraint reflects the practical power budget at each node. We can see that the above optimization problem is non-convex.

B. Gradient-based Power Control Algorithm

Since it is algorithmically difficult to achieve a globally optimal solution for such a non-convex optimization problem, a local optimum, if not computationally complex to find, would still be of interest. Therefore, we design an algorithm to approach a locally optimal solution for the max-min throughput problem, by adjusting the transmit powers towards the direction given by the gradient of the minimum throughput among all the destinations.

Note that with the DE framework, the network is modeled as a continuous system. As a result, we adjust transmit powers continuously as well. At time t, we first identify the node with the minimum throughput among the destinations which have not reached full rank for the current session, $\mathcal{R}$, i.e., to find k such that

$$k = \arg\min_{j\in\mathcal{R}} \dot{V}_j. \tag{9}$$

By definition, the gradient of $\dot{V}_k$ is determined by

$$\nabla\dot{V}_k(\mathbf{P}_{\text{Tx}}) = \left(\frac{\partial \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\partial P_{\text{Tx},1}}, \frac{\partial \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\partial P_{\text{Tx},2}}, ..., \frac{\partial \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\partial P_{\text{Tx},N}}\right). \tag{10}$$

If $\mathbf{P}_{\text{Tx}}$ increases towards the direction of $\nabla\dot{V}_k(\mathbf{P}_{\text{Tx}})$, $\dot{V}_k(\mathbf{P}_{\text{Tx}})$ will increase. Therefore, the adjustment of transmit power is then given by the following equation:

$$\dot{\mathbf{P}}_{\text{Tx}} = a' \cdot \nabla\dot{V}_k\left(\mathbf{P}_{\text{Tx}}\right). \tag{11}$$

where a' is the gain.

C. Estimation of Gradient

If we know a priori, the analytic expression for $P_{i,j}(\mathit{SINR}_{(i,j)})$ for all pairs of $(i,j) \in \mathcal{N} \times \mathcal{N}$, equation (10) can be evaluated. However, there are two reasons to avoid this approach. First, the analytic expression may be unknown or too complicated to calculate. Second, the expression may even change as the environment changes, due to mobility, etc. Therefore we seek to estimate rather than exactly evaluate (10). Specifically, by adopting a sufficiently small parameter Δq, we have the following approximation for the partial derivative

$$\frac{\partial \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\partial P_{\text{Tx},i}} \approx \frac{\dot{V}_k(\mathbf{P}_{\text{Tx}} + \Delta q \mathbf{e}_i) - \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\Delta q}. \tag{12}$$

where $\mathbf{e}_i$ is a vector of length N with ith element being 1 and 0 elsewhere,

Plugging (2) in (12), and letting $\mathbf{q}_i$ be the transmit power vector with small increment Δq at ith node, i.e.

$$\mathbf{q}_i = \mathbf{P}_{\text{Tx}} + \Delta q \mathbf{e}_i. \tag{13}$$

we obtain

$$\frac{\partial \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\partial P_{\text{Tx},i}} = \frac{1}{\Delta q} \cdot \left(\sum_{j \neq k}(z_{j,k}(\mathbf{q}_i) - z_{j,k}(\mathbf{P}_{\text{Tx}}))I(V_{j\cup k} - V_k)\right). \tag{14}$$

Assume each node $i \in \mathcal{N}$ has a power budget $0 \leq P_i \leq P_i^{\max}$, and let a be a gain parameter. Then we propose that the transmit power will be adjusted based on the following differential equation:

$$\dot{P}_{\text{Tx},i} = \begin{cases} 0, & \text{if } P_{\text{Tx},i} = P_{\text{Tx},i}^{\max} \text{ and } g_i > 0, \\ & \text{or if } P_{\text{Tx},i} = 0 \text{ and } g_i < 0; \\ a g_i, & \text{otherwise}, \end{cases} \tag{15}$$

where

$$g_i = \sum_{j \neq k}(z_{j,k}(\mathbf{q}_i) - z_{j,k}(\mathbf{P}))I(V_{j\cup k} - V_k). \tag{16}$$

When $a = 1/\Delta q$, $\dot{P}_{\text{Tx},i} = \frac{\partial \dot{V}_k(\mathbf{P}_{\text{Tx}})}{\partial P_{\text{Tx},i}}$.

To sum up, the power control algorithm is characterized by the following set of equations

$$\boxed{\begin{aligned} & k = \arg\min\nolimits_{j \in \mathcal{R}} \dot{V}_j. \\ & \mathbf{q}_l = \mathbf{P}_{\text{Tx}} + \Delta q \mathbf{e}_l, \ \forall l \in \mathcal{N}. \\ & g_l = \textstyle\sum_{i \neq k}(z_{i,k}(\mathbf{q}_l) - z_{i,k}(\mathbf{P}_{\text{Tx}}))I(V_{i\cup k} - V_k). \\ & \dot{P}_{\text{Tx},l} = \begin{cases} 0, & \text{if } (P_{\text{Tx},l} = P_{\text{Tx},l}^{\max} \text{ and } g_l > 0) \\ & \text{or if } (P_{\text{Tx},l} = 0 \text{ and } g_l < 0); \\ a g_l, & \text{otherwise}, \end{cases} \end{aligned}} \tag{17}$$

Our algorithm essentially works in the following fashion. It continuously identifies the destination k which has the minimum throughput, and it then estimate the gradient of $\dot{V}_k$ with respect to $\mathbf{P}_{\text{Tx}}$. The power is adjusted in the direction of the gradient. By continuing this operation, the algorithm is approaching a maximum throughput value, even though it is not necessarily globally optimal. Besides, since this algorithm dynamically adapts powers and takes into account the latest topology information, it is capable of optimizing the network throughput in spite of any network changes, such as nodes being moved or switched off.

V. Numerical Results

We show the simulation results for the max-min throughput algorithm based on the RNC and interference model. Consider again the topology in Figure 1 with source being node 1 which has 2000 packets to send to destinations $4, 5, 6$. Again, we assume no packet loss due to network congestion or buffer overflow and no routing operations of any kind is performed in this network, i.e., each node simply receives packets and sends out linear combination of received packets. Assume each node is transmitting 1 packet per millisecond. We adopt the ITU model for the calculation of path loss *PL* [13]

$$\mathit{PL} = 20\log f + 10n\log d + P_f(n) - 28. \tag{18}$$

We choose the transmitted signal frequency f to be 2.4GHz, path loss exponent n to be 3, and floor penetration factor $P_f(n)$ to be 11, for the purpose of simulating a wireless network with all nodes placed in the same floor inside a closed building. Although our algorithm does not presume any particular bit error model, for the sake of simulation, we assume BPSK signaling and a Gaussian interference model. Therefore the bit error rate for sender-receiver pair (i,j) is given by

$$p_{i,j} = Q\left(\sqrt{\frac{P_{\text{Tx},i} \cdot h_{ji}}{\sum_{k \neq i,j}(P_{\text{Tx},k} \cdot h_{jk}/g_k) + \sigma^2}}\right). \tag{19}$$

We choose the initial transmit power at each node to be equally 13dBm, and $P_{\text{Tx},i}$ of node i subject to the power budget $0 \leq P_{\text{Tx},i} \leq 15$dBm. The power control gain, a is set to 0.1, and the processing gain is chosen to be 8. The complete dynamical system is evaluated with a numerical solver.

Figure 4 shows the rank evolution process and Figure 5 shows destinations' throughput, both in the case of with power control and without power control. With power control, the rank at each destination grows faster than without power control. Also, as can be readily seen in Figure 5, with power control gain $a = 0.1$, throughput at $t = 500$ms roughly doubles that at $t = 0$ms and the throughput at each destination is gradually approaching a stable state. As an illustration of the role of the gain, the convergence is also shown in Figure 5 for a gain value of $a = 0.05$. Figure 6 shows how the corresponding powers are adapted by the policy. As seen, all the powers at the various nodes have converged by the time t=1600ms.

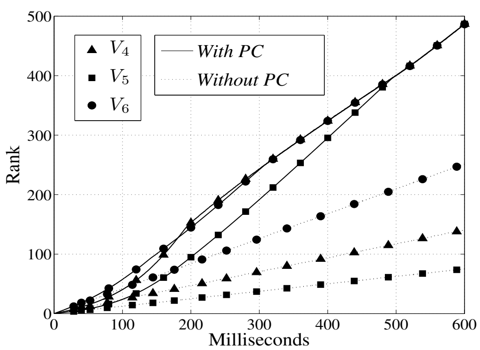

Fig. 4. Rank evolution with and without throughput

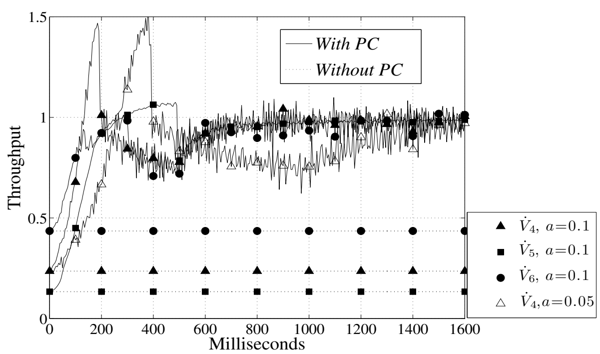

Fig. 5. Throughput with and without power control

VI. Conclusion

We studied power control for random network coding in wireless networks as an example of exploring the impact of lower layers on coding of packets at the network layer. By using a differential equation framework (DE) as an analytic tool to model random network coding performance, we presented an algorithm to dynamically adapt transmit power at each node to achieve higher throughput in a multicast session. Our results reveal that our algorithm is effective in speeding up the rank evolution process and achieving the desired objective.

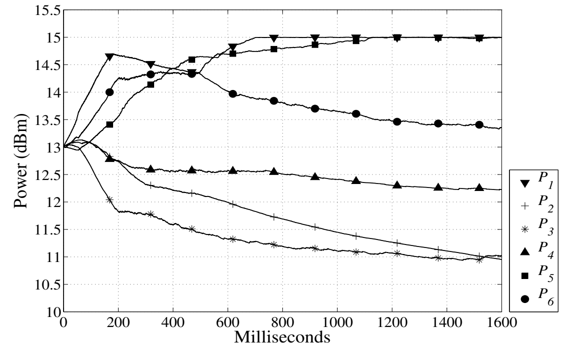

Fig. 6. Power levels at each node

The approach presented here can also be used for studying and designing other lower layer parameters to improve network coding performance.

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