

# Bandwidth Exchange for Fair Secondary Coexistence in TV White Space

Dan Zhang and Narayan Mandayam

WINLAB, Rutgers University  
671 US-1 South, North Brunswick, NJ 08902  
{bacholic, narayan}@winlab.rutgers.edu

**Abstract.** The recent ruling by the FCC has mandated a geo-location database approach to regulate the coexistence of primary and secondary users in TV white space. However, mechanisms for secondary coexistence have been left unspecified and prompts the design and study of the same. In this paper, we propose a mechanism to incentivize fair and efficient secondary user cooperation. Specifically, we assume the existence of cognitive radio equipped secondary users capable of OFDMA with the ability to dynamically exchange subcarriers among themselves. Based on a previously proposed incentive mechanism called Bandwidth Exchange (BE), we further leverage the capability of the mandatory geo-location database to enable fast negotiation between potential cooperation partners to realize a Nash Bargaining Solution (NBS) for secondary coexistence. We illustrate how this mechanism works when secondary users in an OFDMA access network form cooperation through BE for which the NBS is calculated based on information obtained from the database.

**Key words:** Nash bargaining, White Space, bandwidth exchange, geo-location database, OFDMA

## 1 Introduction

On Sept. 23, 2010, FCC released Second Memorandum Opinion and Order [1] which announced the official advent of TV White Space. Two classes of users are defined to operate in white space: the primary users such as TV stations and licensed wireless microphones; the secondary users that include many different white space devices (WSD) conforming to a number of rules to avoid interfering primary users. In the new ruling, plenty of protections and precautions are imposed to guarantee undisturbed operation of primary users. Though spectrum sensing is no longer required, a geo-location database that registers the locations of primary users has become a mandate. Every secondary user is required to query the database through internet to make sure it would not produce interference to nearby primary users before it starts transmission. The query is periodic in case some primary users want to initiate operation in the vicinity. These requirements provide a reliable shield between primary users and secondary users for the purpose of their coexistence. However, the new ruling

does not explicitly designate how coexistence is to be managed among the secondary users. This includes, but not limited to, how to control the interference between secondary users and how to incentivize cooperation among them. The latter issue is of critical importance since the usefulness of TV white space can only be realized if secondary users can coexist to form networks for carrying information.

Among other cooperation forms, cooperative forwarding [2] [3] is an essential technique to enhance connectivity and throughput. As forwarding usually incurs some sort of cost, incentive must be implemented in a certain way. In [4] the authors proposed an incentive mechanism called Bandwidth Exchange (BE) which essentially enables a user to delegate a portion of its bandwidth in exchange for forwarding. While some advantages of BE was outlined, such as its ability to find an efficient and proportionally fair joint cooperation strategy, the lack of central management forces each user to estimate the necessary bargaining [5] parameters through a lengthy process. With such slow estimation, bargaining can only be carried out pairwise while ignoring the fact that existence of other users may affect the bargaining power. This leads to approximation even for pairwise bargaining. In TV white space, with the presence of a geo-location database, it is possible to obtain the bargaining parameters using a reliable geometric channel model, so as to shorten or avoid the estimation process. This idea will form the basis for the study in this paper. We will discuss the database-assisted BE-NBS algorithm as an extension to the work presented in [4] for the specific scenario of white space.

While in [4] BE is introduced by exchanging generic bands of frequency, in this paper we assume an OFDMA system where users execute BE by exchanging (possibly noncontiguous) OFDM subcarriers for cooperation.

## 2 System Model and Bandwidth Exchange

### 2.1 System Model

Suppose we have  $M$  white space users (labeled  $1, 2, \dots, M$ ) transmitting to an AP (labeled as 0) using OFDMA. Each user is assigned  $N$  consecutive subcarriers. The OFDMA system is only generic thus we assume transmit power is evenly allocated to all subcarriers and only one modulation scheme is allowed for each of them. Each user has a minimum required rate  $R_i^{\min}$  which with the generic OFDMA assumption translates into a minimum number of usable subcarriers. Let  $L(d)$  denote the path loss (including fading) a subcarrier experiences in a slot, where  $d$  is the transmission distance. A subcarrier is usable if and only if  $L(d) < L_{th}$  for some threshold  $L_{th}$ . The path loss is by assumption a random variable which are independent across subcarriers as well as across slots, yet it is identically distributed only across subcarriers of the same user. The identical distribution assumption is justified by noticing that for any frequency dependent channel model, the statistics vary little over a few Mega Hertz, which is the amount of bandwidth presumably allocated to a user in the target system.

Our objective is to maximize the average throughput from any node  $i$  to the AP, possibly by means of cooperative forwarding.

In what follows, subscript  $ij$  always stands for the link or request from  $i$  to  $j$ .

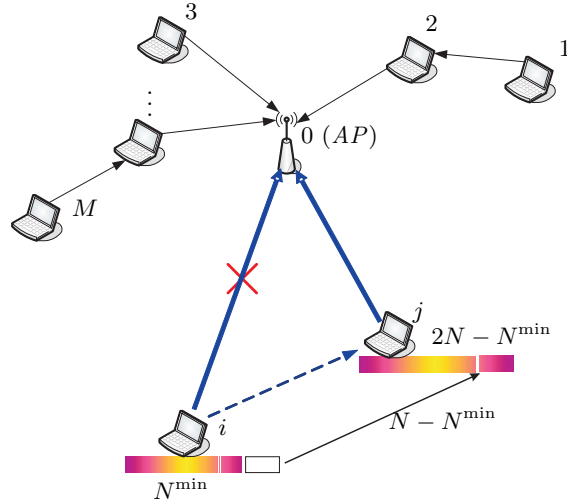


Fig. 1: When the direct link fails, user  $i$  tries to incentivise forwarding by delegating  $N - N^{\min}$  subcarriers to user  $j$ .

## 2.2 Bandwidth Exchange

At the beginning of every slot, user  $i$  first attempts to transmit directly to the AP, with  $X_{i0}$  ( $\leq N$ ) usable subcarriers. If  $X_{i0} < N^{\min}$ , it broadcasts a cooperation request to its neighbors, expecting one of them to forward its data to AP, by means of BE. Specifically, it involves the following steps:

1. User  $i$  offers to delegate  $N - N^{\min}$  subcarriers to user  $j$  as long as the remaining  $N^{\min}$  subcarriers are usable.
2. With the offered subcarriers,  $j$  examines the number of usable subcarriers available to it and determine if a request is supportable. A request is considered supportable in two cases: (i) if initially  $j$ 's direct link is dead, i.e.,  $X_{j0} < N^{\min}$ , but with the added subcarrier, the direct link becomes alive, i.e.,  $X'_{j0} \geq N^{\min}$ ; (ii) if initially  $j$ 's direct link is alive  $X_{j0} \geq N^{\min}$ , with the added subcarriers the direct link should be at least as good as to support both  $i$  and  $j$ , i.e.,  $X'_{j0} \geq 2N^{\min}$ .
3. If the request is not supportable, the request is rejected; otherwise  $j$  chooses to cooperate with a probability  $P_{ij}^c$ . We assume there is no flow splitting and every forwarder serves at most one source.

4. If  $i$  receives multiple acknowledgements, it randomly picks one to follow in this slot.

This procedure is repeated for each slot.

### 3 Database Assisted Nash Bargaining for Bandwidth Exchange

We use NBS to determine  $P_{ij}^c$  in the course of BE. To simplify, we restrict ourselves to pairwise bargaining that regards the other users as (opportunistic) communication resource rather than bargaining participants. This is desired because literal  $N$ -user bargaining entails searching for the solution in a strategy space that scales exponentially with the number of users.

From the perspective of NBS, each slot corresponds to a stage game for user  $i$  and  $j$  is a supportable request is sent from  $i$  to  $j$  or vice versa. The potential forwarder  $j$  must decide whether the request can be granted, which depends on a number of factors:

- the probability  $P_{ij}$  of a supportable request from  $i$  to  $j$  and vice versa;
- the expected utilities available to both source and forwarder contingent on its decisions.

When request granted, the expected utility for source  $i$  is always  $N^{\min}$ , no matter how many positive acknowledgement  $i$  receives at last. When request rejected, the expected utility for forwarder  $j$  is always 0, which follows from an assumption that we will make shortly. We then denote the expected utility for  $j$  when request granted as  $u_{ij}^f$ , the expected utility for  $i$  when request rejected as  $u_{ij}^s$ . These notations are shown in Table 1

Table 1: Expected utilities for source  $i$  and forwarder  $j$  if request supportable.

	utility for $i$	utility for $j$
request granted	$N^{\min}$	$u_{ij}^f$
request rejected	$u_{ij}^s$	0

The pairwise NBS as presented in [4] follows the same methodology. However, it has several obvious drawbacks. First, the bargaining parameters such as  $P_{ij}$  and  $u_{ij}^f$  can only be estimated over time. This limits the applicability of BE to, at best, stationary or slowly moving users. If  $P_{ij}$  is very small, estimation can even fail in practice. Second, bargaining is restricted to pairwise, totally ignoring the effect the other users might have on the bargaining. For example,  $u_{ij}^s = 0$  in [4] even though  $i$  may still get cooperation from other users. While this significantly reduces the computation burden compared to the exact  $N$ -user NBS, it leads to incomplete consideration of bargaining power of different users as they interact. Now that a geo-location database is added and WSD's

are required to consult it before and during operation, we have a better way to implement BE that partially alleviate the two issues associated with the original BE implementation. Specifically, we may resort to a geometric channel model possibly customized for the local transmission environment that enables a user to calculate the interesting bargaining parameters by itself. For this approach to be practical, it is necessary that WSD's engaging in cooperation based on BE register their (and the AP's) locations in the database. It would be more desirable that the registry contains additional information such as the frequency band a user is assigned to. The additional information makes applicable more sophisticated channel models such as those that are frequency dependent. At this point, FCC has not completely decided what information should/could be registered in the database. It is conceivable that a good deal of additional information other than the locations of primary users will be eventually allowed or incorporated in the database. Moreover, if the location information of secondary user is updated sufficiently frequently, BE as described here has a better chance to further support mobility.

As we focus on pairwise bargaining for the sake of its simplicity, the effect of existence of other users can be accounted for as bargain parameters instead of bargain participants. Suppose  $i$  is a requester and  $j$  is a potential forwarder. Intuitively, this implies even if  $j$  refuses to cooperate there is still a chance for  $i$  to get cooperation from other forwarders. Likewise, even if  $j$  agrees to forward for  $i$ , it is possible that  $i$  secures cooperation from another user, thus  $j$ 's positive decision may bring it nothing in return.

However, the issue is complicated by the fact that for any potential forwarder, there could be multiple source users competing for its cooperation. To simplify, we notice that for a practical WSD, the outage probability of its direct link should be fairly low. Besides, the outage probabilities of different WSD's should be independent. This means the chance that user  $i$  needs to compete for cooperation is fairly low. Based on this approximate assumption, we also neglect the probability that a potential forwarder receives multiple request or two users send request to each other in a slot. This also explains when the request from source  $i$  is rejected, the expected utility for forwarder  $j$  is always 0.

### 3.1 Using Database to Obtain Bargaining Parameters

We demonstrate how to use the database and a channel model to calculate the pairwise request probability and the utility obtained by the forwarder if cooperation forms. These parameters do not depend on the existence and number of other users in the system.

First we note as the random path loss  $L_i(d)$  is i.i.d. across the subcarriers of a user  $i$ , probability  $q_d^i = P(L_i(d) < L_{\text{th}})$  is the probability that any subcarrier of  $i$  is usable. Given a number  $X$  of these subcarriers, the probability  $k$  of them are usable is of Binomial distribution, whose cumulative distribution function is

$$F(k, q_d^i, X) = \sum_{\ell=0}^k \binom{X}{\ell} (q_d^i)^\ell (1 - q_d^i)^{X-\ell}, \quad (1)$$

and the probability mass function is

$$\begin{aligned} P(k, q_d^i, X) &= \Delta F(k, q_d^i, X) \\ &= F(k, q_d^i, X) - F(k-1, q_d^i, X). \end{aligned} \quad (2)$$

This helps us write out various probabilities. For example, the probability that the direct link of  $i$  fails is given by  $1 - F(N^{\min}, q_{i0}^i, N)$ .

Define two disjoint events

$$A_{ij} = X'_{j0} \geq 2N^{\min} \wedge X_{j0} \geq N^{\min}, \quad (3)$$

$$B_{ij} = X'_{j0} \geq N^{\min} \wedge X_{j0} < N^{\min}. \quad (4)$$

We may calculate

$$P(A_{ij}) \quad (5)$$

$$\begin{aligned} &= P(X_{j0} \geq N^{\min} \wedge \Delta X_{j0} + X_{j0} \geq 2N^{\min}) \\ &= \sum_{k=N^{\min}}^N P(X_{j0} = k) \sum_{\ell=2N^{\min}-k}^{N-N^{\min}} P(\Delta X_{j0} = \ell) \\ &= \sum_{k=N^{\min}}^N \Delta F(N, q_{j0}^j, k) (1 - F(2N^{\min} - 1 - k, q_{j0}^j, N - N^{\min})), \end{aligned} \quad (6)$$

$$P(B_{ij}) \quad (7)$$

$$\begin{aligned} &= P(X_{j0} < N^{\min} \wedge \Delta X_{j0} + X_{j0} \geq N^{\min}) \\ &= \sum_{k=0}^{N^{\min}-1} P(X_{j0} = k) \sum_{\ell=N^{\min}-k}^{N-N^{\min}} P(\Delta X_{j0} = \ell) \\ &= \sum_{k=0}^{N^{\min}-1} \Delta F(N, q_{j0}^j, k) (1 - F(N^{\min} - k - 1, q_{j0}^j, N - N^{\min})). \end{aligned}$$

We then have

$$\begin{aligned} P_{ij} &= P(X_{i0} < N_i^{\min} \wedge X_{ij} \geq N_i^{\min} \wedge (A_{ij} \vee B_{ij})) \\ &= P(X_{i0} < N_i^{\min}) P(X_{ij} \geq N_i^{\min}) (P(A_{ij}) + P(B_{ij})) \\ &= F(N_i^{\min} - 1, q_{i0}^i, N) \\ &\quad \cdot (1 - F(N_i^{\min} - 1, q_{ij}^i, N)) (P(A_{ij}) + P(B_{ij})). \end{aligned} \quad (7)$$

Next we calculate  $v_{ij}^f$ , defined as the utility obtained by  $j$  if cooperation forms between  $i$  and  $j$ , First note

$$v_{ij}^f = E[\Delta X_{ij} | A_{ij} \wedge B_{ij}] - N^{\min}, \quad (8)$$

then we calculate

$$E[\Delta X_{ij}|A_{ij} \wedge B_{ij}] = \frac{P(A_{ij})E[\Delta X_{ij}|A_{ij}] + P(B_{ij})E[\Delta X_{ij}|B_{ij}]}{P(A_{ij}) + P(B_{ij})}, \quad (9)$$

with

$$\begin{aligned} & P(A_{ij})E[\Delta X_{ij}|A_{ij}] \quad (10) \\ &= \sum_{k=0}^{N-N^{\min}} kP(\Delta X_{ij} = k \wedge X_{j0} \geq N^{\min} \\ & \quad \wedge X_{j0} + \Delta X_{ij} \geq 2N^{\min}) \\ &= \sum_{k=0}^{N-N^{\min}} kP(\Delta X_{ij} = k \wedge X_{j0} \geq \max(N^{\min}, 2N^{\min} - k)) \\ &= \sum_{k=0}^{N-N^{\min}} k\Delta F(k, q_{j0}^i, N - N^{\min}) \\ & \quad \cdot (1 - F_{j0}(\max(N^{\min}, 2N^{\min} - k) - 1, q_{j0}^j, N - N^{\min})), \end{aligned}$$

and similarly

$$\begin{aligned} & P(B_{ij})E[\Delta X_{ij}|B_{ij}] \quad (11) \\ &= \sum_{k=0}^{N-N^{\min}} k\Delta F(k, q_{j0}^i, N - N^{\min})(F_{j0}(N^{\min} - 1, q_{j0}^j, N - N^{\min}) \\ & \quad - F_{j0}(N^{\min} - k - 1, q_{j0}^j, N - N^{\min})). \end{aligned}$$

### 3.2 Effect of Existence of Other Users

Let  $P_{i,-j}^c$  be the probability that  $i$  secures cooperation from some user  $\ell$ ,  $\ell \neq i, j$  and  $\alpha_{i,\ell}$  be the probability that  $i$  secures cooperation from user  $\ell$ . Then

$$\begin{aligned} \alpha_{i,\ell} &= P(X_{i\ell} \geq N^{\min} \wedge (A_{i\ell} \vee B_{i\ell}))P_{i\ell}^c \\ &= P(X_{i\ell} \geq N^{\min})(P(A_{i\ell}) + P(B_{i\ell}))P_{i\ell}^c, \end{aligned} \quad (12)$$

$$P_{i,-j}^c = 1 - \prod_{\ell \neq i, j} (1 - \alpha_{i,\ell}). \quad (13)$$

where

$$P(X_{i\ell} \geq N^{\min}) = 1 - F(N^{\min} - 1, q_{i\ell}^i, N) \quad (14)$$

and  $A_{i\ell}, B_{i\ell}$  are defined in the same way as  $A, B$ . Let  $u_{ij}^r$  be the expected utility of  $i$  if  $j$  refuses to cooperation. Based on (13), we have

$$u_{ij}^s = P_{i,-j}^c N^{\min}. \quad (15)$$

The computation of expected utility for  $j$ , denoted by  $u_{ij}^f$ , when it agrees to forward for  $i$  is more complicated, because whether  $i$  takes this offer or not depends on how many acknowledgement it receives from all the potential forwarders. Recall  $i$  would randomly select one according to the rule. Let  $U = \{1, 2, \dots, M\}$  denote the set of all users. We first calculate the probability that  $i$  takes  $j$ 's offer, denoted by  $P_{ij}^o$ ,

$$\begin{aligned} P_{ij}^o &= \sum_{k=0}^{M-2} \left[ \sum_{\substack{S \subset U \setminus \{i,j\} \\ |S|=k}} \prod_{\ell \in S} \alpha_{i\ell} \prod_{m \in S^c} (1 - \alpha_{im}) \right] \frac{1}{k+1} \\ &= \int_0^1 \prod_{\ell \in U \setminus \{i,j\}} (1 - \alpha_{i\ell} + \alpha_{i\ell}x) dx. \end{aligned} \quad (16)$$

Then

$$u_{ij}^f = P_{ij}^o v_{ij}^f \quad (17)$$

where  $v_{ij}^f$  is given by (8).

### 3.3 Pairwise Nash Bargaining Solution

Now we can draw the extensive form of the stage game as shown in Fig. 2, each leaf representing the expected utilities resulted from the respected decision –  $c$  for “cooperation” and  $n$  for “noncooperation”.

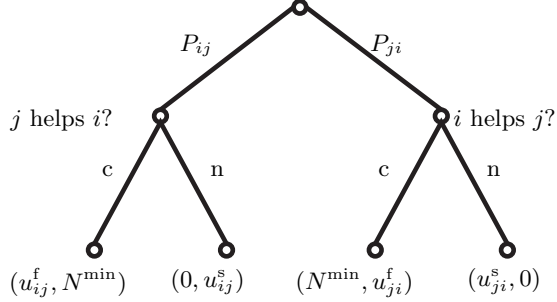


Fig. 2: Extensive form of the two-user stage game.

The normal form of the game, as shown in Table 2, consists of four strategy profiles and their associated payoff profiles, denoted by  $\langle n, c \rangle$ ,  $\langle c, c \rangle$ ,  $\langle c, n \rangle$  and  $\langle n, n \rangle$ , where  $\langle n, c \rangle$  (abbreviation for  $\langle \text{noncooperation}, \text{cooperation} \rangle$ ) means user  $j$  would choose not to forward for  $i$  if  $i$  requests its cooperation while  $i$  would choose to forward for  $j$  if  $j$  requests its cooperation. The two-user NBS is then a mixed strategy profile of these four that maximizes the proportional fairness metric, i.e.,



Table 2: Normal form of the stage game. The first component is  $j$ 's average utility, the second component  $i$ 's average utility, corresponding to the specific strategy profile.

$j \backslash i$	cooperation (c)	noncooperation (n)
c	$\left( \begin{array}{c} P_{ij}u_{ij}^f + P_{ji}N^{\min} \\ P_{ij}N^{\min} + P_{ji}u_{ji}^f \end{array} \right)$	$\left( \begin{array}{c} P_{ij}u_{ij}^f + P_{ji}u_{ji}^s \\ P_{ij}N^{\min} \end{array} \right)$
n	$\left( \begin{array}{c} P_{ji}N^{\min} \\ P_{ij}u_{ij}^s + P_{ji}u_{ji}^f \end{array} \right)$	$\left( \begin{array}{c} P_{ji}u_{ji}^s \\ P_{ij}u_{ij}^s \end{array} \right)$

$$\begin{aligned}
 & \max_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} && u_i u_j, && (18) \\
 \text{s.t.} &&& u_j = \lambda_1 (P_{ij}u_{ij}^f + P_{ji}N^{\min}) + \lambda_2 (P_{ij}u_{ij}^f \\
 &&& + P_{ji}u_{ji}^s) + \lambda_3 P_{ji}N^{\min} + \lambda_4 P_{ji}u_{ji}^s, \\
 &&& u_i = \lambda_1 (P_{ij}N^{\min} + P_{ji}u_{ji}^f) + \lambda_2 P_{ij}N^{\min} \\
 &&& + \lambda_3 (P_{ij}u_{ij}^s + P_{ji}u_{ji}^f) + \lambda_4 P_{ij}u_{ij}^s, \\
 &&& \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1, \quad \lambda_i \geq 0, \quad i = 1, 2, 3, 4.
 \end{aligned}$$

The cooperation probabilities are then given by

$$P_{ij}^c = \lambda_1 + \lambda_2, \quad P_{ji}^c = \lambda_1 + \lambda_3. \quad (19)$$

Due to the recursive form as shown in (12), in practice the pairwise NBS may need to be evaluated repeatedly until it converges. This process is summarized in the following algorithm. The simplicity of this algorithm is in contrast with

---

**Alg. 1** Algorithm for Computing NBS Based on BE

---

- Require:** initialize  $\{P_{ij}^c\}_{i \neq j}$
- 1: retrieve location information from database
  - 2: compute  $\{q_{ij}^i\}_{i \neq j}$  using a geometric channel model
  - 3: compute  $\{P(A_{ij})\}_{i \neq j}$  and  $\{P(B_{ij})\}_{i \neq j}$  with (5) and (6)
  - 4: compute  $\{P_{ij}\}_{i \neq j}$  with (7)
  - 5: compute  $\{v_{ij}^f\}_{i \neq j}$  with (8)
  - 6: **repeat**
  - 7:   compute  $\{\alpha_{ij}\}_{i \neq j}$  with (12)
  - 8:   compute  $\{P_{i,-j}^c\}_{i \neq j}$  with (13), then  $\{u_{ij}^s\}_{i \neq j}$  with (15)
  - 9:   compute  $\{P_{ij}^o\}_{i \neq j}$  with (16), then  $\{u_{ij}^f\}_{i \neq j}$  with (17)
  - 10:   compute  $P_{ij}^c$  by solving (18)
  - 11: **until**  $\{P_{ij}^c\}_{i \neq j}$  converge
- 

the complexity of the original BE algorithm presented in [4]. Initialization of  $P_{ij}^c$  can be arbitrary as experiments show that it has little effect on the outcome. In experiment, it has been observed that convergence of Alg. 1 is fast, usually

taking less than 100 iterations for  $\leq 50$  users, though the proof or condition under which the convergence is unique is still ongoing work.

## 4 Performance Improvement

Performance improvement is measured in terms of outage probability and the average number of usable subcarriers (proportional to rate). Without BE, the outage probability for an arbitrary user  $i$  is given as

$$P_i^{\text{out}} = P(X_{i0} < N_{\min}) = F(N_{\min} - 1, q_{i0}^i, N). \quad (20)$$

With BE, the situation can be improved if, during a normal outage, any other user  $j$  could provide cooperation, which happens with probability  $\alpha_{ij}$ . Therefore, with cooperation, the outage probability is given as

$$P_i^{\text{out, c}} = P_i^{\text{out}} \prod_{j \neq i} (1 - \alpha_{ij}), \quad (21)$$

i.e., the improvement is by a factor of  $\prod_{j \neq i} (1 - \alpha_{ij})$ . Without BE, the average number of usable subcarriers for an arbitrary user  $i$  is given as

$$m_i = \sum_{k=N_{\min}}^N k P(X_{i0} = k). \quad (22)$$

With BE, this number is

$$\begin{aligned} m_i &= P(X_{i0} \geq N_{\min}) E[X_{i0} | X_{i0} \geq N_{\min}] \\ &\quad + P(X_{i0} < N_{\min} \wedge \text{no cooperation}) N_{\min} \\ &= m_i + F(N_{\min} - 1, q_{i0}^i, N) \left( 1 - \prod_{i \neq j} (1 - \alpha_{ij}) \right) N_{\min}, \end{aligned} \quad (23)$$

i.e., the improvement is given as

$$m_i^c - m_i = F(N_{\min} - 1, q_{i0}^i, N) \left( 1 - \prod_{i \neq j} (1 - \alpha_{ij}) \right) N_{\min}. \quad (24)$$

## 5 Numerical Results

### 5.1 Simulation model

For the purpose of illustration, we consider an infrastructure network in white space using OFDMA. We put 10 to 50 secondary users randomly in a 2000m

$\times 2000\text{m}$  area where AP sits at the center. Each user is allocated with 40 subcarriers with 10KHz spacing. The total bandwidth  $N \times 400\text{KHz}$  will be taken from upper end of white space, i.e., from 698MHz down, i.e., user 1 is allocated 697.6-698MHz, user 2 allocated 697.2-697.6MHz, and so on. With the maximum number of 50 secondary users, TV white space channels 48 – 51 will provide enough bandwidth for our simulation.

We use the Hata urban model to simulate frequency dependent path loss. Given the antenna height at the AP ( $h_A = 10\text{m}$ ) and antenna height at the secondary user ( $h_U = 1\text{m}$ ), the path loss is modeled as

$$L(f) = 69.55 + 26.16 \log_{10} f - 13.82 \log_{10} h_A - C_A \\ + (44.9 - 6.55 \log_{10} h_A) \log_{10} d + \Delta L, \quad (25)$$

$$C_A = 0.8 + (1.1 \log_{10} f - 0.7)h_U - 1.56 \log_{10} f \quad (26)$$

where  $d$  is the distance over which we make the measurement,  $\Delta L$  is the log-normal shadowing term with zero mean and 8dB standard deviation,  $C_A$  is the correction term. Since each user has only (consecutive) 400KHz bandwidth, we can simplify the simulation by assuming the same path loss statistics for all subcarriers that belong to a single user.

We consider a subcarrier usable if the associated path loss is less than 130dB, a link usable if the number of usable subcarriers is no less than  $N^{\min} = 10$ .

## 5.2 Simulation Results

Fig. 3 shows the average outage probability with and without BE-based cooperation. As the number of users increase, the outage probability scales down exponentially demonstrating the power of user cooperation diversity incentivised by BE. Fig. 4 shows that BE also helps improve the rate. With BE, each user has on average additional 4 usable subcarriers. Considering that every user requires 10 subcarriers to have a sustainable connection, this improvement is substantial. Fig. 5 shows that the efficiency achieved by BE does not come at the cost of fairness. Specifically, with different number of total users in the system, we find the user with the highest/lowest outage probability and plot it against the left  $y$ -axis. At the same time we plot against the right  $y$ -axis the average number of additional subcarriers made available to it through BE and NBS. It is seen that the highest outage user receives nearly 10 additional subcarriers that help it almost always have a sustainable connection; the lowest outage user however does not benefit as much resulting in a higher social welfare as we expected.

## 6 Conclusion and Discussions

In the absence of a predefined coordination infrastructure, TV white space and WSD lack the mechanism that systematically brings self-conscious secondary users into synergy. In this paper we discussed a strategy to incentivize cooperative forwarding that constitutes an essential issue in secondary coexistence

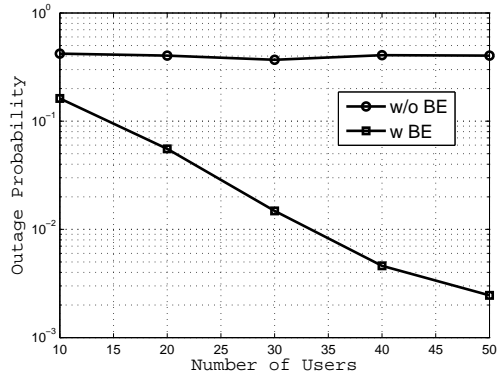


Fig. 3: Average outage probability with and without BE.

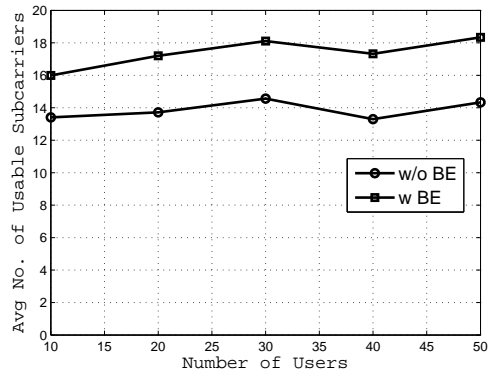


Fig. 4: Average available subcarriers with and without BE.

in white space. The strategy has been built with the particular notion of geo-location database in mind, which is mandatorily required to be consulted by all white space devices. Specifically for secondary users equipped with OFDM enabled radios, we showed that the database combined with a reliable channel model can be used to realize a form of cooperation called bandwidth exchange which promotes fair and efficient operation via a Nash bargaining framework. Though discussed in previous studies, the channel and network state information enabled by the database makes the bargaining process much faster and more accurate by taking into consideration the effect of existence of many other users. The numerical results show that bandwidth exchange dramatically improves the system performance in terms of outage and rate without compromising fairness.

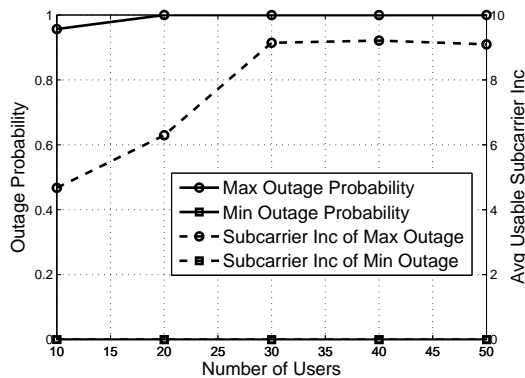


Fig. 5: User with the highest/lowest outage probability and the average extra number of subcarriers made available to it via BE.

## References

1. FCC, Second Memorandum Opinion and Order, [http://www.fcc.gov/Daily\\_Releases/Daily\\_Business/2010/db0923/FCC-10-174A1.pdf](http://www.fcc.gov/Daily_Releases/Daily_Business/2010/db0923/FCC-10-174A1.pdf) (2010)
2. Biswas, S., Morris, R.: Opportunistic Routing in Multi-Hop Wireless Networks. In: ACM SIGCOMM (2004)
3. Dianati, M., Ling, X., Naik, K., Shen, X.: A Node-Cooperative ARQ Scheme for Wireless ad hoc Networks. *IEEE Trans. Veh. Technol.*, vol. 55, no. 3, pp. 1032–1044 (2006)
4. Zhang, D., Shinkuma, R., Mandayam, N.: Bandwidth Exchange: An Energy Conserving Incentive Mechanism for Cooperation. *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2055–2065 (2010)
5. Owen, G.: *Game Theory*, 3rd ed. Academic Press (1995)