Multipath Beamforming UWB Signal Design Based on Ternary Sequences

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Abstract

We study an adaptive transmission approach that exploits the increased phase coherence of ultra-wideband (UWB) channels with a "discrete" number of reflectors relative to narrowband wireless channels. The proposed multipath beamforming technique suggests selecting a ternary or multilevel sequence for an impulsive direct sequence UWB signal waveform based on the signs of the reflection coefficients corresponding to a few strongest paths so that the signal energy at the output of a non-adaptive correlator is enhanced. The approach to beamforming signal design is illustrated by several example designs.

1 Introduction

We study an adaptive approach that exploits the increased phase coherence of ultra-wideband (UWB) channels [1] with a "discrete" number of reflectors relative to narrowband wireless channels. The proposed multipath beamforming technique suggests selecting the transmitted signal waveform based on limited multipath channel state information so that the signal energy at the output of a non-adaptive correlator is enhanced. The approach falls into the general category of adaptive signal design where the transmitter has a full or partial knowledge of the channel. Narrowband literature on this topic is abundant; nevertheless, even though somewhat related UWB literature exists (see, e.g. [2]), an approach that explicitly exploits the distinct phase coherence characteristic of impulsive UWB signaling has not been found in the literature.

It is clear that unlike carrier-based technologies, UWB impulse-based technologies allow for turning the transmitter's power off during short intervals of time. Ternary based signaling which includes epochs of zero (power off) signal amplitude is a natural extension of binary antipodal signaling for this type of technology. We suggest ternary direct sequence impulsive UWB (TS-UWB) signaling as a unifying descriptor of a number of impulse-based UWB signaling schemes. When the number of non-zero chips within a ternary spreading code is not much smaller than its length N, TS-UWB retains the low peak-to-average power ratio and low probability of detection characteristics of binary DS-UWB (the former is helpful in enabling low-cost designs). On the other hand, allowing for some of the chips to be 0 enables significant improvement in the autocorrelation properties of the employed signaling. In fact, one can construct a number of perfect ternary sequences with perfect periodic autocorrelation properties (i.e., their out-of-phase periodic auto-correlation values are all equal to 0 [3, 4]). Signals with good correlation properties are of special importance for high rate/low power UWB communications for several reasons. In particular, synchronization efficiency, multi-path resolution, inter-symbol-interference (ISI) suppression, and path channel estimation can be significantly degraded when the number of chips per symbol (i.e., the processing gain) and the corresponding autocorrelation sidelobe suppression are reduced.

In view of target high data rate applications and corresponding simple receiver design, our approach focuses on signal designs that enhance the performance of the non-adaptive correlator.

The first approach exhaustively searches ternary beamforming sequences that maximize any of two criteria. The optimal ternary beamforming sequences minimize detector's error probability, the suboptimal one enhances the signal energy at the output of a matched filter based on the sign of the reflection coefficients of several strongest reflectors. Here we assume the signs of the reflection coefficients are known, as would be natural in a TDD system. It is obvious that the exhaustive computer search is only suitable for relatively short sequences.

We also provide a constructive approach for multilevel beamforming sequence design. This method is based on the autocorrelation properties of perfect ternary sequence, the idea of pre-RAKE [5] and Lüke's approach to the construction of binary Alexis sequences [6]. In [5], the authors suggested concentrating all the processing required for the RAKE combination at the transmitter and keeping the receiver as simple as a non-combining single path receiver. In [6], Lüke suggested an approach to the construction of binary Alexis sequences [7] for various length. The aperiodic autocorrelation function (ACF) of Alexis sequences vanishes in a broad window. We combined these ideas to achieve beamforming gain in a multipath environment by employing a multilevel beamforming sequence and a receiver correlation sequence at transmitter and receiver side respectively. Note that, the only knowledge we need to construct the beamforming sequence is the signs of several strongest multipath coefficients.

The paper is organized as follows. A channel model is introduced in section 2. According to the characteristics of a UWB channel, some important and reasonable assumptions we made here. In section 3, we provide approaches to obtain the beamforming sequences. Simulation results demonstrating the beamforming gains are presented in section 4. Finally, in section 5, we draw some concluding remarks.

2 Signal Model

The received signal model is:

$$r(t) = \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \alpha_l b_k p(t - kT - \tau_l) + n(t)$$

where

$$p(t) = \sum_{n=0}^{N-1} c_n \psi(t - nT_c)$$

b_k are binary antipodal symbols, $T = NT_c$ is the symbol period, K is the number of symbols, N is the length of the spreading sequence $\{c_n\}$ whose elements are in $\{-1, +1, 0\}$, $\psi(t)$ is the signaling pulse assumed here to be equal to zero outside an interval which is equal to or smaller to the chip interval $[0, T_c]$ and assumed known to the receiver.

L is the number of multipath components. The α_l are real channel coefficients, each with a random sign. The delays τ_l are either fixed or chosen randomly. When sufficient multipath resolution is available small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients can be modelled as real numbers whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [8]. In addition, in slowly changing environments determining, tracking, and conveying the signs of reflection coefficients to the transmitter would require a non-significant overhead. In fact, due to symmetry in a UWB TDD system no receiver/transmitter feedback is necessary.

It is assumed that the transmitter knows the sign of the first L_c paths (e.g., based on receiver feedback) and can select the spreading sequence which attempts to maximize the energy at the output of the matched filter. The resulting effect is that the first L_c paths are combined coherently in a beamforming manner. The crosscorrelation between any one of the L_c path components times the channel coefficient and the line of sight path component is positive and, thus, enhances the received signal energy.

3 Beamforming Sequence Design

3.1 Exhaustive computer search for ternary beamforming sequence

We address exhaustive computer search maximization of two criteria only for relatively short sequences.

The energy-based criterion for beamforming sequence selection is:

$$\begin{aligned} &\textstyle\sum_{l=0}^{L_c-1} sign(\alpha_l) E\{|\alpha_l|\} R(l) - \sum_{l=L_c}^{N-1} E\{|\alpha_l|\}|R(l)| \\ &- \textstyle\sum_{l=0}^{L_c-1} E\{|\alpha_l|\}|R(N-l)| - \sum_{l=L_c}^{N-1} E\{|\alpha_l|\}|R(N-l)| - B \end{aligned} \tag{1}$$

where

$$\begin{aligned} &B = \textstyle\sum_{k=1}^{\lfloor (L-1)/N \rfloor} E\{|\alpha_{kN}|\}|R(0)| \\ &+ \textstyle\sum_{l=N+1, l \mathtt{mod} N \neq 0}^{L-1} E\{|\alpha_l|\}(|R(l \mathtt{mod} N)| + |R(N - l\mathtt{mod} N)|) \end{aligned} \tag{2}$$

$R(\tau)$ is the autocorrelation function of $p(t)$ with T_c being suppressed (assumed to be equal to 1) for notational simplicity.

Here we assume that the mean values of the channel coefficients are known. Simply setting these values to be 1, we obtain another criterion for the case when we only know the signs of first several strongest channel coefficients.

The optimal bit-error rate criterion for sequence selection is based on the probability of detection error.

$$p(error) = E_{\boldsymbol{\alpha},\mathbf{b}}\{p(error|\boldsymbol{\alpha}, \mathbf{b})\} \tag{3}$$

where the expectation is over $L-1$ random channel coefficients $\boldsymbol{\alpha} = \{\alpha_1, \ldots, \alpha_{L-1}\}$ (here, note that the sign of the first L_c coefficients might be known) and $\lfloor L/N \rfloor$ interfering symbols b $= \{b_1, \ldots, b_{\lfloor L/N \rfloor}\}$. For a correlator detector, the conditional probability of error is:

$$p(error|\boldsymbol{\alpha}, \mathbf{b}) = \begin{cases} \frac{1}{2}\mathrm{erfc}(\sqrt{E_b/N_0}) & A_b \geq 0 \\ 1 - \frac{1}{2}\mathrm{erfc}(\sqrt{E_b/N_0}) & A_b < 0 \end{cases} \tag{4}$$

where erfc(.) is the complementary error function. Here, $E_b = A_b^2$, where

$$\begin{aligned} A_b = \textstyle\sum_{l=0}^{N-1} \alpha_l R(l) + \sum_{l=1}^{N-1} b_1 \alpha_l R((N-l)) \\ + \textstyle\sum_{l=N+1, l\bmod N \neq 0}^{L-1} b_{\lfloor l/N \rfloor} \alpha_l R(l \bmod N) \\ + b_{1+\lfloor l/N \rfloor} \alpha_l R(N - l \bmod N) + \textstyle\sum_{k=1}^{\lfloor (L-1)/N \rfloor} b_k \alpha_{kN} R(0) \end{aligned} \tag{5}$$

Note that the energy-based criteria, as defined above, is based on the mean values of channel coefficients, whereas, the probability of error criteria requires the knowledge of the coefficient probability distributions. A particularly important component of the sequence design is the peak to average ratio (PAR) of the signal. For ternary based signaling, arbitrarily padding the transmitted sequence with zeros is helpful to achieving good ACF properties. The tradeoff is an increase in PAR. A reasonable approach is to find the best sequence under a constraint on the number of zeros (or, more generally, under a PAR constraint). Even so, as will be demonstrated in Section 4, the single pulse (maximum PAR solution) sequence is not always optimum for signaling in multipath channels when beamforming is aided with limited channel state information at the transmitter.

3.2 Constructive design of multilevel beamforming sequence

The construction of multilevel beamforming sequence is based on the properties of perfect ternary sequence, the idea of pre-RAKE and Lüke's approach to the binary Alexis sequences design. A mother perfect ternary sequence $\{c_0, c_1, \ldots c_{N-1}\}$, for which

$$\sum_{i=0}^{N-1} c_i c_{i+k} = \begin{cases} M & \text{if } (k \bmod N) \equiv 0 \\ 0 & \text{if } (k \bmod N) \neq 0 \end{cases} \tag{6}$$

where M is the number of non-zero elements in the sequence, is linearly combined with its first $L_c - 1$ left cyclic shifts where the combining coefficients are the signs of the first L_c paths. Thus, the multilevel beamforming sequence $\{\tilde{c}_0, \tilde{c}_1, \ldots, \tilde{c}_{N+\tilde{L}-1}\}$ is obtained by appending $\tilde{L} < L - 1$ zero guard chips to

$$\tilde{c}_i = \sum_{l=0}^{L_c - 1} sign(\alpha_l) c_{(i+l) mod N}, i = 0, 1, \ldots N - 1 \tag{7}$$

By periodically extending the mother perfect ternary sequence up to length $N + \tilde{L}$, we obtain the receiver correlation sequence. By employing this pair of sequences and a simple correlation receiver, the signal replicas corresponding to the first L_c paths are coherently combined at the output of the correlator. For example, assume that the multipath number is 11, that the signs of the first three paths (including the direct path) are {+, -, +} respectively, and that the mother sequence is the Hoholdt's perfect ternary sequence {+ + + + + - + 0 + 0 - + + - 0 0 + - 0 - -} [3]. Then, the transmitter beamforming sequence of length 31 is {1 1 1 -1 3 -2 2 -1 0 2 -1 -1 2 -1 1 -2 2 -2 0 1 -1 0 0 0 0 0 0 0 0 0 0 } and the received correlation sequence is {+ + + + + - + 0 + 0 - + + - 0 0 + - 0 - - + + + + + - + 0 + 0}.

We note that the transmitter beamforming sequence is a multilevel signal with a number of levels no larger than $2L_c + 1$, while the received correlation sequence is still ternary. In this example, the energy from the first, second, and the third path is coherently combined. The multipath interference from other paths will be suppressed due to the perfect ACF properties of the mother perfect ternary sequence. The noise penalty increases since we employ different beamforming and receiver correlation sequence. Thus, this approach trades multipath interference suppression and beamforming gain for SNR loss and an increase in the number of levels of the beamforming sequence. The simulation result in the next section shows that even with no zero guard chips, multilevel beamforming sequence can still achieve lower BER than single pulse sequence.

4 Analysis

Two sets of experiments have been conducted. Bit energy for all signaling schemes is normalized, so that, for example, a ternary sequence of length seven with three zero pulses have pulses with a peak 30% higher and single impulse sequence pulses 160% higher than the corresponding m-sequence pulses. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration T_c. It is assumed that each bin can contain only one path; the sign of the reflected path coefficient is modelled as a uniformly distributed random variable [9]. Effects of interchip interferences have been assumed negligible. Employed multipath beamforming was based on the sign of first L_c path coefficients. For each simulation block, one of 2^{L_c} optimum ternary sequences was selected based on the sign of the first L_c reflected path coefficients.

In the first set of experiments, we focus on high rate direct sequence ultra-wideband schemes with low spreading gain $N = 7$, which, for a chip time of $T_c < 1$ nanoseconds would correspond to data rates of over 100 Mbits. Note that such high rate systems employed in channels with significant multipath can experience significant inter-symbol interference which can dominate the performance degradation, in particular, at high SNRs. We have employed two sets of channel coefficients. One is an exponentially decaying profile, the other is a deterministic set of coefficients based on the indoor line of sight (LOS) measurements (performed in 23 homes) [10]. In the latter case, channel coefficients are chosen to be equal to average values given in [10] where it is observed that the line of sight component and the first 10 paths account for 33% and 75% of the total power, respectively. In both cases, the distribution of the arrival times of individual multipath components follows a modified Poisson model (namely, the $\Delta - K$ model) [11]. Here, we have used k=0.5 (see [12]). Two scenarios have been considered. In the first, the delay spread was restricted to be less than 8 bins, and in the second to be less than 11 bins We study correlator receiver performance for beamforming, m-, single impulse and ternary sequences.

Figures 1 and 2 depict the results based on the exponential multipath profile decreasing 10dB (in power) over a 4 nanosecond excess delay and the decreasing 10dB over a 2.8 nanosecond excess delay, respectively. Results in Figure 3 is based on the profile in [10] truncated to 2.4 nanoseconds. Figure 1 demonstrates the gain of the multipath beamforming approach which requires knowledge of the first four indirect path coefficient, means and signs over the non-adaptive single impulse, m-, and perfect ternary sequence. For a range of SNRs both single pulse and the beamforming approach can overcome the inter-symbol interference induced error floor observed when either perfect ternary or m-sequences are used. Figure 2 demonstrates that beamforming based on two path coefficients which constrains the number of zeros to be less or equal to three can still improve over the non-adaptive single pulse sequence. Figure 3 demonstrates that correlator receiver with multipath beamforming and ternary sequences can capture as much energy as the maximum ratio combining (MRC) receiver when m-sequences are used.

The second set of experiments focuses on the long beamforming sequences and assumes that each bin contains exactly one multipath component (emulating a dense multipath environment) and an exponential profile decreasing 10dB in power over a 4.4 nanosecond excess delay. Employed pair of transmitter multilevel beamforming and receiver correlation sequences is based on a Hoholdt's perfect ternary sequence and has been introduced in Section 3.2. Figure 4 depicts that a 2dB beamforming gain can be achieved by employing beamforming sequences over m- and single pulse sequences. In Figure 5, demonstrates that the beamforming approach benefits from the multipath interference suppression and beamforming gain exceed the noise penalty due to difference in transmitting and receiving sequences when the excess delay is larger than 1.6ns. In Figure 6, we shorten the transmitter multilevel beamforming sequence by reducing the number of zero guard chips. At the receiver, the correlation sequence will accordingly shrink. The more zeros we reduce, the higher the multipath and inter-symbol interference. Even in the case of zero guard chips, the SNR required for achieving the BER of 10^{-3} is smaller for the beamforming approach than that for the single pulse sequence.

5 Conclusions

We have proposed TS-UWB signaling as a unifying descriptor of a number of impulse-based ultra-wideband signaling schemes. In a LOS environment, based on the signs (and/or mean) of a few strongest multipaths' coefficients, the transmitter adaptively selects the beamforming sequence so that the signal energy of these paths coherently combines at the output of a receiver correlator. By employing both the ternary and multilevel beamforming sequences, better performance can be achieved than by using a single pulse sequence while still keeping a low PAR.

References

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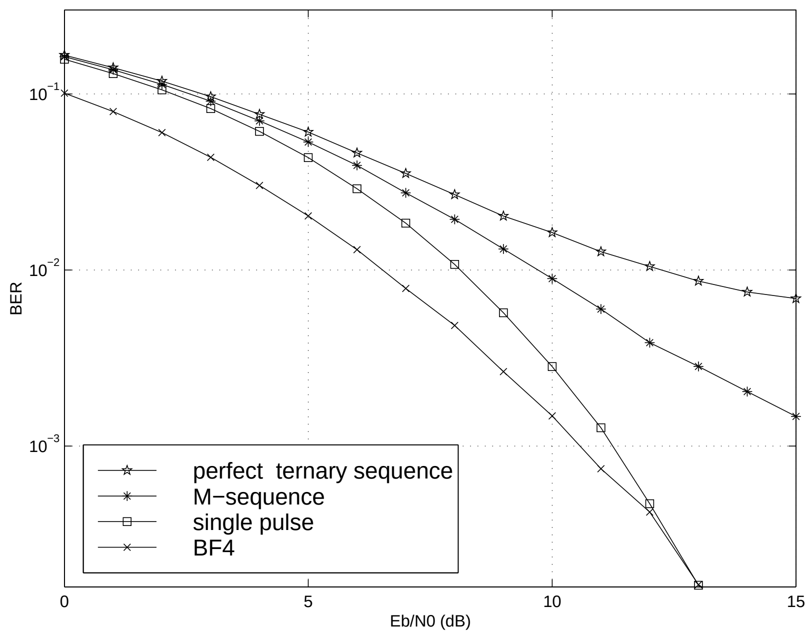

Figure 1: Short sequences BER performance for an exponential channel profile and supersymbol path delays.

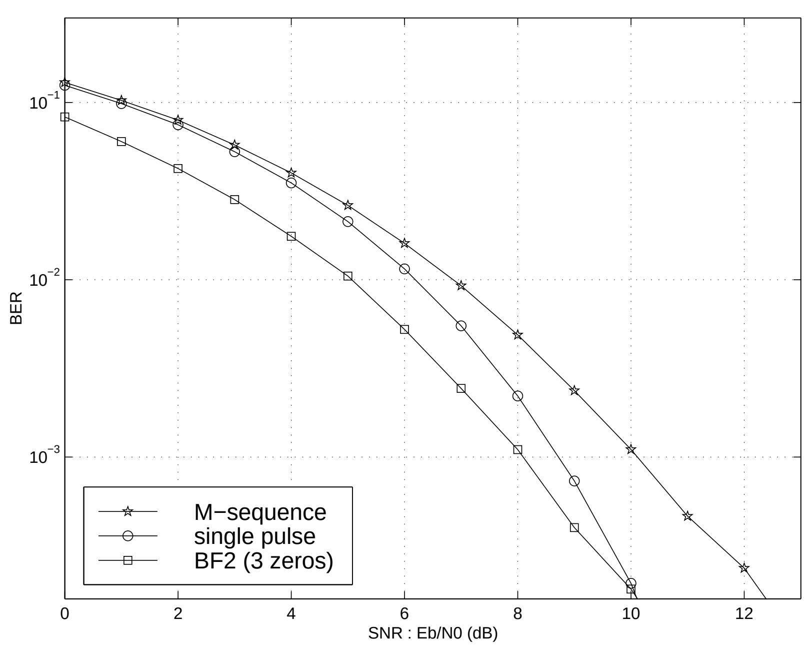

Figure 2: Short sequences BER performance for an exponential channel profile and subsymbol path delays: PAR optimized sequences.

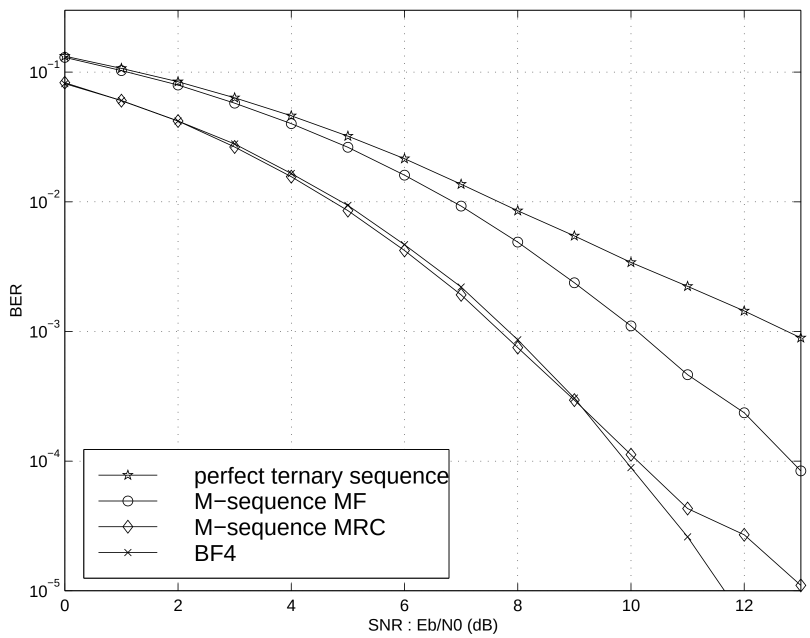

Figure 3: Short sequences BER performance for the channel based on [10].

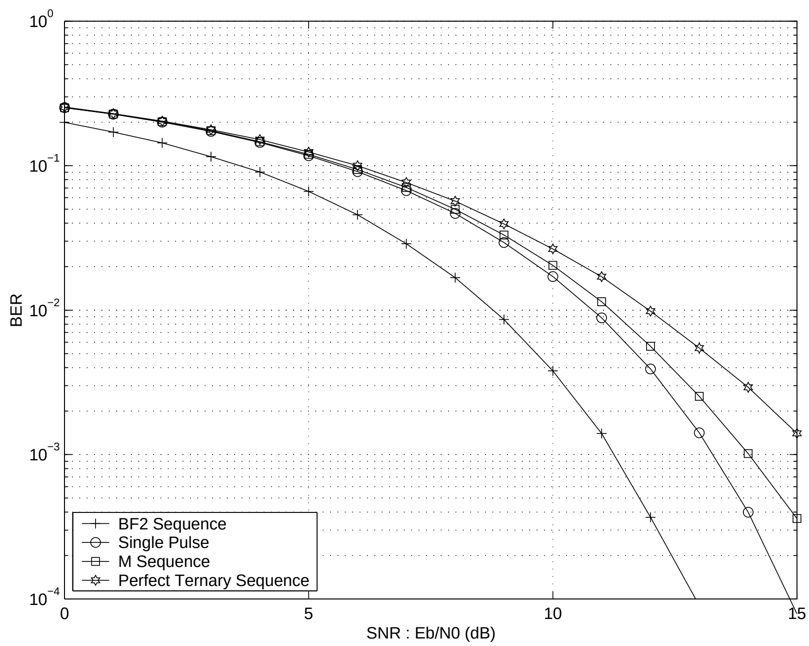

Figure 4: Long sequences BER performance for an exponential channel profile.

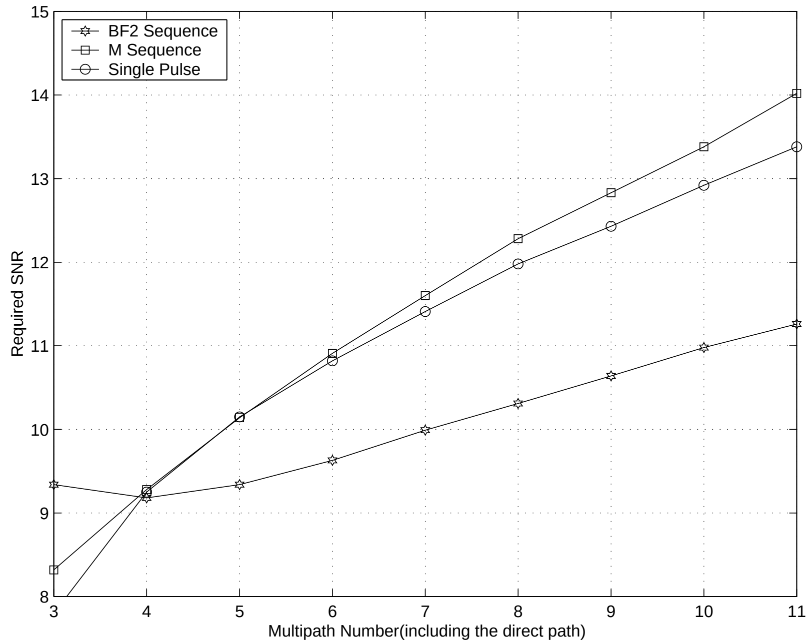

Figure 5: Required SNR vs number of multipath components for BER=10^{-3}

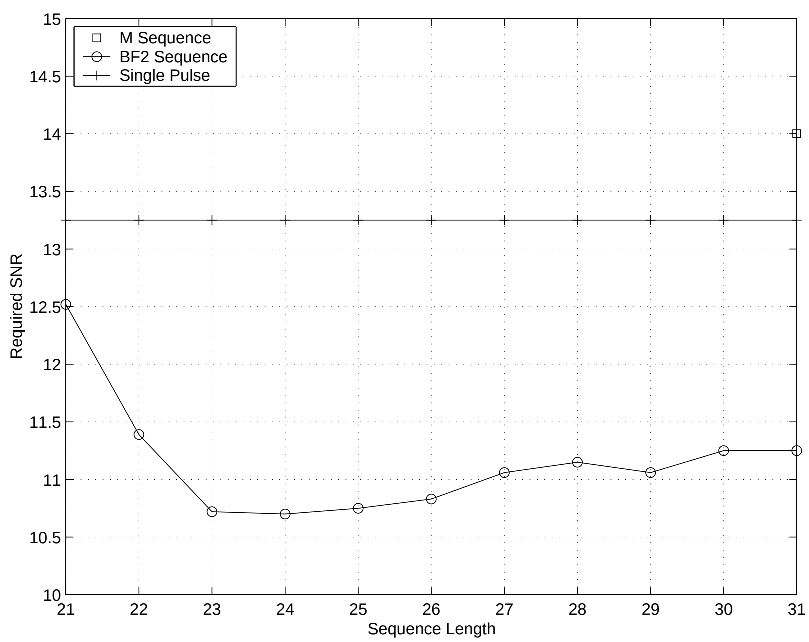

Figure 6: Required SNR vs sequence length for BER=10^{-3} (Increasing ISI experienced with a decrease in zero padding for beamforming sequences.)