Is User-Cooperation in Wireless Networks Always Beneficial?

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1. Motivation and Context
2. Coalitional Game theory overview
3. Receiver cooperation in an interference channel
4. Transmitter cooperation
5. Summary and Future work
Motivation

- Cooperation in wireless networks promises improved sharing of time and bandwidth resources
  - Advent of cognitive radio promises to make this a reality

- Does cooperation come with conflict?
  - Do all users always gain from cooperation?
  - Are shared resources utilized better when users cooperate?

- Can cooperative protocols induce the formation of disjoint groups of users that are "stable"?
  - Coalitional game theory can provide answers.
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A simple example

- 3 users A, B and C communicating with their receivers (assume co-located)

- Receivers can cooperate by jointly decoding their received signals.

- Suppose sum-rate achieved by a coalition is apportioned equally.
  - What cooperative behavior emerges?
  - What coalitions are formed?
Figure: Stable coalition structures when recd. SNR of user 3 is fixed while those of A and B are varied.
A coalitional game with transferable utility \( \langle S, v \rangle \)

- finite set of players \( S \)
- value function \( v : G \rightarrow \mathbb{R} \) \( \forall G \subseteq S \)

Payoff: Share of the value \( v(G) \) to each player.

Characteristic function form game: \( v(G) \) is unaffected by the "strategy" of users not in \( G \).

When \( v(G) \) can be flexibly apportioned between cooperating players, the game is said to have transferable utility (TU).
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A coalitional game with TU is cohesive if 
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A game with TU is **superadditive** if for any two disjoint coalitions \( G_1 \) and \( G_2 \) we have
\[ v(G_1 \cup G_2) \geq v(G_1) + v(G_2). \]
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Example:

\[
\begin{array}{c}
A \quad B \\
2 \quad 2 \\
v(G_1) = 4
\end{array} \quad \begin{array}{c}
A \quad B \quad C \quad D \\
1 \quad 1 \\
v(G_2) = 2
\end{array}
\]

\[
\begin{array}{c}
A \quad B \quad C \quad D \\
1.75 \quad 1.75 \quad 1.75 \quad 1.75
\end{array}
\]

**Equal Apportioning**

- Not everyone is better off
- NOT STABLE

\[ v(G_1 \cup G_2) = 7 \]

(superadditive)

\[
\begin{array}{c}
A \quad B \quad C \quad D \\
2.25 \quad 2.25 \quad 1.25 \quad 1.25
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**Transferable Payoff**

- Everyone is better off
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**Example:**

\[ \begin{array}{c|c|c|c|c}
   & A & B & C & D \\
\hline
A & 2 & 2 & 1 & 1 \\
B & 1 & 1 & & \\
C & & & & \\
D & & & & \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
   \text{Equal Apportioning} & & & & \\
\hline
A & 1.75 & 1.75 & 1.75 & 1.75 \\
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D & & & & \\
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\[ v(G_1 \cup G_2) = 7 \] (superadditive)

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\hline
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Everyone is better off

\[ \text{NTU} \quad \text{TU} \]
The core $C(\nu)$ of a coalitional game is the set of feasible payoffs for which no coalition $\mathcal{G}$ has incentive to defect by achieving a greater payoff for all its members.
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The core can be empty!
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The core can be empty!

Empty core $\Rightarrow$ No stable form of cooperation.
An example with an empty core

A simple 3-player game:

\[ S = \{1, 2, 3\} \]
\[ v(\{i\}) = 0, \quad i = 1, 2, 3. \]
\[ v(G) = \alpha, \quad \forall|G| = 2 \]
\[ 0 < \alpha < 1 \]
\[ v(S) = 1 \]

Any feasible payoff profile in the core must satisfy:

\[ x_1 \geq v(\{1\}) = 0 \]
\[ x_2 \geq v(\{2\}) = 0 \]
\[ x_3 \geq v(\{3\}) = 0 \]
\[ x_1 + x_2 \geq v(\{1, 2\}) = \alpha \]
\[ x_2 + x_3 \geq v(\{2, 3\}) = \alpha \]
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- Existence of a non-empty core \(\iff\) feasibility of an LP.
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- Existence of a non-empty core \( \iff \) feasibility of an LP.
- Core is non-empty only if \( \alpha \leq \frac{2}{3} \).
  - Game is superadditive. Superadditivity does not guarantee non-empty core.
Outline

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When different types of devices/networks coexist, Tx cooperation may not always be possible.

- Rx cooperation may be the only feasible way
- Central entity required → Spectrum server [Ileri & Mandayam 2005], [Raman, Yates & Mandayam, 2005].
A spectrum server serves as a central entity that enables disparate devices to jointly decode their received signals.
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Receiver cooperation turns IC into a Gaussian SIMO-MAC.

Under TU, if value $v(G) = \max$ information-theoretic sum-rate for users in $G$

- Are there stable coalitions?
- Does the grand coalition form?
Transmitters use Gaussian signalling.
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$C_G =$ Capacity region of the SIMO-MAC formed by a coalition of links $G$ is given by $C_G = \left\{ R_G : \sum_{k \in A} R_k \leq I(X_A; Y_G | X_G A; \forall A \subset G) \right\}$
Setting up the receiver cooperation game

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- Value $v(G) = \max_{R_G} \sum_{i \in G} R_i = \max_{p_{X_G}} I(X_G; Y_G)$. 
Setting up the receiver cooperation game

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- We are interested in maximum sum-rate.
  - Let $D(C_G)$ denote the dominant face of the capacity region $C_G$ where $\sum_{i \in S} R_i = v(S)$
Receiver cooperation coalitional game is superadditive.
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**Theorem**

The grand coalition (coalition of all links) maximizes spectrum utilization for the receiver cooperation IC coalitional game.
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**Theorem**
The core of the receiver cooperation IC coalitional game is non-empty. In fact, every point on the dominant face $D(C_S)$ of the capacity region $C_S$ of the grand coalition belongs to the core.

[Mathur, Sankaranarayanan & Mandayam, ISIT 2006]
All points on $D(C_S)$ lie in the core.

Core is **non-unique** → Can we assign fairness criteria to particular points?
Fair Allocations - Bargaining for Rates

- All points on $D(C_S)$ lie in the core.
- Core is non-unique → Can we assign fairness criteria to particular points?

Nash Bargaining Solution over IC performance

$$R_{S}^{NBS} = \arg \max_{\{R_{S} : R_{m} > R_{m}^{IC}\}} \prod_{m=1}^{M} \left( R_{m} - R_{m}^{IC} \right)$$
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Nash Bargaining Solution over IC performance

$$R^\text{NBS}_S = \arg \max_{\{R_S : R_m > R^\text{IC}_m\}} \prod_{m=1}^{M} \left( R_m - R^\text{IC}_m \right)$$

Proportional fairness

$$R^\text{PF}_S = \arg \left\{ \max_{\{R_S \in C_S\}} \prod_{m=1}^{M} R_m \right\}$$
An example topology

Figure: A skewed topology.

Table: Rate allocations NBS, PF and ER (NTU)

<table>
<thead>
<tr>
<th>Coalition Structure</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
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<th>Sum-rate</th>
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</thead>
<tbody>
<tr>
<td>{1, 2, 3}_NBS</td>
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<tr>
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<td>3.7703</td>
</tr>
<tr>
<td>Stable ER Coalition: {1, 2}, {3}</td>
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Table: Rate allocations NBS, PF and ER (NTU)
Transmitters are allowed to cooperate:
- Through ideal noise-free inter-user links.
- Cooperating transmitters do joint encoding: by appropriately structuring their transmit covariance matrices.
- All receivers always jointly decode their recd. signals.
- Transmitters can form coalitions.

Is user-cooperation always beneficial?
Coalitions of transmitters - The value of a coalition

- Such cooperation turns the channel into a **MIMO-MAC**.
- Virtual MIMO with individual power constraints.
- Value of a coalition $\mathcal{G}$ of transmitters can be defined again as maximum sum-rate achievable by $\mathcal{G}$.

$$ v(\mathcal{G}) = \max_{\mathcal{X}_G} I(\mathcal{X}_G; Y_S) $$

- Interference seen by $\mathcal{G}$ depends on whether signals from users outside $\mathcal{G}$ coherently combine at Receivers.
- $v(\mathcal{G})$ depends on the actions of players outside $\mathcal{G}$!
- Tx cooperation game not of characteristic function form!
- Difficult to analyze the game in the present form.
Transmitter Jamming Game

- Users in $\mathcal{G}^c$ jam the coalition $\mathcal{G}$ by jointly transmitting the worst case interference signal $X_{\mathcal{G}^c}$ [La & Anantharam, 2002]

$$v(\mathcal{G}) = \min_{X_{\mathcal{G}^c}} \max_{X_\mathcal{G}} \left\{ \log \frac{|I + H_G Q_G H_G^\dagger + H_{G^c} Q_{G^c} H_{G^c}^\dagger|}{|I + H_{G^c} Q_{G^c} H_{G^c}^\dagger|} \right\}$$

$$Q_{ii} \leq P_i \quad i = 1, \ldots, N$$

- $H_G = \text{Channel from users in } \mathcal{G} \text{ to receivers.}$
- $H_{G^c} = \text{Channel from users in } \mathcal{G}^c \text{ to receivers.}$
- $Q_G = \text{Transmit covariance matrix of coalition } \mathcal{G}.$
- $Q_{G^c} = \text{Transmit covariance matrix of users in } \mathcal{G}^c.$

- The log function above is strictly concave in $Q_G$, strictly convex in $Q_{G^c}$.

$\bullet$ $v(\mathcal{G})$ has a saddle point [Diggavi & Cover, 2001].
Transmitter cooperation - Results

**Theorem**
The coalitional game with TX cooperation is **cohesive**.

Proof: Follows from saddle point property.

**Corollary**
Since the game has TU, the grand coalition is the only candidate coalition structure for the core.

- The core cannot be guaranteed to be non-empty
  - We have counter-examples to show that the grand coalition cannot always be guaranteed to be stable.
Example with an empty core

Consider a cooperative MAC (1 receiver) with 5 users with channel gains given by

\[
H = \begin{pmatrix}
4.6 \times 10^{-2} \\
5.4 \times 10^{-3} \\
2.3 \times 10^{-6} \\
1.2 \times 10^{-3} \\
1.5 \times 10^{-2}
\end{pmatrix}
\]

- **Recall**: Existence of a non-empty core \(\equiv\) feasibility of an LP
- Check feasibility of the LP for this example
- Infeasible LP \(\Rightarrow\) Core is empty \(\Rightarrow\) grand coalition not stable
- Further, cohesiveness \(\Rightarrow\) no stable coalition exists.

We find when there are users with widely disparate link gains, cooperation need not be stable.
Motivation and Context

Coalitional Game theory overview

Receiver cooperation in an interference channel

Transmitter cooperation

Summary and Future work
Summary

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  - Is the grand coalition always stable?
  - Are there any stable coalitions?
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  - Grand coalition is the only candidate for the core
  - Not always guaranteed to have stable forms of cooperation.
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Transmitter cooperation as a jamming game:
  - Grand coalition is the only candidate for the core
  - Not always guaranteed to have stable forms of cooperation.

When there are costs to cooperation, cohesiveness cannot be guaranteed and disjoint stable coalitions may result.
  - Coalitional games in linear multiuser detectors.

[Mathur, Sankaranarayanan & Mandayam, CISS 2006]