Capacity of Hierarchical Wireless Networks

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Wireless Ad Hoc Networks

- Network of \( n \) wireless source nodes (SN) [GK, ’00]
- Node throughput ↓ as sources ↑ -- cooperative routing
- \( m > n^{1/2} \) wired access points (AP) reverses throughput scaling [LLT, ’03]
Motivation for Hierarchy

- **Typical Ad Hoc Networks:**
  - Direct communication is **power hungry** (battery) or requires expensive infrastructure (**wired** AP)
  - Cooperative routing between sources reduces **node lifetime**

- **Hierarchical Wireless Networks:**
  - use **wireless** forwarding nodes (FNs) between sources and AP
  - FNs have intermediate processing/energy resources between sources and APs
Capacity of Hierarchical Networks

- **Prior Art**
  - [DML ’03] Sensor networks: *Interference-limited* capacity increases on using FNs
  - Capacity of general HSNs not known
- **Our Work**
  - Simple HSN: a cluster of sources communicating with an AP via an FN
  - Capacity bounds and cooperative strategies for a simple HSN using *constrained* MARC model
  - Examples illustrating capacity-achieving strategies
  - Compare rates against *multi-hopping* rates
Two models for simple HSN

1. Multiple-access relay channel (MARC) (ideal) [KvW ’00]
   - Relay transmits and receives in the same time and bandwidth

2. Constrained MARC (realistic)
   - Relay receives and transmits in time (or frequency) fraction $\alpha$
     and $(1 - \alpha)$ respectively
Cooperative Strategies

- Two basic strategies for a relay channel [Cover, El Gamal ’79] extended to MARCs and C-MARCs as:
  - **Decode-and-forward (DF)**
    - relay decodes $Y_{M+1}$: IT multi-hopping
  - **Compress-and-forward (CF)**
    - relay compresses $Y_{M+1}$

We also consider
- **Amplify-and-forward (AF)**
  - relay amplifies $Y_{M+1}$

- **Outer bounds (OB)** are obtained using cut-sets
Decode-and-Forward Strategy

- Superposition block Markov encoding [CEG ’79]
- Sources transmit a new message in each block
- Example: two-sensor MARC (new messages in red)

\[
\begin{array}{cccc}
\text{S1} & \text{S2} & \text{FN} \\
\text{Frac. } \alpha & \text{Frac. } 1 - \alpha & \text{Frac. } \alpha & \text{Frac. } 1 - \alpha \\
\frac{1}{2} x_1 (w_{11}) & \frac{1}{2} x_1 (w_{12}, w_{11}) & x_1 (w_{13}) & \frac{1}{2} x_1 (w_{14}, w_{13}) \\
\frac{1}{2} x_2 (w_{21}) & \frac{1}{2} x_2 (w_{22}, w_{21}) & x_2 (w_{23}) & \frac{1}{2} x_2 (w_{24}, w_{23}) \\
x_3 (w_{11}, w_{21}) & x_3 (w_{11}, w_{21}) & x_3 (w_{13}, w_{23}) & x_3 (w_{13}, w_{23}) \\
\end{array}
\]

- This strategy uses *multi-hopping* (relay partially decodes)
Compress-and-Forward Strategy

- Relay compresses received $Y^{(\alpha)}_{M+1}$ as $\hat{Y}^{(\alpha)}_{M+1}$.
- Relay forwards $\hat{Y}^{(\alpha)}_{M+1}$ to destination via $X^{(1-\alpha)}_{M+1}$.
- At destination:
  - Decode $\hat{Y}^{(\alpha)}_{M+1}$
  - Decode source messages using $\left(\hat{Y}^{(\alpha)}_{M+1}, Y^{(\alpha)}_{M+2}\right)$

- Noisy MIMO (multiple receive antenna) model
- Useful strategy when relay cannot decode
Amplify-and-Forward Strategy

• Suppose $\alpha = \frac{1}{2}$.
• In transmit fraction $1 - \alpha$ relay sends
  \[ X_{M+1,i}^{1-\alpha} = c Y_{M+1,i}^\alpha \]
• At destination: a two-symbol ISI channel over fractions $\alpha$ and $1 - \alpha$
• Appropriate if relays must be simple
Illustration of Results – Geometries

\[ P_1 = P_2 = P_3 = 3 \text{ dB} \]
\[ \gamma = 4 \]

Case 1

\( d_{31} = d_{41} = 1 \)
\( d_{32} = d_{42} = 1 \)

Case 2

\( d_{31} = d_{32} = d \)
\( d_{41} = d_{42} = 1 \)
Channel Model – Details

• Complex white Gaussian channel with noise $Z_{ji} \sim \mathcal{CN}(0,1)$

• Fading parameters $h_{jmi}$ (time-invariant)
  – fade of $m^{th}$ source known only at $j^{th}$ receiver

1. no fading: $h_{kmi} = \frac{1}{\sqrt{d_{km}^\gamma}} \quad \gamma : \text{path-loss exponent}$
2. ergodic phase-fading: $h_{kmi} = e^{j\theta_{km}}/\sqrt{d_{km}^\gamma} \quad \theta_{km} \sim \mathcal{U}(-\pi, \pi)$

• Plot $R_1 + R_2$ for $\alpha = \frac{1}{2}$ and optimal $\alpha^*$

• Compare with multi-hop routing

$$(S_1, S_2) \xrightarrow{\alpha} R \xrightarrow{1-\alpha} D$$

• Sources sleep in $1 - \alpha$ fraction
C-MARC: No Fading -- $\alpha = 1/2$

- If relay and destination are clustered: CF $\rightarrow$ OB
- If relay and sources are clustered: DF $\rightarrow$ OB
- MH: destination does not use both fractions
- Relay useful for $\alpha = \frac{1}{2}$ when rates better than FD direct
C-MARC: Ergodic Phase-fading

- Random unknown phase fading $\rightarrow$ no cooperation in $1 - \alpha$ fraction [KGG, '03], [SKM, '04]

- Sources sleep in $1 - \alpha$ fraction
  - same rates as no-fading case
  - Capacity achieved for source-relay clustering

- Better rates achieved when sources transmit over both fractions
  - without cooperation (synchronization not required)
  - for same average power
Conclusions

- This work shows that in a simple HSN
  - Network throughput is increased by using a relay
  - Source power requirement reduced by using a relay

- Capacity achieving strategies for certain channels
  - Decode-and-forward desirable for R-S clustering
  - Compress-and-forward best when R-D cluster

- Cooperative strategies achieve better rates than multi-hopping