Channel State Feedback in MIMO Systems

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**Multiple Antenna Solutions**

- Rely on [Tse-03]
  - Spatial multiplexing
  - Diversity

- **High Spectral Efficiency**
  - MIMO
    - Capacity gains understood under idealized assumptions [Foschini-96]
    - Need to understand
      - Performance for multiuser systems
      - Frequency selective and time varying channels

- Multiple antenna transmitter optimizations in multiuser systems
  - Coherent [Farrohi-98, Visotsky-Madhow-99, Shamai-Caire-03, Goldsmith-03]
    - Rely on availability of CSI at transmitter
    - Very sensitive to CSI mismatch
  - Opportunistic beamforming [Laroia-Viswanath-03]
  - Dynamic subsectorization/fixed beams [Pedersen-03, Alamuti-04]
Talk Outline

- Downlink multiple antenna multiuser optimization schemes
- Effects of CSI delay on the sum data rate
- Channel state prediction based on the MMSE criterion
- How to efficiently transmit CSI?
- We propose ‘zero-delay’ unquantized and uncoded (UQ-UC) CSI feedback
**TX Optimization - System Model**

- **Transmitter**
- **Transform S**
- **MIMO Channel H**

Assumed to be known at the TX

- Mobile 1
  - $y(1)$
- Mobile 2
  - $y(2)$
- Mobile 3
  - $y(3)$
Constraints and Simplifications

• Received signal

\[ y = H S x + n \]

• \( S \) – spatial pre-filter is simplified as

\[ S = A P \]

\( A \) is a linear transformation
\( P \) is a diagonal matrix

• Constraint total average power where power does not have to be equal among users

\[ \text{trace}(AP P^H A^H) = P_{max} \]

\( P \) is diagonal and selected to maximize sum rate
Linear Transformation: Three Solutions

1. Zero Forcing
   
   $$A = H^H (H H^H)^{-1}$$
   
   • Zero the interference

2. Modified ZF
   
   $$A = H^H \left( H H^H + \left( \frac{No}{P_{av}} \right) I \right)^{-1}$$
   
   • Tame zeroing of the interference

   Using uplink/downlink duality we proved equivalence to the MMSE receiver.

3. Triangularization
   
   $$A = H^H R^{-1}$$
   
   • $H = (QR)^H$ - Orthogonal-triangular decomposition
   • $H A = L$ – lower triangular matrix
   • Preparing for Dirty Paper Coding

   We proved the optimality for high SNR.
$M = 3, N = 3$, Rayleigh Fading, Average Rate

DPC approaches the MIMO case

MZF vs. ZF similar behavior to MMSE vs. ZF receiver

Significant gains versus conventional user multiplexing
ZF and MZF Scheme, $M = 3$, $N = 3$, $SNR = 10dB$

Average Rate for Correlated Channels vs. CSI Delay

- More spatially correlated – more immune to the CSI delay.

- Worse than if no TX optimization is applied.
**MMSE Channel State Prediction**

- Goal of the transmitter is to predict future channel state $h(\tau)$ based on the CSI $h(t)$
- Prediction is to be done using a linear predictor $W$
- To be designed based on minimum mean square criterion

$$MMSE = \mathbb{E}[|W^H h_u - h(\tau)^H|^2]$$

where is $h_u$ vector $h_u = [h(0) \ldots h(-(K-1)T)]^T$

- Let us denote

$$U = \mathbb{E}[h_u h_u^H] \quad V = \mathbb{E}[h_u h(\tau)]$$

Estimated based on previous $h(t)$

$$W = U^{-1} V$$

- Exploits **spatial and temporal correlations** of the channel in MISO system
### MMSE Prediction Implementation

- Assuming stationary system, the estimates are used

\[
\hat{R} = \frac{1}{L} \sum_{n=-L+1}^{0} h_u(-n) h_u(-n)^H \\
\hat{c} = \frac{1}{L} \sum_{n=-L+1}^{0} h_u(-n) h(\tau - nT)^H \\
\hat{W} = \hat{R}^{-1} \hat{c}
\]

- Predicted CSI is

\[
\hat{h}(\tau) = \hat{W}^H h_u(0)
\]

- Assumptions and limitations
  - For duration of estimation \(LT_{\text{update}}\) stationary is assumed
  - Nyquist condition \(T_{\text{update}} < 1/(2f_{\text{doppler}})\)
ZF and MZF Scheme with MMSE Prediction, $M = 3$ and $N = 3$, $SNR = 10dB$, Average Rate for Correlated Channels vs. CSI Delay

- $T_{\text{update}} = \text{CSI delay (worst case)}$
- MMSE prediction extends life of the CSI
CSI Feedback

- How to efficiently transmit CSI?
  - Reliable feedback with minimum delay

- We propose ‘zero-delay’ unquantized and uncoded (UQ-UC) CSI feedback
  - Optimality
  - Application in wireless systems

- UQ-UC CSI feedback on correlated channels
  - Auto regressive-moving average (ARMA) channel model
  - Performance bounds
  - Linear CSI feedback receiver design
How to Code CSI - Digital Dogma

• (Vector) quantize continuous source (in this case it is channel state)

• Take the quantized values and apply channel coding

• At the receiver perform the channel decoding, and then reconstruct the continuous signal

• If the channel capacity is $C$, one can code the source with the rate $R = C$ achieving the distortion

$$D^* = E[\| s(t) - \bar{s}(t-T_d) \|]$$

• To achieve it, theoretically infinite source and channel coding delay is needed
**Digital Approach– Rate and Distortion** [Berger-71]

- Quantize a source whose output corresponds to a white Gaussian process with variance 1.
- For the given mean square error $D$, average rate needed to quantize the source is

$$R(D) = \log\left( \frac{1}{D} \right)$$

- Rate $R(D)$ is the minimal average number of bits per each source output needed to achieve distortion equal or less to $D$ (on average).

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**Diagram:**

- **Analog Source** $s(t) \sim N(0, 1)$
- **Quantization** $s_q(t)$
- **Virtual AWGN channel**
  - **Digital Source** $s(t) \sim N(0, D)$
  - **Channel Encoding** $s_q(t) \sim N(0, 1)$
  - **Additive Noise** $n \sim N(0, D)$
  - **Output** $C_{virtual} = R(D) = \log\left( \frac{1}{D} \right)$
Shannon Approach– Matching Source and Channel

- Channel capacity
  \[ C = \log\left(1 + \frac{P}{N_o}\right) \]

- Match the rate of the source coding \( R(D) \) and channel capacity \( C \)
  \[ R(D) = \log\left(\frac{1}{D}\right) = C = \log\left(1 + \frac{P}{N_o}\right) \]

- Distortion \( D \) is
  \[ D^* = \left(1 + \frac{P}{N_o}\right)^{-1} \]
Uncoded and Unquantized Transmission

[Goblick-65, Berger-02, Gastpar-Vetterli-03]

\[D^{\text{mmse}} = \text{MSE} = E \left[ (w^* (s \sqrt{P} + n) - s)^2 \right] \rightarrow w = \frac{\sqrt{P}}{P + N_o}\]

- MMSE solution

- Distortion (MMSE) is \(D^{\text{mmse}} = D^* = \left(1 + \frac{P}{N_o}\right)^{-1}\)

- UQ-UC scheme is completely matched in a Shannon sense, and it achieves MMSE with no quantization and coding delay.
• If downlink is \textit{iid} Rayleigh and uplink is AWGN and CSI estimate is perfect, then the UQ-UC CSI feedback is optimal

• Clearly not the case for typical (correlated) wireless channels $\rightarrow$ the UQ-UC CSI transmission may not be optimal
  • Performance bounds – upper and lower bounds?
  • Enhancements to performance?
Aside: The UQ-UC Scheme is **NOT** Analog Transmission

- **CDMA System Example** -
Upper and Lower Bound on Distortion

• Upper bound on the MSE

\[ MSE_{ub} = E_h \left[ \frac{1}{1 + \frac{|h| P_{csi}}{N_o}} \right] \]

• Lower bound on the MSE for ARMA channel model

\[ MSE_{lb} = c^2 2^{-C_{ul}} \]

\[ C_{ul} = E_h \log \left( 1 + \frac{|h|^2 P}{N_o} \right) \]

• ARMA channel model

\[ h_{dl}(i) = \sum_{j=1}^{L} c_j h_{dl}(i-j) + c_0 n_{dl}(i) \]

- \( n_{dl}(i) \) is a complex random variable with distribution \( N_C(0, 1) \)
- Coefficients \( c_j \) (\( j = 0, \ldots, L \)) determine the correlation properties of the channel
- \( n_{dl}(i) \) is the innovation sequence

• Lower bound is derived assuming that the above model and previous channel states \( h_{dl}(i-j) \) (\( j = 1, \ldots, L \)) are known at the CSI feedback transmitter and receiver
**Enhancements to UQ-UC CSI Feedback on Correlated Channels: Linear MMSE Receiver**

- Use UQ-UC CSI feedback in conjunction with linear MMSE receiver
- Form a received vector at the uplink CSI receiver

\[ y = [y(i) \ y(i-1) \ldots y(i-K+1)]^T \]

- Apply linear filter \( w \), such that it is minimizing the MSE distortion as

\[ w = \arg \min_v \ E[|v^H y - h_{dl}(i)|^2] \]

\[
R = E_{y|h_{ul}} [yy^H] \quad c = E_{h_{dl},y|h_{ul}} [h_{dl}^* y] \quad w = R^{-1} c
\]

- \( w \) implicitly takes into account the following correlations:
  - temporal correlations in the downlink channel
  - temporal correlations in the uplink channel
  - correlations between the uplink and the downlink
**MSE Distortion Performance**

- Downlink channel is modeled as an ARMA process whose coefficients are chosen to correspond to Jake's model for a carrier frequency of 2GHz and different mobile terminal velocities.

\[
\text{SNR}_{ul}^{\text{csi}} = 10 \log \left( \frac{P_{ul}^{\text{csi}}}{N_o} \right) = 10 \text{dB}
\]

\[
v = 10 \text{kmph}
\]
**UQ-UC CSI Feedback for ZF Spatial Pre-Filtering**

- Base station obtains CSI corresponding to each downlink channel state $h_{nm}$
- CSI is obtained from each mobile terminal using the UQ-UC CSI feedback
- At time instant $i$, terminal $n$ ($n=1, \ldots, N$) is transmitting the corresponding CSI $h_{nm}(i)$ via the uplink CSI feedback channel
- Instead of the ideal channel state $h_{nm}(i)$, the spatial pre-filter applies the estimate of $\hat{h}_{nm}(i)$ obtained from the uplink CSI feedback receiver
Conclusion

- Transmitter optimization schemes were presented
- MMSE prediction described showing its effectiveness
- We presented the ‘zero-delay’ UQ-UC CSI feedback scheme on correlated wireless channels
- ‘Zero-delay’ UQ-UC CSI feedback optimal for mutually independent $iid$ Rayleigh downlink and AWGN uplink
- We described the ARMA correlated channel model and presented the corresponding performance bounds for the UQ-UC CSI feedback scheme
- Performance limits of the scheme in the context of downlink multiple antenna multiuser transmitter optimization
- Attractive choice for CSI feedback
Future Directions

• Transmitter optimization with delayed CSI
  • TDD uplink/downlink multiplexing
  • Environments that offer narrow angular spread should be considered because they may allow a base station to obtain downlink CSI without explicit feedback from a mobile terminal
  • Channel state prediction schemes -validation using real propagation measurements may be of interest

• UQ-UC CSI feedback scheme
  • Understanding the trade-off between resources (e.g., power, time and spectrum) allocated to the pilots and the CSI feedback versus the resources of the data carrying signals on the downlink and
  • To compare the presented UQ-UC CSI feedback scheme to different schemes that use quantization and channel coding optimized for a given delay constraint