Power Control in a Multicell CDMA Data System Using Pricing

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IAB Meeting
October 17-18, 2000

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Outline

• Introduction
  – Motivation
  – Microeconomics: utility, game, price

• Utility

• Multicell power control game

• Efficiency through pricing

• Numerical Results


**Introduction**

- **Cellular voice telephony**
  - Successful evolution of technology and business of systems
  - Effective Radio Resource Management for system quality and efficiency

- **Wireless multimedia communications**
  - Voice and data have different quality of service (QoS) objectives
  - RRM techniques for voice not necessarily efficient for data

- **Concepts and mathematics of microeconomics/game theory for RRM in wireless networks**
  - Utility, Non-cooperative games, Nash equilibria, Pareto efficiency, Pricing
  - Focus on power control
Microeconomics Concepts

- **Utility**
  - The level of satisfaction received from consumption of resources

- **Non-cooperative game**
  - Player chooses strategy/action to maximize own utility in a distributed fashion

- **Nash equilibrium**
  - Fixed point reached as a result of non-cooperative game
  - Does not necessarily exist

- **Pareto efficiency**
  - Describes socially desirable solution
  - Nash equilibrium not necessarily Pareto efficient

- **Pricing**
  - Promotes choice of more Pareto efficient strategies/actions
Utility: Voice versus Data

- Utility function measures quality of service

- Power control for voice
  - Studied extensively (e.g. [Grandhi][Zander][Yates])
  - Objectives
    * Provide each signal with adequate QoS
    * Avoid unnecessary interference
    * Minimize battery drain
  - Service considered unacceptable below SIR target, however no extra benefit above target
    * Implicit assumption of a step utility function as a function of SIR
Utility: Voice versus Data

- Several specific definitions possible in wireless networks
- Different from voice due to different QoS objectives for data traffic
- QoS measure for wireless data in (Goodman, Mandayam, IEEE Pers. Comm., April 00)
- Factors affecting utility of wireless data systems?
  - Signal-to-interference ratio (SIR)
    - Frame Success Rate: data intolerant to errors
    - Throughput: rate of reception of correct data
  - Power consumption
    - Battery life: Inversely proportional to power drain
  - SIR and power strongly interdependent:
Properties of Data Utility Function

- Low SIR $\Rightarrow$ High error rate $\Rightarrow$ Low utility
- High SIR $\Rightarrow$ High throughput $\Rightarrow$ High utility
- Very high SIR $\Rightarrow$ Utility approaches constant asymptotically
- High transmit power $\Rightarrow$ Fast battery drain $\Rightarrow$ Low utility
System Model

- $K$ base stations serving $N$ terminals where each terminal $j$
  - transmits $L$ information bits in $M$ bit frames
  - transmits at $R$ bits/second over $W$ Hz with AWGN ($\sigma^2$)
  - is located $d_{ij}$ meters from base station $i$ with path gain $h_{ij}$

- SIR for terminal $j$ at base station $i$:
  \[
  \gamma_{ij} = \frac{W}{R} \frac{h_{ij}p_j}{\sum_{k=1, k \neq j}^{N} h_{ik}p_k + \sigma^2}.
  \]
The Utility Function

• Utility is the number of bits transmitted successfully per unit energy
  
  Utility of terminal $j$ obtained at base station $i$ is
  
  $$u_{ij}(p_j, p_{-j}) = \frac{RLf(\gamma_{ij})}{Mp_j} \text{ [bits/Joule]}$$  \hspace{1cm} (2)

  * $p_{-j}$ vector of transmit powers except user $j$
  * $f : \mathbb{R}_+ \rightarrow [0, 1]$ is the Efficiency Function
    · Approximates the frame success rate (FSR)
    · Depends on modulation format, channel coding
Efficiency Function

- Efficiency function: Approximation of Frame Success Rate
- Assuming independent bit errors

\[
FSR = (1 - BER)^M
\]  \hspace{1cm} (3)

- FSR > 0 if \( p = 0 \). With FSR, \( \lim_{p \to 0} u = \lim_{p \to 0} \frac{RLf(\gamma)}{Mp} = \infty \)
- Efficiency function approximated as

\[
f(\cdot) = (1 - 2BER)^M
\]  \hspace{1cm} (4)

which has the property \( \lim_{p \to 0} u = \lim_{p \to 0} \frac{RLf(\gamma)}{Mp} = 0 \)
Multicell Power Control Game

- In MCPG based on Maximum Received Signal Strength (MRSS),

\[
\max_{p_j \in P_j} u_j(p_j, p_{-j}) = u_{ajj}(p) \quad \text{for all } j = 1, \ldots, N
\]  \hspace{1cm} (5)

- assigned base station \( a_j = \arg \max_i h_{ij} \equiv \arg \min_i d_{ij} \)

- First order necessary optimality condition,

\[
f'(\gamma_{ajj}) - \gamma_{ajj}f'(\gamma_{ajj}) = 0
\]  \hspace{1cm} (6)

- \( \gamma_{ajj} = \tilde{\gamma} \) for all \( j \) and \( \tilde{\gamma} \) is unique.

- Similar to target SIR based power control for speech communications, but

  - Value of \( \tilde{\gamma} \) dictated by system (modulation, frame length)
  - Target SIR determined by considerations of subjective speech quality
  - Also, Nash of MCPG-MRSS inefficient
Nash Equilibrium in MCPG-MRSS

• What happens as a result of distributed self-optimizing behavior?
  – Nash equilibrium: no single user can improve its utility by unilateral change in its power

• Nash equilibrium exists in MCPG-MRSS due to quasiconcave utility functions and compact strategy spaces

• At the Nash equilibrium for MCPG-MRSS,

$$p_j = \min \left( \overline{p}, \frac{\tilde{\gamma}(\sum_{k \neq j} h_{a_j k} p_k + \sigma^2)}{G h_{a_j j}} \right)$$  \hspace{1cm} (7)

  – Terminals with better channel (high $h_{a_j}$) achieve $\tilde{\gamma}$
  – Terminals with poor channel quality (low $h_{a_j}$) do best with $\overline{p}$
Efficiency of the MCPG Equilibrium

- Power vector $x$ is more Pareto efficient than $y$
  - if $u_j(x) \geq u_j(y)$ for all $j$ and
  - if $u_j(x) > u_j(y)$ for some $j$

- Force all terminals to reduce powers at equilibrium of MCPG-MRSS
  $\Rightarrow$ All terminals receive increased utility
  $\Rightarrow$ MCPG-MRSS equilibrium is Pareto inefficient
  - Fixed target type power control not efficient for data communications
  - Introduce pricing to encourage lower power
MCPG with Pricing under MRSS Assignment

- In MCPGP-MRSS, terminal $j$ optimizes net utility:

$$\max_{p_j \in P_j} u_j^c(p) = u_{ajj}(p) - c_{aj}p_j$$  \hspace{1cm} (8)

- $c_{aj}$ is the pricing factor imposed by base station $a_j$

- Nash equilibrium exists in MCPGP-MRSS
  - Due to supermodularity of the utility functions.

- Iterative power updates result in Nash equilibrium
  - Equilibrium utilities with pricing, higher than MCPG-MRSS
  - Equilibrium transmit powers with pricing, lower than MCPG-MRSS
Each terminal $j$ solves

$$\max_{p_j,a_j} u_{a_j j}(p_j, p_{-j}).$$  \hspace{1cm} (9)$$

We find that joint problem equivalent to

$$\max_{p_j} u_j(p_j, p_{-j}) = u_{a_j j}(p)$$ \hspace{1cm} (10)$$

where $a_j = \arg \max_i u_{ij}(p) \equiv \arg \max_i \gamma_{ij}$

Resulting assignment referred to as Maximum SIR (MSIR)

MCPG-MSIR has Nash equilibrium ($u_j(p)$ in (10) is quasiconcave)

- Equilibrium is inefficient
MCPG with Pricing under MSIR Assignment

- In MCPGP-MSIR, terminal $j$ optimizes net utility:

$$\max_{p_j} u_j^c(p) = u_j(p) - c_{aj}p_j$$  \hspace{1cm} (11)$$

where $a_j = \max_i \gamma_{ij}$

- Experiments suggest existence of equilibrium

- Heuristic local pricing (LP) scheme:
  - $c_i = \alpha N_i$ where $N_i$ is the number of terminals in cell $i$
  - In experiments $\alpha = R$
## Numerical Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$, number of terminals</td>
<td>28</td>
</tr>
<tr>
<td>$K$, number of base stations</td>
<td>4</td>
</tr>
<tr>
<td>$M$, total number of bits per frame</td>
<td>80</td>
</tr>
<tr>
<td>$L$, number of information bits per frame</td>
<td>64</td>
</tr>
<tr>
<td>$W$, spread spectrum bandwidth</td>
<td>$10^6$ Hz</td>
</tr>
<tr>
<td>$R$, bit rate</td>
<td>$10^4$ bits/second</td>
</tr>
<tr>
<td>$\sigma^2$, AWGN power at the receiver</td>
<td>$5 \times 10^{-15}$ Watts</td>
</tr>
<tr>
<td>modulation technique</td>
<td>non-coherent FSK</td>
</tr>
<tr>
<td>$\bar{p}$, maximum power constraint</td>
<td>1 Watt</td>
</tr>
</tbody>
</table>
Equilibrium Utilities with MSIR

Terminal distance from base 1 (meters)

- Utilities in cell 1 (bits/Joule)
  - MCPG/MSIR
  - MCPGP/MSIR(LP)

Terminal distance from base 2 (meters)

- Utilities in cell 2 (bits/Joule)
  - MCPG/MSIR
  - MCPGP/MSIR(LP)

Terminal distance from base 3 (meters)

- Utilities in cell 3 (bits/Joule)
  - MCPG/MSIR
  - MCPGP/MSIR(LP)

Terminal distance from base 4 (meters)

- Utilities in cell 4 (bits/Joule)
  - MCPG/MSIR
  - MCPGP/MSIR(LP)
Equilibrium Transmit Powers with MSIR

- **MCPG/MSIR**
- **MCPGP/MSIR(LP)**

### Cell 1
- **Terminal Distance from Base 1 (meters)**: 300 to 700
- **Powers in Cell 1 (Watts)**: $10^{-4}$ to $10^0$

### Cell 2
- **Terminal Distance from Base 2 (meters)**: 0 to 800
- **Powers in Cell 2 (Watts)**: $10^{-4}$ to $10^0$

### Cell 3
- **Terminal Distance from Base 3 (meters)**: 300 to 500
- **Powers in Cell 3 (Watts)**: $10^{-3}$ to $10^1$

### Cell 4
- **Terminal Distance from Base 4 (meters)**: 0 to 800
- **Powers in Cell 4 (Watts)**: $10^{-4}$ to $10^0$
Equilibrium Base Assignment for MCPG-MSIR
Equilibrium Base Assignment for MCPGP-MSIR
Summary

- Studied power control for utility maximization in wireless multicell data networks
  - Leads to voice type power control
  - Inefficient for data
- Pricing improves efficiency
  - Benefit due to decreased power
- In addition to resulting in increased utilities and decreased transmit powers, pricing may also help relieve loaded cells