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Based on Utility and Pricing
Error Correction & Power Control
Battery energy, $E$

Data rate, $R$

Powers, $P$

Channel gains, $h$

Distances, $d$

$K$ users

A CDMA System
- Strategic & algorithm development
- Limited battery life
- Mutual interference
- Distributed users

Power Control Problem
- Voice is delay sensitive, error tolerant
- Data is error sensitive, delay tolerant
- Voice and data are different
- Proliferation of data type services
- Power control has been addressed for voice type applications
Pricing

Game theory

Utility theory

Microeconomic theories applied to wireless data power control

A New Look at the Power Control Problem for Data
(1) \[
\frac{z^0 + \int d^3 \eta \frac{1}{\gamma^2} \left( \frac{1}{M} \right)}{\int d^3 \eta} = \gamma
\]

Signal-to-Interference Ratio (SIR) -

Transmitter power $P$ - 

Curicenics of the wireless data system -

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Information: files, faxes, email, etc. -

Commodity of the wireless data system -

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Definition: Utility is the level of satisfaction that a user gets from consuming a good or undertaking an activity -

Utility for Wireless Data
necessary be efficient for data.

Power control algorithms developed for voice may not
Battery Energy (Joules) $\mathcal{E} \equiv \text{Battery Energy}$

Channel Error Correction

Frequency of protocol (function of SIR)

$\mathcal{E} \equiv [0, 1] \in (t, \gamma, \Omega) \frac{dI}{dE} = (\gamma, \Omega) \Omega$

transmitted correctly in the lifetime of the battery

Definition: Utility $U_R(\rho, \Omega)$ is the total number of bits

\[
\frac{T}{W - C - T} = (t) I
\]

$W$ - parity bits correct $t$ errors per frame

$C$ - CRC check bits

$T$ - total bits per frame

Code Rate - Ratio of information bits in each frame

Formulation of a Utility Function with RC
Example of a Utility Function
• Definition: A Nash equilibrium for the non-cooperative power control game is a power vector $P^*$ such that no user can improve its utility by a unilateral change in its power.

$$\max_{P^*} \left\{ \max_{\substack{P\geq 0, V_i = 1, \ldots, K}} U_i(P_1, P_2, \ldots, P_K) \right\}$$

(3)

• Formulated as a non-cooperative game.
non-cooperative power control game based on utility. 

**Theorem:** There exists a unique Nash equilibrium to the

\[ I + \frac{\lambda R}{M} \geq Y \]

capacity

Feasible power vector \( \mathbf{p} \) exists if \( Y \) is less than the system

\[ \mathbf{Y} \mathbf{d} = \mathbf{f}, \ \mathbf{\lambda} = \mathbf{f} \lambda \]

such that

\[ \mathbf{Y} \mathbf{d} = \mathbf{f}, \ \mathbf{\lambda} = \mathbf{f} \lambda \]

Equilibrium solution and power vector \( \mathbf{p} \)

\[ (\mathbf{f}, \mathbf{\lambda}) \mathbf{f} \mathbf{\lambda} = (\mathbf{f}, \mathbf{\lambda}) \mathbf{f} \mathbf{\lambda} = Y \mathbf{d} \]

A necessary condition for equilibrium is

\[ \frac{\partial P}{\partial \lambda} \]
Closer users do better

\[ \mathcal{X} \cdots I_i = \mathcal{J} \quad \forall \mathcal{J} \in \mathcal{P} \quad P_i = P \]

Equal received power at the base station

Equal SINR for each user

Properties of the Equilibrium Solution
By scaling their powers in concert by a fixed amount \( a > 1 \),

At the equilibrium solution, all users can increase their utility
efficient for any level of error correction.

• **Theorem:** The non-cooperative equilibrium is not Pareto

In other words, there exists no power vector \( \mathbf{p} \) such that

the same or improve.

achieves higher utility while the other users' utilities remain

there exists no such vector \( \mathbf{p} \) such that at least one user

• **Definition:** A power vector \( \mathbf{p}^* \) is Pareto efficient if and only if
Proposition: If the path gains of the $M$ users in a cell are in the order $\eta_1 < \eta_2 < \cdots < \eta_M$, then the following is true:

$$ \eta_1 > \cdots > \eta_2 > \eta_M $$

Definition: Define the harm caused by user $j$ by acting as an interfering to user $i$ as follows:

$$ \forall j \neq i \quad \int_{0}^{\frac{d_{ij}}{\theta}} \int_{0}^{D} d_{ij} \, d\theta $$

Formulation of a Pricing Function
Pareto improvements (everbody has higher utility).

Numerical results show that equilibrium exists and provides

- Solution hard to characterize analytically

\[
\max _{p, \beta} \prod _{i=1}^n Y_i \\
\widetilde{Y} - \mathcal{H} \cup \mathcal{H}
\]

Non-cooperative power control game with pricing

Distributed power control based on utility and pricing
system.

Pareto improvements more pronounced for a "fully loaded"

Favors users closer to the base station

Unequal received powers

Unequal SIR's

Properties of the Solution with Pricing
All the users

Optimal amount of error correction provides the most utility to

Provides Pareto improvements to the equilibrium solution

Increases the number of users

Reduces the equilibrium SIR,

FEC and the Equilibrium Solution
Transmitter power

Forward error control increases utility and decreases

User Utility vs Distance without Pricing

User Power vs Distance without Pricing

Numerical Results without Pricing
Pricing increases utility and decreases transmitter power for all users.

**Numerical Results with Pricing**
the equilibrium solution

Picking transmitter power can achieve Pareto improvements for
data.

Příční increases system capacity and utility at lower transmitter

utility

Formulated power control as a non-cooperative game based on

Wireless data.

Used economic theory to address the power control problem for


d

Conclusions
Extensions of concept to Internet traffic, congestion control and other resource management areas.

- Rate optimization
- Incorporating delay, traffic models, fading channels, etc.
- Researching pricing functions that result in Pareto efficiency.

Heterogeneous users and integrating services (voice and data, video, etc.)