RESOURCE MANAGEMENT FOR DOWNLINK WIRELESS SYSTEMS

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ABSTRACT OF THE THESIS

Resource Management for Downlink Wireless Systems

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In this thesis, we investigate radio resource management for the downlink of multimedia wireless networks. Our target is to develop simple practical algorithms utilizing scarce and expensive radio resources in the most efficient way, while meeting quality of service (QoS) requirements of various service classes.

In the first part of the thesis, we investigate optimum rate scheduling on the downlink of a multirate CDMA wireless network. Systems employing orthogonal variable spreading factor codes as well as systems using multiple codes have been studied. Our objective is to maximize the network throughput under constraints on total transmit power, total bandwidth and individual QoS requirements in terms of minimum rates. First, users are ordered based on their transmit energy per bit requirements to achieve the target received energy per bit to interference power spectral density ratio at the receivers. Based on the initial ordering, we prove that for systems employing multiple codes, the greedy rate scheduling is optimal, and therefore it yields maximum network throughput. For systems employing OVSF codes, the greedy rate scheduling is optimal

only if the minimum rate requirement of a user is larger than or equal to the minimum rate requirement of any other user with a larger transmit energy per bit requirement. Simulation results show that the greedy algorithm, even when it is suboptimal, is a good heuristic yielding average throughput which is very close to the optimal achievable throughput in OVSF-CDMA systems.

In the second part of the thesis, we investigate joint power control and orthogonal code selection (rate control) in frequency selective multipath channels. We show that the standard power control framework can be extended to include a form of rate control as well. Using this framework, we prove that a joint power and rate control algorithm converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient SINR. Specifically, for a CDMA wireless network, we observe that the optimal solution is achieved when each user selects those spreading codes from the Walsh set whose frequency domain responses match to the channel response of the user.

Finally, we show how to apply combinatorial network flow models in wireless resource management problems. Network flows are well-known subject with many applications in various fields of computer science, engineering, management and operations research. The minimum cost flow problem, a fundamental problem of network flows, deals with determining a least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes. Here we model the communication through a wireless network as a network flow, and we aim to minimize the cost of information flow through the network under constraints on demands (minimum rate) of mobiles and supply (bandwidth) of the base station. This model helps us to deal with practical discrete system constraints, and to improve fairness by enforcing minimum rate constraints on each mobile.

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Chapter 1

Introduction

Unlike voice-based second generation cellular networks, third and fourth generation mobile networks will provide multimedia data services in addition to classical voice service. The characteristics of data services are quite different from those of voice. While voice users require constant bit rate transmission with a fixed QoS target in terms of bit error rate (BER), or equivalently in terms of signal to noise plus interference ratio (SINR), data users may receive multiple rates and may require multiple QoS targets depending on the applications. In addition, data service may tolerate transmission delays while voice service requires real time continuous transmission. As a result, the tools developed for efficient utilization of radio resources for voice networks have to be revisited for wireless data.

To this end, this thesis examines resource management for wireless data networks. Our focus is the downlink of the system, which is supposed to carry the main traffic load due to the asymmetric nature of multimedia applications. We consider optimization and control of two important physical layer parameters, transmission power and rate. We investigate optimum power control and rate scheduling algorithms so as to allocate the resources in the most efficient way, while meeting QoS requirements of each user class within a total transmit power and bandwidth budget.

In Chapter 2, we consider throughput maximization on the downlink of a CDMA wireless network. Both systems employing orthogonal variable spreading factor codes (OVSF-CDMA) and multiple codes (Multicode CDMA) have been studied. Our objective is to maximize the network throughput under constraints on total transmit power, total bandwidth and individual QoS requirements specified in terms of minimum rates. First, users are ordered based on their transmit energy per bit requirements to achieve the target received energy per bit to interference power spectral density ratio at the receivers. Based on the initial ordering, we prove that for systems employing multiple codes, the greedy rate scheduling is optimal, and therefore it yields maximum network throughput. For systems employing OVSF codes, the greedy rate scheduling is optimal if the minimum rate requirement of a user is larger than or equal to the minimum rate requirement of any other user with a larger transmit energy per bit requirement. Simulation results show that the greedy algorithm, even when it is suboptimal, is a good heuristic yielding average throughput which is very close to the optimal achievable throughput in OVSF-CDMA systems.

In the downlink simulations and analyses, the effect of loss of orthogonality of transmitted waveforms is characterized by the orthogonality factor OF, which is defined as the fraction of received downlink power converted by multipath into multi-access interference [1–3]. We used the same convention in Chapter 2, and modeled the multiaccess interference using an average value of the orthogonality factor. On the other hand, the orthogonality factor may vary with time depending on the instantaneous multipath link gains as well as the receiver structures and the spreading codes employed. In Chapter 3, the multi-access interference characterization takes all these parameters into account. Since, in frequency selective channels, different waveforms are filtered in different ways by each user's channel, the SINR of user j decoding bit i will depend on both the waveform $\mathbf{s}_i$ and the channel $\mathbf{H}_j$. Therefore, it is crucial to determine which orthogonal waveforms provide a given set of rate assignments. To this end, we study joint power control and orthogonal code selection (rate control) in Chapter 3. We show that the power control framework [4] can be extended to include rate control as well. Using this framework, we prove that a joint power and rate control algorithm converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient SINR. Specifically, for a CDMA wireless network, we observe that the optimal solution is achieved when each user selects those spreading codes from the Walsh set whose frequency domain responses match to the channel response of the user.

Finally, in Chapter 4, we illustrate how to apply combinatorial network flow models in wireless resource management problems. Network flows are well-known combinatorial subject with many applications in various fields of computer science, engineering, management and operations research. The minimum cost flow problem, a fundamental problem of network flows, deals with determining a least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes [5]. Here we model the communication through a wireless network as a network flow and we aim to minimize the cost of information flow through the network under constraints on demands (minimum rate) of mobiles and supply (bandwidth) of the base station. This model helps us to deal with practical discrete system constraints, and to improve fairness by enforcing minimum rate constraints on each mobile. We first apply the network flow model in the case of an OFDM system where the channel is flat on each subcarrier and frequency selective across the subcarriers. Next, we propose an iterative minimum cost flow algorithm for CDMA networks and analyze its convergence properties. Finally, we consider the downlink of a CDMA network with linear receiver processing, and apply the network flow model in this system.

We summarize our work and give further research directions in Chapter 5.

Chapter 2

Optimum Rate Scheduling on the Downlink of a CDMA Wireless Network

In this chapter, we investigate optimum rate scheduling on the downlink of a multirate CDMA wireless network. Both systems employing orthogonal variable spreading factor codes and multiple codes have been studied. Our objective is to maximize the network throughput under constraints on total transmit power, total bandwidth, and individual QoS requirements specified in terms of minimum rates.

The chapter starts with a brief background on the topic including the relevant literature. In Section 2.2, we describe the system model and define the problem. The optimum rate scheduling algorithms for both OVSF CDMA and Multicode CDMA systems are discussed in Section 2.3, and the optimality of the proposed algorithms are proven in Section 2.4. Examples and simulation results are given in Section 2.6. We conclude the chapter with a brief summary and discussion.

2.1 Introduction

The multimedia oriented next generation wireless networks will provide multirate services with various service classes in addition to classical voice service. In CDMA wireless networks, multiple users share a common communication medium by means of spreading codes. There are two ways to assign spreading codes in such systems. First, in

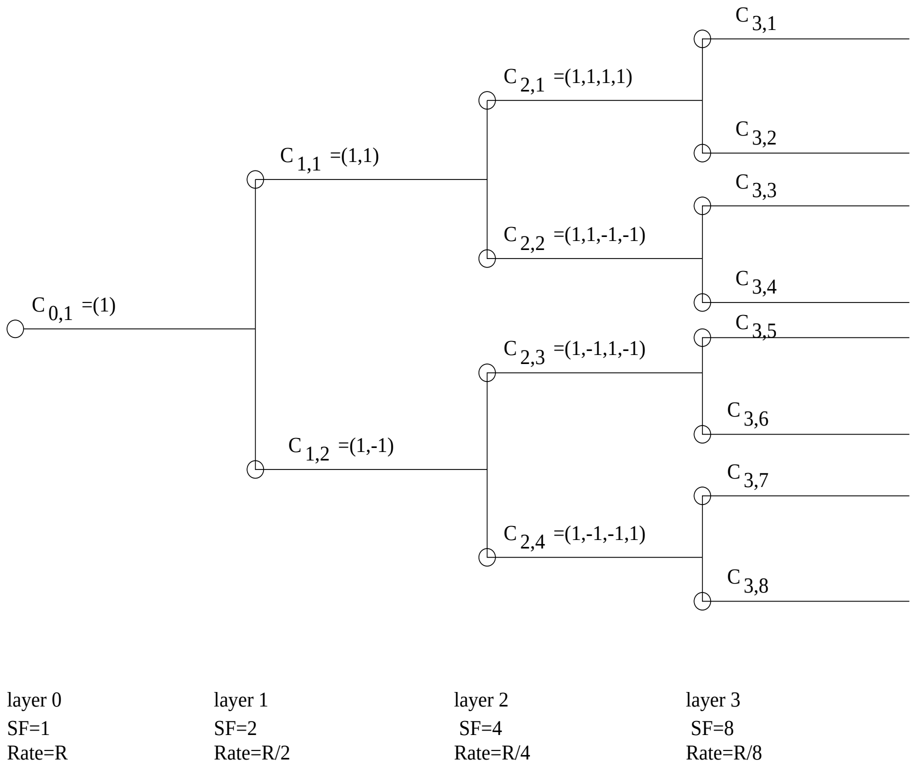

Figure 2.1: OVSF Code Tree. $C_{i,j}$ represents node j on layer i and has a length of SF=2^i

the third generation W-CDMA standard, multirate data service is provided by assigning each user a single spreading code with variable length [6]. In such a scheme, the spreading codes are obtained from a binary tree structure (Figure 1) and are called Orthogonal Variable Spreading Factor (OVSF) codes [7]. On the other hand, in Multicode CDMA systems, each user can be provided with multiple spreading codes of fixed length, depending on the users' rate requests.

Figure 2.1 shows how OVSF codes are obtained from a binary tree structure. In the figure, $C_{i,j}$ represents jth node on layer i on the binary tree and it corresponds to a unique signature sequence of length 2^i and rate $R_0/2^i$ where R_0 is the root rate corresponding to the node $C_{0,1}$. Moreover, orthogonality of the assigned signature sequences is guaranteed by the fact that none of the parent-child node pairs is assigned at the same time. As an example, the nodes $C_{1,2}$ and $C_{2,4}$ in Figure 1 cannot be in use at the same time since $C_{1,2}$ is a prefix of $C_{2,4}$. Therefore, the resulting set of assigned signature sequences must have the prefix-free property.

Accordingly the prefix-free condition imposes a constraint on the set of spreading codes that can be assigned to active users in variable spreading CDMA systems. It is a well-known fact that the Kraft inequality determines whether a set of codes with specified lengths can be placed on the binary tree as a prefix-free set [8]. Denoting the length of the branch from the root node ($C_{0,1}$ in Figure 1) to the uth user's node by l_u and number of users by N, the Kraft Inequality

$$\sum_{u=1}^{N} 2^{-l_u} \leq 1 \tag{2.1}$$

must be satisfied to obtain a prefix-free set.

In the context of CDMA spreading codes, the Kraft inequality can be interpreted as a bandwidth constraint. Since R_0 denotes the root rate and represents the maximum total rate or the bandwidth of the system, it follows that the rate of user u is

$$R_u = \frac{R_0}{2^{l_u}} \tag{2.2}$$

Multiplying both sides of (2.1) by R_0, the Kraft inequality becomes

$$\sum_{u=1}^{N} R_u \leq R_0 \tag{2.3}$$

which states that R_0 is an upper bound on the total data rate of all users in variable spreading CDMA case. It is trivial to generalize the constraint in (2.3) to the multicode

CDMA case and R_0 represents the system bandwidth in this case.

Recent studies on multirate CDMA systems focus on efficiency of dynamic spreading code assignment schemes, especially for systems employing OVSF codes [9–12]. The basic question these studies attempt to answer is how to accommodate an incoming user's rate request on the OVSF code tree. For a multicode CDMA system it is easy to handle an incoming user's request, if the requested rate is within available network resources, as many spreading codes as needed to meet the requested amount are assigned to the incoming user. On the other hand, in variable spreading CDMA systems the inherent binary tree structure of OVSF codes complicates code management. For example, assume there are 2 users in the system and their spreading codes are located at $C_{2,1}$ and $C_{2,3}$ on the binary code tree in Figure 1. Thus the total used system bandwidth is $R/2$ and half of the bandwidth is still available. In case a new user requests $R/2$, the system cannot locate a spreading code for the new user since $C_{1,1}$ ($C_{1,2}$) is not orthogonal to the existing code $C_{2,3}$ ($C_{2,1}$), although the system bandwidth is available for this request. This is known as *code blocking* [9].

In [9, 11, 12], dynamic code assignment schemes are proposed to minimize the code blocking probability and to minimize the number of existing spreading codes relocated in case of an incoming user. In [10], the authors propose a protocol which uses a credit-based reservation scheme to prioritize users and attempts to provide fairness to each user while providing per-connection bandwidth guarantees to bursty data applications. However, except for total available bandwidth, throughput limiting system resources such as total transmit power is neglected in the above studies.

For the uplink of multirate CDMA systems, resource allocation problems are studied extensively in [13–22]. The common objective in these studies is either to maximize the sum of rates (or utility), or to minimize the total transmit power, while there may be constraints on individual transmit power assignments, total bandwidth, the quality of reception in terms of SINR targets, minimum service requirement of each user (fairness), or delay. In [13], both the problem of minimizing the total transmit power and maximizing the sum of rates are considered. For the total power minimization problem, a closed form solution in terms of a matrix equation is developed. For the throughput maximization problem, a gradient projection algorithm is proposed, while its convergence to a global optimum solution is not proved. In [14], throughput maximization problem for a dual class CDMA system is considered. The objective in this study is to maximize the throughput of delay tolerant users while minimizing the interference caused by constant bit rate delay intolerant users. In [15], throughput is defined as the sum of correctly received bits. Since there are no constraints on minimum rates in this study, the optimum solution becomes unfair, allocating the whole bandwidth to the user with the largest link gain. Processing gain selection and transmission power control for multiple class CDMA networks are considered in [16–18]. The delay performances of the proposed algorithms are also analyzed. Throughput maximization problems with discrete set of rates are considered in [19, 20]. In [19], distributed control of rate and power for the best effort data services is studied. The proposed greedy rate assignment algorithm is claimed to maximize the number of users supported with minimum rate in case of nonzero minimum rate requirements. Here we will show that, for the downlink, the optimality of the greedy algorithm is in fact dependent on the minimum rates in case of geometric set of rates, and the greedy rate scheduling is optimal only if the minimum rate requirement of a user is larger than or equal to the minimum rate requirement of any other user with a larger transmit energy per bit requirement to achieve the target received SINR per bit at the receiver. Finally, CDMA power control problems are analyzed in a game theoretic framework in [21, 22].

Since voice based networks, such as IS-95, provide real-time constant bit rate service with QoS requirement in terms of target SINR at the base station, most of the initial CDMA resource allocation research is focused on power control algorithms in the uplink direction. On the other hand, multimedia oriented wireless networks provide data services, such as web applications, wireless video etc., which require heavy traffic load in the downlink direction. In [23–29], resource allocation problems for the downlink CDMA systems are studied. A hierarchical SIR and rate control algorithm is proposed in [23]. First, the mobiles determine their SIR targets using mobile specific information, and then the base station determines rate assignments using the limited feedback from the mobiles. Our analysis in this chapter will answer how the base station algorithm in the hierarchical structure should work. Joint power control and intracell scheduling for nonreal time data is studied in [24]. It is shown that a time division scheme in which users transmit one by one fashion within each cell provides energy efficiency and increased capacity. In [25], we study joint power control and orthogonal spreading code selection in frequency selective multipath channels. We show that an iterative algorithm achieves the optimal solution in which the total transmit power is minimized, while each user selects the spreading codes from the Walsh set whose frequency domain responses match to the channel response of the user. Finally, game theoretic pricing models are applied for downlink CDMA systems in [26–29].

There are basic differences between the downlink and the uplink of CDMA systems. First, orthogonal spreading codes are used in the downlink while random spreading codes are used in the uplink. Thus, in a channel with no multipath, both the transmitted and the received waveforms are orthogonal in the downlink direction. Second, both the desired signal and the interferer signals go through the same channel in the downlink, which means that both the interference powers and the desired signal power are filtered equally by the mobile's channel. We will explore these two important observations. Using the orthogonality of the spreading codes, we model the overall system as a number of parallel channels under the assumption of a flat channel. Using the same channel argument, we are able to derive a closed form expression for the transmit energy per bit requirement of each mobile to achieve the target SINR per bit at the receiver in a frequency selective multipath channel. Once the transmit energy per bit targets are determined, the problem formulation is simplified greatly as the interference constraints are accounted for in the energy per bit expressions, and the problem formulation is the same as that in the flat channel case.

Most of the literature on radio resource allocation assumes continuous rate and power assignments, instead of realistic discrete system parameters, which simplifies the problem at the expense of an approximate solution. Here, we assume practical discrete rates corresponding to systems employing either multiple codes or variable spreading codes (OVSF codes). In this case, the throughput maximization becomes a discrete optimization problem. We show that for systems employing multiple codes, the problem can be solved efficiently by a simple greedy algorithm with polynomial complexity. Moreover, for systems employing variable spreading codes, we show that the greedy algorithm is optimal if the minimum rate requirement of a user is larger than or equal to the minimum rate requirement of any other user with worse channel quality[1];

[1]By channel quality, we refer to the transmit energy per bit requirements to achieve the target SINR per bit at the receiver. Thus, a user with a smaller transmit energy per bit requirement is the user with the better channel.

otherwise the greedy algorithm is only a heuristic yielding a suboptimal solution.

We note that the greedy algorithm in this chapter orders users based on transmit energy per bit requirements which are shown to be not only a function of the path loss, but also a function of other user specific parameters such as the orthogonality factor, which is related to the multipath dispersion of the channel [1], and application specific target received SINR per bit. Thus, an ordering by transmit energy per bit requirements may not result in the same ordering as by path loss. We identify such cases and show that, in short range wireless systems such as WLANs or Infostations in which the average path loss is small compared to a cellular layout, the optimum scheduling is based only on the orthogonality factor (multipath dispersion or multipath delay profile) and the target received SINR per bit, and is independent of the path loss.

In the following section, we describe the system model, derive closed form expressions for the transmit energy per bit requirements, and define the problem.

2.2 System Model and Problem Statement

We consider a single cell CDMA downlink. There are N active mobiles in the system. Each mobile has a different application running and therefore each has a specific QoS requirement in terms of a minimum service rate. For each transmitted bit of user u, there is a target received energy per bit to interference power spectral density ratio, or SINR per bit, denoted by $(E/I)_u^r = \gamma_u$, with which the receiver can decode the transmitted bits with an acceptable bit error rate BER.

The channel between the base station and each mobile can be modeled either as a frequency flat single path channel, or as a frequency selective multipath channel. These two models have quite different implications on the problem definition. Since orthogonal spreading codes are used on the downlink of the system, orthogonality of the transmitted waveforms is preserved at each receiver under frequency flat channel assumption. On the other hand, in a frequency selective multipath channel, the orthogonality of the transmitted waveforms is lost at the receivers. Our problem definition will be general enough to account for both channel models. We accomplish this by determining the transmit energy per bit requirements for both channel models.

Since both the transmitted and the received waveforms are orthogonal in a frequency flat channel, the overall system is modeled as a number of parallel channels, and the question is how to assign rates on each parallel channel in the presence of total transmit power and bandwidth (sum of rates) constraints at the base station. Since there is no interference across parallel channels, the received E_u^r only compensates for the background noise, i.e. $(E/I)_u^r = (E/N_0)_u^r = \gamma_u$, and therefore $E_u^r = \gamma_u N_0$ where N_0 is the noise power spectral density. In this case, the transmit energy per bit is given by $E_u^t = \gamma_u N_0 / h_u$ where h_u denotes the link gain for user u.

When there is multipath in the channel, delayed versions of orthogonal spreading codes arrive at each receiver, leading to multi-access interference due to the loss of orthogonality between spreading codes. In the CDMA downlink, the effect of the loss of orthogonality can be characterized by the orthogonality factor (OF), which is defined as the fraction of received downlink power converted by multipath into multi-access interference [1–3]. As noted in [1–3], the orthogonality factor is a time-varying parameter that depends on the instantaneous multipath channels as well as the receiver structure and the spreading codes employed. On the other hand, in the analysis of downlink, the time average value of the orthogonality factor is traditionally employed [1–3]. Here we follow the same convention, and assume that user u has an average orthogonality factor of $\bar{\beta}_u$. In this case, we can express $(E/I)_u^r$ as

$$\left(\frac{E}{I}\right)_u^r = \frac{W}{R_s}\frac{p_u h_u}{\left(\bar{\beta}_u \sum_{i\neq u} p_i h_u + N_0 W\right)} = \frac{W}{R_s}\frac{p_u h_u}{\left(\bar{\beta}_u h_u \sum_i p_i - \bar{\beta}_u p_u h_u + N_0 W\right)} = \gamma_u \quad (2.4)$$

where W denotes the spreading bandwidth, R_s denotes the rate corresponding to a single spreading code (one of the multicodes) and p_u is the transmit power. Assuming that $\sum_i p_i = P$, i.e. the base station transmits at its peak power and any nonzero residual power is used either to increase the throughput with some nonzero rate or for some users to achieve higher received SINR per bit and better reception quality[2,3], we rewrite (2.4) in the form

$$\begin{aligned} p_u h_u &= \frac{R_s}{W}(\bar{\beta}_u h_u P + N_0 W)\gamma_u - \frac{R_s}{W}\gamma_u \bar{\beta}_u p_u h_u \\ &= \frac{\frac{R_s}{W}(\bar{\beta}_u h_u P + N_0 W)\gamma_u}{1 + \frac{R_s \gamma_u \bar{\beta}_u}{W}} \end{aligned} \quad (2.5)$$

Using the fact that the received power, $p_u h_u$, equals the received energy per bit multiplied by the rate, $E_u^r R_s$, we obtain

$$E_u^r = \frac{p_u h_u}{R_s} = \frac{(\bar{\beta}_u h_u P + N_0 W)\gamma_u}{W + R_s \bar{\beta}_u \gamma_u} \quad (2.6)$$

Notice that when $\bar{\beta}_u = 0$, (2.6) reduces to $E_u^r = \gamma_u N_0$ which we derived before in the case of a frequency flat channel. In a system with a large number of users, each having nonzero minimum rate requirements, none of the users can dominate the whole available bandwidth by itself, i.e $W \gg R_s$. In this case we can further approximate

[2] Here, we deal with a practical case where the power constraint is the bottleneck on system throughput.

[3] See Theorem 1 of [23] and Proposition 1 of [28] for a similar analysis.

(2.6) as

$$E_u^r = \frac{p_u h_u}{R_s} \approx \frac{(\bar{\beta}_u h_u P + N_0 W)\gamma_u}{W} \tag{2.7}$$

A design based on (2.7) is more conservative than a design based on (2.6). While the approximation is quite accurate for very low data rates, users with relatively high rates achieve larger SINR per bit when constraint (2.7) is satisfied. Notice that E_u^r is a decreasing function of R_s (2.6). Finally, the transmit energy per bit in the case of a frequency selective channel is given by $E_u^t = E_u^r / h_u$. We note that E_u^t is required for one of the multicodes corresponding to rate R_s. Assignment of multiple codes to a user generates self interference. In this case, each spreading code needs to be treated separately, and E_u^t has to be calculated for each of them as if every other spreading code is an interferer signal.

An interesting observation is that when the received multiaccess interference power is much larger than the receiver noise, i.e. $\bar{\beta}_u h_u P \gg N_0 W$, the transmit energy per bit becomes

$$E_u^t = \frac{(\bar{\beta}_u h_u P + N_0 W)\gamma_u}{h_u(W + R_s \bar{\beta}_u \gamma_u)} \approx \frac{\bar{\beta}_u P \gamma_u}{W + R_s \bar{\beta}_u \gamma_u} \tag{2.8}$$

which is independent of the path loss. We will explore this fact in Section 2.5 to schedule users in short range wireless systems such as WLANs or Infostations where the path loss is small (h_u is large) and the term $\bar{\beta}_u h_u P$ dominates $N_0 W$.

2.2.1 Problem Definition

We first examine OVSF CDMA systems. Given the minimum rate requirement of each user and the constraint on the total BS power P, our problem is to assign each user u a data rate R_u corresponding to a node on the OVSF code tree such that the Kraft inequality (2.1), each user's individual data rate and total BS power requirements are satisfied and the total data rate of all users (network throughput) is maximized. The problem in the multicode CDMA case is similar; given the minimum rate requirement of each user and the constraint on the total BS power, we determine the number of spreading codes that will be assigned to each user such that each user's individual data rate and total BS power requirements are satisfied and the network throughput is maximized.

OVSF CDMA Rate Scheduling Problem

Let R_0 denote the root rate, R_u and $R_{u,min}$ denote the rate assignment and the minimum rate requirement for user u respectively, and l_u denote the length of the branch from the root node ($C_{0,1}$ in Figure 1) to the uth user's node. Remember that there is a one-to-one relationship between l_u and R_u given by (2.2). In this case, R_0 corresponds to $l_0 = 0$, and the minimum rate constraint $R_{u,min}$ corresponds to maximum branch length constraint $l_u \leq l_{u,max} = L_u$. The problem is to find $\mathbf{l}$=$[l_1, \ldots, l_N]$ solving

$$\max_{\mathbf{l}} \quad \sum_{u=1}^{N} R_0 2^{-l_u} \tag{2.9}$$

$$\text{subject to} \quad \sum_{u=1}^{N} E_u^t R_0 2^{-l_u} \leq P \tag{2.9a}$$

$$\sum_{u=1}^{N} 2^{-l_u} \leq 1 \tag{2.9b}$$

$$l_u \in \{0, 1, \ldots, L_u\} \tag{2.9c}$$

In the above problem formulation, $E_u^t R_0 2^{-l_u}$ is the transmit power for user u and (2.9a) represents the total transmit power constraint. On the other hand, the Kraft inequality

(2.9b) is necessary to obtain a set of orthogonal codes, and represents the bandwidth constraint.

Multicode CDMA Rate Scheduling Problem

The problem formulation for the multicode system is similar. Let R_s denote the rate corresponding to a single spreading code, R_0 denote the sum of rates of all spreading codes, n_u and R_u denote the number of spreading codes and the rate assignment for user u respectively, thus $R_u = R_s n_u$. Each user requires at least $n_u^{'}$ spreading codes as the minimum QoS requirement. The problem is to find $\mathbf{n}$=$[n_1, \ldots, n_N]$ solving

$$\max_{\mathbf{n}} \quad \sum_{u=1}^{N} R_s n_u \tag{2.10}$$

$$\text{subject to} \quad \sum_{u=1}^{N} E_u^t R_s n_u \leq P \tag{2.10a}$$

$$\sum_{u=1}^{N} R_s n_u \leq R_0 \tag{2.10b}$$

$$n_u \in \{n_u^{'}, n_u^{'} + 1, \ldots, R_0/R_s\} \tag{2.10c}$$

2.3 Optimum Rate Scheduling Algorithms

Before the details of the algorithms, we first summarize our results. First, users are ordered by their transmit energy per bit, E_u^t, requirements (from smallest to largest) in order to achieve the target $(E/I)_u^r = \gamma_u$ at the receivers. Based on the initial ordering:

1. For multicode systems, greedy rate scheduling is optimal, achieving maximum network throughput.

2. For OVSF CDMA systems, greedy rate scheduling is optimal if the minimum rate

requirement of a user is larger than or equal to the minimum rate requirement of any other user with a larger transmit energy per bit requirement, i.e. when $E_i^t < E_j^t$ implies $L_i \leq L_j$ for all i and j. [4]

3. For OVSF CDMA systems in which minimum rates are not ordered by transmit energy per bit requirements, the greedy rate scheduling achieves maximum throughput if there is an optimal set of rate assignments ordering rates by the transmit energy per bit requirements. Otherwise the greedy rate scheduling is only a heuristic yielding a suboptimal solution.

We will show in the following sections that in cases where the greedy algorithm is optimal, the set of rate assignments by any existing optimal algorithm can be made more "greedy" by reordering and reassigning the user rates in a way to favor the users with better channels. Moreover, as the rate assignments by the optimal algorithm look more like the greedy assignments, the total power is reduced while keeping the total throughput constant.

To see how reordering reduces the total power, assume N users with energy per bit requirements $\mathbf{E}_b^t = [E_1^t, E_2^t, \ldots, E_N^t]$ and rate assignments $\mathbf{R} = [R_1, R_2, \ldots, R_N]$. If $R_i < R_j$ and $E_i^t < E_j^t$, we can swap the rates of user i and j without changing the total sum of rates, as long as the new assignments do not violate the minimum rate constraints. Denoting the total transmit power before and after swapping R_i and R_j

[4] The statement includes the case where there is no minimum rate constraint on user rates, i.e. $L_u = \infty$ for all u, or all users have the same minimum rate constraint, i.e. $L_u = L$ for all u.

Input : R_0, P, $\pounds_u = \{0, 1, \ldots, L_u\}$, $\mathbf{E}_b^t = [E_1^t, E_2^t, \ldots, E_N^t]$ (in increasing order)
Output : $\mathbf{l}$=$[l_1, \ldots, l_N]$
Initialization : $l_u = L_u$, $u = 1, \ldots, N$
$P_t = \sum_{u=1}^{N} E_u^t R_0 2^{-L_u}$
for u = 1 : N
$l_u = \min\{l \in \pounds_u |\ 2^{-l} + \sum_{v \neq u} 2^{-l_v} \leq 1, \quad 2^{-l} - 2^{-L_u} \leq \frac{P - P_t}{E_u^t R_0}\}$
$P_t = P_t + E_u^t R_0 (2^{-l_u} - 2^{-L_u})$
end

Figure 2.2: Optimal Greedy Rate Scheduling for OVSF CDMA System

by P and P^s respectively, it follows that

$$\begin{aligned} P - P^s &= [E_i^t R_i + E_j^t R_j] - [E_i^t R_j + E_j^t R_i] && (2.11) \\ &= (R_j - R_i)[E_j^t - E_i^t] && (2.12) \\ &> 0 && (2.13) \end{aligned}$$

where (2.13) follows from the fact that $E_j^t > E_i^t$ and $R_j > R_i$.

2.3.1 OVSF CDMA Case: The Greedy Algorithm

The algorithm we propose for the OVSF rate assignment problem initially provides the minimum QoS requirement of each user. The rest of the algorithm is greedy in nature, and the objective is to increase (to double in the binary tree case) the rate of the user who spends minimum energy per bit. Based on the initial ordering by transmit energy per bit E_u^t requirements, the algorithm attempts to maximize the rate of a user at each greedy step within the total transmit power (2.9a) and the bandwidth (2.9b) constraints. We summarize the algorithm in Figure 2.2.

An important fact is that the resulting rate assignment $R_0 2^{-l_u}$ at the end of each iteration can be obtained by repeatedly doubling the user's initial rate as long as the constraints (2.9a) and (2.9b) permit.

Notice that uth user's spreading code resides on layer l_u of the binary code tree in Figure 2.1. Although l_u uniquely determines the rate assignment for uth user (2.2), it does not tell us which node on layer l_u of the code tree should be assigned to user u. On the other hand, satisfying the Kraft Inequality (2.1) guarantees the fact there is at least a set of N spreading codes on the binary code tree such that uth user's spreading code is placed on layer l_u, the rates of all other users are not affected by this placement (although spreading codes might shift on the same layer) and all spreading codes in the set are mutually orthogonal as a result of the prefix free property. The shifts or replacements of spreading codes on the same layer on the binary code tree are implementation issues and such shifts do not affect the assigned rate of a user. Thus, the way the spreading code replacements or shifts occur at each step of the algorithms, the subject of [9, 11] and [12], is not addressed in this study.

2.3.2 Multicode CDMA Case: The Greedy Algorithm

Similar to the OVSF system, the greedy approach solves the rate assignment problem in multicode systems. However, in this case the greedy rate assignment only favors the user with the smallest transmit energy per bit requirement. After the algorithm assigns minimum rates and allocates corresponding spreading codes to each user, only the rate of the user with the minimum energy per bit requirement is maximized using the remaining power budget.

We summarize the algorithm above in Figure 2.3.

Input : P, R_s, R_0, $\aleph_u = \{n_u^{'}, n_u^{'}+1, \ldots, R_0/R_s\}$, $\mathbf{E}_b^t = [E_1^t, E_2^t, \ldots, E_N^t]$ (in increasing order)
Output : $\mathbf{n}$=$[n_1, \ldots, n_N]$
Initialization : $n_u = n_u^{'}$, $u = 1, \ldots, N$
$P_t = \sum_{u=1}^{N} E_u^t R_s n_u^{'}$
$n_1 = \max\{n \in \aleph_1 |\ n + \sum_{v \neq u} n_v \leq R_0/R_S,\ \ (n - n_1^{'}) \leq \frac{P-P_t}{E_1^t R_s}\}$
end

Figure 2.3: Optimal Greedy Rate Scheduling for Multicode CDMA

2.4 Correctness and Proof of the Algorithms

2.4.1 Greedy Optimality in Multicode CDMA Systems

Theorem 1. *The greedy algorithm solves any instance of the multicode rate assignment problem (2.10).*

Proof. Consider an optimal multicode assignment vector $\mathbf{n}^* = [n_1^*, n_2^*, \ldots, n_N^*]$ yielding maximum total throughput. Our greedy algorithm yields the vector $\hat{\mathbf{n}} = [\hat{n}_1, \hat{n}_2, \ldots, \hat{n}_N]$ where $\hat{n}_u = n_u^{'}$ for $u = 2, \ldots, N$. We now rearrange the user rates in $\mathbf{n}^*$ to obtain another set of assignments $\tilde{\mathbf{n}}^*$ yielding the same network throughput as $\mathbf{n}^*$ such that $\tilde{n}_u^* = n_u^{'}$ for $u = 2, \ldots, N$ and

$$\tilde{n}_1^* = n_1^* + \textstyle\sum_{u=2}^{N} (n_u^* - n_u^{'}) \tag{2.14}$$

From $\mathbf{n}^*$ to $\tilde{\mathbf{n}}^*$, all assignments other than the first are reduced to the minimum required levels and the total rate reduction is assigned to the first user.

It is easy to show that $\tilde{\mathbf{n}}^*$ requires less power than $\mathbf{n}^*$ while they both offer the same optimal throughput. Let P^* and $\tilde{P}^*$ denote the power required by $\mathbf{n}^*$ and $\tilde{\mathbf{n}}^*$

respectively, then

$$\begin{aligned} P^* - \tilde{P}^* &= E_1^t R_s(n_1^* - \tilde{n}_1^*) + \sum_{u=2}^{N} E_u^t R_s(n_u^* - \tilde{n}_u^*) \\ &\geq E_1^t R_s[(n_1^* - \tilde{n}_1^*) + \sum_{u=2}^{N} (n_u^* - \tilde{n}_u^*)] \\ &= 0 \end{aligned} \tag{2.15}$$

The last inequality follows from the fact that user 1 requires the smallest energy per bit, and replacing E_u^t by E_1^t upperbounds $\tilde{P}^* - P^*$ since $(n_u^* - \tilde{n}_u^*) \geq 0$.

Comparing $\tilde{\mathbf{n}}^*$ and the greedy $\hat{\mathbf{n}}$, they agree on all rate assignments except the first user. On the other hand, for mobile 1, the greedy algorithm makes a locally maximum choice and maximizes its rate within power and bandwidth constraints, while assuming that all other users receive the minimum rates. Therefore $\hat{n}_1 \geq \tilde{n}_1^*$. On the other hand, since $\tilde{\mathbf{n}}^*$ obtains maximum total throughput by assumption, $\tilde{n}_1^* \geq \hat{n}_1$. We conclude that $\tilde{n}_1^* = \hat{n}_1$ and $\tilde{\mathbf{n}}^* = \hat{\mathbf{n}}$. □

The intuition behind the optimality of the greedy algorithm is that if a spreading code is to be assigned, it is better if it is assigned to the user who can receive it with the smallest power (the smallest contribution to the total power). On the other hand, unlike uplink where the multiaccess interference at the receiver depends on the channels of all users and the individual power assignments, the multiaccess interference on the downlink is a function of the user's own channel and the power assignments of interferer spreading codes. Thus, a reduction in the power of any interferer signal is beneficial to all users no matter who the spreading code is assigned to.

2.4.2 Greedy Optimality in OVSF CDMA Systems

We first prove the optimality of the greedy algorithm in the two user case. We then generalize the proof to any number of users.

Theorem 2. *Given that the user with smaller transmit energy per bit requirement has also larger (or equal) minimum rate requirement, the greedy algorithm solves the OVSF rate assignment problem for $N = 2$ users.*

Proof. A general form of the problem in the 2 user case is as follows

$$\max \quad R_0(2^{-l_1} + 2^{-l_2}) \tag{2.16}$$

$$\text{subject to} \quad E_1^t R_0 2^{-l_1} + E_2^t R_0 2^{-l_2} \leq P' \tag{2.16a}$$

$$2^{-l_1} + 2^{-l_2} \leq \rho \tag{2.16b}$$

$$l_i \in \{0, 1, \ldots, L_i\}, \; i = 1, 2 \tag{2.16c}$$

for any $0 < P' \leq P$ and $0 < \rho \leq 1$. Assume that the first user requires smaller transmit energy per bit $E_1^t < E_2^t$.

Consider an optimal vector $\mathbf{l}^* = [l_1^*, l_2^*]$ yielding maximum total throughput. Given $L_1 \leq L_2$, we can always choose the optimal vector $\mathbf{l}^*$ in such a way that $l_1^* \leq l_2^*$; otherwise if $L_1 \leq L_2$ and $l_1^* > l_2^*$, user 1 and user 2 can exchange the assigned values so that the total throughput remains the same and the new assignments even save some power. Note that such an exchange may violate individual minimum rate constraints if $L_1 > L_2$. Our greedy algorithm yields the vector $\hat{\mathbf{l}} = [\hat{l}_1, \hat{l}_2]$

$$\hat{l}_1 = \min\{l \in \pounds | 2^{-l} + 2^{-L_2} \leq \rho, \quad E_1^t R_0 2^{-l} + E_2^t R_0 2^{-L_2} \leq P'\} \tag{2.17}$$

$$\hat{l}_2 = \min\{l \in \pounds | 2^{-l} + 2^{-\hat{l}_1} \leq \rho, \quad E_2^t R_0 2^{-l} + E_1^t R_0 2^{-\hat{l}_1} \leq P'\} \tag{2.18}$$

We compare $\mathbf{l}^*$ and $\hat{\mathbf{l}}$

(i) Assume $l_1^* < \hat{l}_1$: For user 1, the greedy algorithm finds the minimum layer number $\hat{l}_1$ while power and bandwidth constraints are satisfied and second user's spreading code resides on layer L_2. Thus $l_1^* < \hat{l}_1$ is impossible; otherwise $\hat{l}_1$ would not be the local minimum choice.

(ii) Assume $l_1^* > \hat{l}_1$: In this case the smallest possible value of l_1^* is $\hat{l}_1 + 1$. Since $l_1^* \leq l_2^*$, we have

$$\hat{l}_1 + 1 \leq l_1^* \leq l_2^* \tag{2.19}$$

Since $\mathbf{l}^*$ is optimal, it must offer the maximum total throughput. This requires

$$R_0(2^{-\hat{l}_1} + 2^{-\hat{l}_2}) \leq R_0(2^{-l_1^*} + 2^{-l_2^*}) \tag{2.20}$$

However, from (2.19),

$$R_0(2^{-l_1^*} + 2^{-l_2^*}) \leq R_0(2^{-(\hat{l}_1+1)} + 2^{-(\hat{l}_1+1)}) = R_0 2^{-\hat{l}_1} < R_0(2^{-\hat{l}_1} + 2^{-\hat{l}_2}) \tag{2.21}$$

since $0 \leq \hat{l}_2 \leq L_2$. Thus we have a contradiction implying $l_1^* > \hat{l}_1$ is also impossible. As a result we conclude that $l_1^* = \hat{l}_1$.

Similar to user 1, the greedy algorithm makes a local minimum choice for user 2 while the first user's spreading code resides on layer $\hat{l}_1$. Because $l_1^* = \hat{l}_1$, and $\hat{l}_2$ is a local minimum, $\hat{l}_2 \leq l_2^*$ must be true. On the other hand if $\hat{l}_2 < l_2^*$, then the optimal $\mathbf{l}^*$ offers smaller throughput compared to the greedy $\hat{\mathbf{l}}$, which is a contradiction. Therefore $l_2^* = \hat{l}_2$ must be true. Since we also proved that $l_1^* = \hat{l}_1$, we conclude $\mathbf{l}^* = \hat{\mathbf{l}}$. □

We next generalize the proof to any number of users.

Theorem 3. *Given that minimum rate requirement of a user is larger than or equal to minimum rate requirement of any other user with a larger transmit energy per bit requirement, the greedy algorithm solves the OVSF rate assignment problem.*

Proof. Here we will prove a more general statement that the greedy algorithm solves rate maximization problem for any power constraint $0 < P^{'} \leq P$ and any bandwidth constraint $0 < \rho \leq 1$, corresponding to a "partial" binary code tree. The general form of the problem is

$$\max \quad \sum_{u=1}^{N} R_0 2^{-l_u} \tag{2.22}$$

$$\text{subject to} \quad \sum_{u=1}^{N} E_u^t R_0 2^{-l_u} \leq P^{'} \leq P \tag{2.22a}$$

$$\sum_{u=1}^{N} 2^{-l_u} \leq \rho \leq 1 \tag{2.22b}$$

$$l_u \in \{0, 1, \ldots, L_u\} \tag{2.22c}$$

Without loss of generality, we can assume that $E_1^t < E_2^t < \cdots < E_N^t$. Consider an optimal vector $\mathbf{l}^* = [l_1^*, l_2^*, \ldots, l_N^*]$ yielding maximum total throughput. Given $L_1 \leq L_2 \leq \cdots \leq L_N$, any set of optimal rate assignments can be reordered in such a way that $l_1^* \leq l_2^* \leq \cdots \leq l_N^*$, without violating individual minimum rate constraints. It is important to notice that, given $E_i^t < E_j^t$ and $l_i^* > l_j^*$, an exchange between assignments of users i and j does not violate minimum rate constraints if $L_i \leq L_j$ [5]. Note that such an ordering always saves power (2.11)-(2.13), therefore the power constraint is not violated while the throughput remains constant. Our greedy algorithm yields the vector $\hat{\mathbf{l}} = [\hat{l}_1, \hat{l}_2, \ldots, \hat{l}_N]$.

The proof goes by induction. We already proved the greedy optimality for the two user case in Theorem 2. Here, we assume that Theorem 3 is true for any system of $\tilde{N} < N$ users. We also assume that there is a feasible rate assignment vector for the problem (2.22). Therefore, optimal and greedy solutions are both feasible and we will consider them below.

Definition 1. *Let A be a user index such that*

$$\begin{aligned} \hat{l}_u &= l_u^* \quad u = 1, \ldots, A-1 \\ \hat{l}_A &\neq l_A^* \end{aligned} \tag{2.23}$$

Due to local optimality of the greedy algorithm, (2.23) can be made more specific

$$\hat{l}_A \leq l_A^* - 1 \tag{2.24}$$

First of all we assume $A > 1$. In this case at least the first user gets the same rate assignment by both algorithms (greedy and optimal). The remaining $N - 1$ users can be assigned in a greedy fashion because our induction hypothesis stipulates that for

[5] In fact, the proof is valid as long as there exists an optimal algorithm whose output rate assignments can be reordered in such a way that $E_i^t < E_j^t$ implies $l_i \leq l_j$, without violating individual minimum rate constraints.

any $\tilde{N} < N$ users, greedy assignments are optimal. This proves Theorem 3 for the case of $A > 1$. Thus, it remains to consider the case of $A = 1$. Since optimal $\mathbf{l}^*$ achieves maximum total throughput, employing (2.24) for the case $A = 1$ we can write that

$$2^{-l_1^*} + \sum_{u=2}^{N} 2^{-l_u^*} \geq 2^{-(l_1^*-1)} + \sum_{u=2}^{N} 2^{-L_u} \tag{2.25}$$

Lemma 1. *For any $\mathbf{l}^* = [l_1^*, l_2^*, \ldots, l_N^*]$ satisfying (2.25), we can always find a set of assignments $\tilde{l}_2^*, \ldots, \tilde{l}_N^*$ such that*

$$2^{-l_1^*} + \sum_{u=2}^{N} 2^{-l_u^*} = 2^{-(l_1^*-1)} + \sum_{u=2}^{N} 2^{-\tilde{l}_u^*} \tag{2.26}$$

$$l_u^* \leq \tilde{l}_u^* \leq L_u, u = 2, \ldots, N \tag{2.27}$$

Proof of Lemma 1. Here we give a constructive proof of Lemma 1 by providing an explicit algorithm which computes the new assignments $\tilde{l}_2^*, \ldots, \tilde{l}_N^*$. The algorithm is presented in Figure 2.4. In the figure, n, $\triangle_n$ and J_n denote iteration index, bandwidth released by decreasing rate of a user at nth iteration and the aggregate bandwidth released up to the nth iteration respectively.

We will show by contradiction that the aggregate bandwidth released J_n equals to $2^{-l_1^*}$ at some point, and that the algorithm falls through Step 3 and always terminates. Let's assume that there is <u>no</u> n such that $J_n = 2^{-l_1^*}$. Together with (2.25) it means that there will be such an iteration t for which

$$J_{t-1} < 2^{-l_1^*} \tag{2.28}$$

$$J_t > 2^{-l_1^*}$$

Input : $\mathbf{l}^* = [l_1^*, l_2^*, \ldots, l_N^*]$, $\mathbf{L} = [L_1, L_2, \ldots, L_N]$, $\mathbf{E}_b^t = [E_1^t, E_2^t, \ldots, E_N^t]$
Output : $\tilde{\mathbf{l}}^* = [\tilde{l}_1^*, \tilde{l}_2^*, \ldots, \tilde{l}_N^*]$
Initialization :
$n := 0$; $J_0 := 0$; $\tilde{l}_u^* := l_u^*$, for $u = 1, \ldots, N$; $\mathbf{S} := \{2, \ldots, N\}$
Step 1 :
$\mathbf{U} := \{u \in \mathbf{S} | \min_{u \in \mathbf{S}} l_u^*\}$;
$u_m := \arg\max_{u \in \mathbf{U}} E_u^t$;
if $\tilde{l}_{u_m}^* = L_{u_m}$ then $\mathbf{S} := \mathbf{S} - \{u_m\}$ and repeat Step 1;
Step 2
$n := n + 1$;
$\tilde{l}_{u_m}^* := \tilde{l}_{u_m}^* + 1$;
$\triangle_n := 2^{-\tilde{l}_{u_m}^*}$;
$J_n := J_{n-1} + \triangle_n$;
Step 3: if $J_n \neq 2^{-l_1^*}$ goto Step 1;
Step4 : **Outputs**
$\tilde{l}_1^* := l_1^* - 1$;
end

Figure 2.4: Algorithm for the proof of Lemma 1

We also observe that

$$J_t = \textstyle\sum_{n=1}^{t} \triangle_n \tag{2.29}$$

$$2^{-l_1^*} \geq \triangle_1 \geq \triangle_2 \geq \cdots \geq \triangle_t \tag{2.30}$$

Partial ordering in (2.30) follows from Step 1 of the algorithm which always chooses the largest rate user to release its bandwidth. Due to the ordering in (2.30) and keeping in mind the definition of t by (2.28), the difference between J_t and $2^{-l_1^*}$ is smaller that the contribution from the last iteration

$$J_t - 2^{-l_1^*} = \sum_{n=1}^{t} \triangle_n - 2^{-l_1^*} < \triangle_t \tag{2.31}$$

We emphasize here that by construction $\triangle_n$ is always some negative integer power of two. Therefore, with the help of (2.30) we conclude that the ratios $\triangle_n / \triangle_t$ are integer numbers for $n = 1, \dots, t$. Next, we divide both sides of (2.31) by $\triangle_t$

$$\left(\sum_{n=1}^{t} \frac{\triangle_n}{\triangle_t}\right) - \frac{2^{-l_1^*}}{\triangle_t} < 1 \tag{2.32}$$

In the above inequality, the left side is a positive integer due to (2.28), while the right side is a fractional number less than one. Therefore (2.32) contains a contradiction, proving that Step 3 terminates the algorithm which results in the set of assignments $\tilde{l}_2^*, \dots, \tilde{l}_N^*$ as in Lemma 1. $\square$

With the help of Lemma 1, we can immediately construct a new rate assignment vector $\tilde{\mathbf{l}}^* = [l_1^* - 1, \tilde{l}_2^*, \dots, \tilde{l}_N^*]$ yielding the same throughput as the optimal assignments $\mathbf{l}^*$, yet because of (2.27), it consumes less energy. Notice that $\tilde{\mathbf{l}}^*$ looks more "greedy" than $\mathbf{l}^*$, i.e. user 1 with the minimum transmit energy per bit requirement gets an enhanced rate while the other users get less than or equal to their rate assignments in $\mathbf{l}^*$ due to (2.27).

To complete the proof of Theorem 3 in the case of $A = 1$, Lemma 1 can be applied to $\tilde{\mathbf{l}}^*$ as well, i.e. starting from the new set of optimal assignments $\tilde{\mathbf{l}}^*$, we can construct another optimal set with the first user receiving $l_1^* - 2$ and all other users receive less than or equal to their assignments in $\tilde{\mathbf{l}}^*$, but more than their minimum requirements. We can continue in this fashion until the first user receives $\hat{l}_1$, the assignment by the greedy algorithm, in an optimal set of assignments. At this point, we already proved the greedy optimality, i.e. for $A > 1$ using our induction hypothesis that greedy assignments are optimal for any $\tilde{N} < N$ users. The greedy optimality proof of Theorem 2 in the two user case is therefore generalized to any number of users by induction. $\square$

2.4.3 Discussion

It is interesting to note that the optimality of the greedy algorithm in OVSF CDMA systems depends on the minimum rate constraints. When users with worse channels require larger minimum rates, the greedy algorithm may not be the optimal way to assign user rates. As a simple example, assume there are two users in the system; the first with $E_1^t = 1$, and the second one with $E_2^t = 1.25$ (for simplicity, we omit the units and assume all units are consistent and scaled appropriately). The power constraint is 11, and minimum rate requirements are $R_{1,min} = 1$ and $R_{2,min} = 4$. Total power to provide the minimum rates is

$$P_{min} = \sum_{u=1}^{2} E_u^t R_{u,min} = 6 \tag{2.33}$$

Since the first user requires smaller transmit energy per bit, the greedy algorithm would double (note the geometric relationship between rates on the binary code tree) the first user's initial rate assignment using the remaining power budget of $11 - 6 = 5$. In this case, the greedy algorithm could at most assign 4 units of rate to user one, which requires an additional 3 units of power. The remaining $11 - 6 - 3 = 2$ units of power would not be enough to double second user's initial assignment from 4 to 8, therefore the greedy algorithm would conclude $R_1 = 4$, $R_2 = 4$ and a total throughput of 8 units of rate. On the other hand, optimal throughput is 9 units of rate which is achieved when $R_1 = 1$ and $R_2 = 8$. In this case, optimal transmission strategy is to favor the user with the worse channel, which contradicts with the opportunistic transmission strategies in which users with better channels are favored.

We note that discrete optimization problems, or integer programming problems, are in general NP-complete so that they cannot be solved by simple algorithms with polynomial complexity such as greedy algorithms. One such intractable problem, which is similar to the throughput maximization problem, is the Knapsack problem in which one wish to fill a knapsack of capacity b with items having the largest possible total utility [30, 31]. Formally,

$$\max \sum_{j=1}^{n} c_j x_j \tag{2.34}$$

$$\text{subject to} \sum_{j=1}^{n} a_j x_j \leq b \tag{2.34a}$$

$$x_j \text{ integers, 0-1 or } 0 \leq x_j \leq 1 \tag{2.34b}$$

where c_j, a_j and x_j denote the utility, the size/capacity and the amount included in the knapsack for jth item respectively. Similarly in resource allocation problems, the power constraint represents the knapsack capacity, and the target is to achieve the largest total utility by filling the knapsack by as much rate (item) as possible. Based on constraints on x_j, the Knapsack problems are classified either as integer knapsack problems, where x_j is constrained to be positive integers, $0-1$ knapsack problems, where x_j is constrained to be 0 or 1 only, or fractional knapsack problems where x_j may be a fractional number and we may take pieces of items [32]. While the integer knapsack and the $0-1$ knapsack problems are NP-complete, the fractional knapsack problems can be solved by greedy algorithms with polynomial complexity [32]. There is a sizeable theory behind the knapsack problems and we will not go into its details here. Instead, we conjecture that those OVSF CDMA problems which are not greedy solvable are NP complete due to similar reasons for NP completeness of some Knapsack problems.

2.5 Examples and Simulations

We apply the algorithms on the downlink of a single cell CDMA wireless network. The system assumptions are consistent with 3G system specifications. Spreading gain is assumed to be in the range from a minimum of 4 upto 512, which corresponds to 8 levels on the binary code tree including the root node. The spreading bandwidth is 3.84 MHz, maximum base station transmission power is 10 Watts, the receiver noise, N_oW, is 10^{-13} Watts, and the target $(E/I)_u^r = \gamma_u$ is 5 (7dB) (different applications may have different γ_u targets, the algorithms are applicable in such cases as well). The x any y coordinates of each mobile are selected uniformly on (0-2000m) in a square cell of 4km^2. The path loss at any given distance d is given by [33]

$$PL = A + 10\epsilon \log_{10}(d/d_0) + s; \qquad d \geq d_0 \tag{2.35}$$

where A is the decibel path loss at distance d_0, ϵ is the path loss exponent, and s is the shadow fading variation. The numerical values are d_0=100 m, A=78 dB, s is lognormal with σ=8 dB, and ϵ=4. The multipath characteristics, or the frequency selectivity, of the channels are represented by the orthogonality factor $\bar{\beta}$; for simplicity we assume that all users have the same average $\bar{\beta}$. The experiments are conducted for $\bar{\beta} = 0$, which corresponds to a flat channel, $\bar{\beta} = 0.1$, $\bar{\beta} = 0.5$ and $\bar{\beta} = 0.8$. Each mobile requires a minimum rate corresponding to one of $\{128, 256, 512\}$ length spreading codes with equal probability. We assume fixed modulation and coding [6].

[6]There is no explicit assumption on modulation and coding format. The only assumption is that they are the same and fixed for all users, which implies that spreading codes with the same lengths correspond to the same rate, and the rate assignments are proportional to the number of spreading code assignments. In numerical calculations, we assumed uncoded QPSK, while the algorithms and analysis are valid for any fixed coding and modulation format.

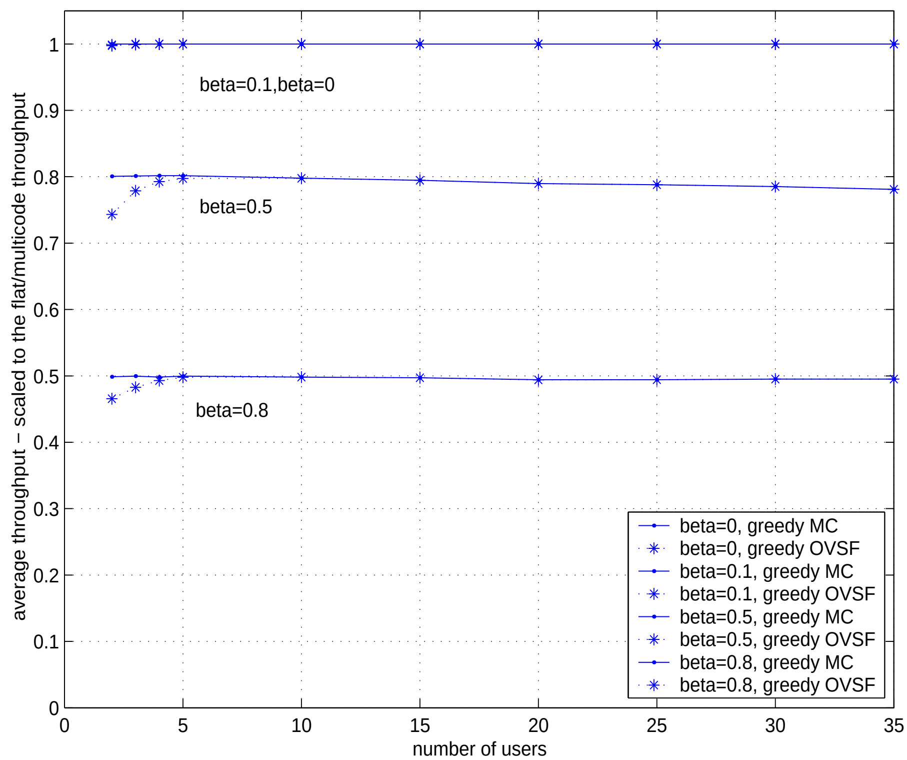

Figure 2.5: Comparison of the Average Throughput Results

The average throughput of the greedy algorithm in an OVSF CDMA system is compared to the average optimal throughput of a system using multiple codes (Multicode CDMA). The average throughput values are obtained over 500 simulation runs for a given number of users. In each simulation run, a set of user locations and a set of minimum rate requirements are generated. In case minimum rate requirements are not feasible, i.e. if providing minimum rates to each user at the target $(E/I)_u^r = \gamma_u$ requires more power than P, a new set of minimum rates are generated by decreasing the minimum rate requirements of the users who require the largest transmit energy per bits.

Figure 2.5 compares the average throughput results in multirate CDMA systems using multiple codes and variable spreading factor codes; the values are scaled with

Path Loss (dB)	71	78	93	97	107	108	110	124	127	129
$R_{u,min}$ (Kbps)	15	60	15	15	15	15	15	60	15	60
R_u (Kbps), $\beta = 0$	960	480	240	60	15	15	15	60	15	60
R_u (Kbps), $\beta = 0.1$	960	480	240	60	15	15	15	60	15	60
R_u (Kbps), $\beta = 0.5$	960	240	120	30	15	15	15	60	15	60
R_u (Kbps), $\beta = 0.8$	480	240	30	30	15	15	15	60	15	60

Table 2.1: Throughput vs Path Loss for a system using OVSF codes, for 10 users. Uncoded QPSK is assumed. The target SINR per bit is $\gamma = 5$ ($\approx$ 7dB), corresponding to a probability of a symbol error of $P_b \approx 5 \times 10^{-3}$.

respect to the average optimal throughput of a multicode system, which is achieved by the greedy algorithm for any set of minimum rates. Since any set of rates offered by variable spreading factor codes can be realized by multiple codes (for example a spreading code of length 128 offers the same rate as 4 spreading codes of length 512), the optimal throughput of a multicode system can be thought of as an upperbound to the average throughput of a variable spreading system.

The results in Figure 2.5 verify the superiority of the greedy algorithm under various system and channel assumptions. In the figure, we observe that the performance gap between multicode throughput and variable spreading throughput closes as the number of users increases and the holes on the binary code tree get filled more efficiently by more users. Notice that it is much easier to manage orthogonal spreading codes in a multicode CDMA system; for example in a 2 user OVSF system, the user with a better channel can achieve at most half of the bandwidth (the node below the root node), while in a multicode CDMA system the same user can have the whole available bandwidth as long as the power constraint permits.

In Table 2.1, individual rate assignments and corresponding path loss values are presented for 10 users in a system using OVSF codes. Each user is assumed to have an

Path Loss (dB)	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
OF (β)	**0.7**	**0.4**	**0.3**	**0.2**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
$R_{u,min}$ **(Kbps)**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
R_u **(Kbps)**	**15**	**15**	**15**	**15**	**960**	**480**	**240**	**60**	**15**	**15**

Table 2.2: Throughput vs Path Loss for a Short Range System

average orthogonality factor of 0.1. Uncoded QPSK is assumed, and the target received SINR per bit is $\gamma = 5$ ($\approx$ 7dB) for each user, corresponding to a probability of a symbol error of $P_b \approx 5 \times 10^{-3}$. Notice that except path loss, all other factors affecting the transmit energy per bit are the same for all users in the example. As a result, users with relatively low path loss values receive high throughputs since they require lower transmit energy per bit.

An interesting case is the scheduling of users in a short range system, such as in the case of a WLAN or an Infostation where the path loss is small compared to a cellular layout. A simple link budget calculation shows that if the path loss is smaller than $100 - 110$ dB, the transmit energy per bit is independent of path loss. For example, if $P = 10$ Watts, $\bar{\beta}_u \approx 0.1$, $N_0 W \approx 10^{-13}$ Watts, and $h_u > 10^{-11}$, then

$$(E_b)_u^t = \frac{1}{h_u} \frac{(\bar{\beta}_u h_u P + N_0 W)\gamma_u}{W + R_s \bar{\beta}_u \gamma_u} \approx \frac{\bar{\beta}_u P \gamma_u}{W + R_s \bar{\beta}_u \gamma_u} \tag{2.36}$$

which is independent of h_u. In this case, users are scheduled based on the SINR per bit target γ_u and the orthogonality factor $\bar{\beta}_u$, which is a function of the multipath dispersion of the channel [1–3]. In Table 2.2, simulation results are presented for such a system. We observe that the user with a moderate path loss (50 dB) but with the smallest orthogonality factor achieves the largest throughput.

Finally, the effect of the orthogonality factor on throughput is analyzed in Figure

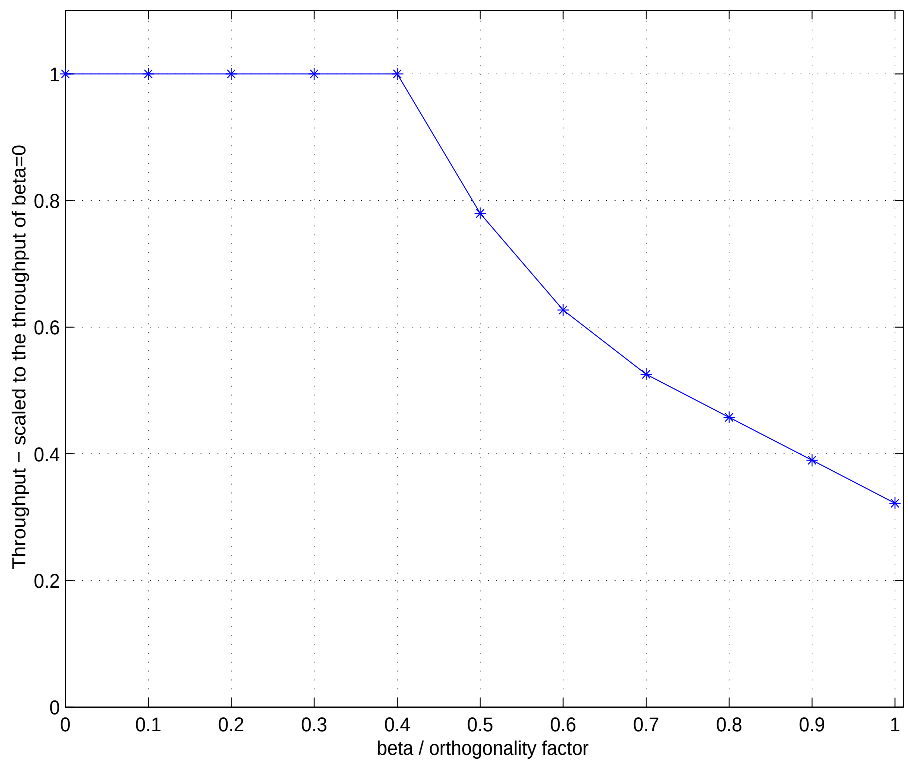

Figure 2.6: Sum Rate vs the Orthogonality Factor

2.6. The experiment is conducted for 10 users in a system using OVSF codes. A sharp decline in total throughput is observed beyond a threshold value of the orthogonality factor.

2.6 Chapter Summary and Conclusion

Next generation mobile networks will provide multimedia services with variable data rates and different service classes in addition to classical voice service. Accordingly efficient usage of limited radio resources such as power and bandwidth, and QoS satisfaction of various service classes is essential in the future systems. In addition, the characteristics of data services requires a large traffic load on the downlink of the system.

In this chapter, we investigated throughput maximization on the downlink of a CDMA wireless network. Both systems employing orthogonal variable spreading factor codes (OVSF CDMA) and multiple codes (multicode CDMA) have been studied. The objective is to maximize the network throughput under constraints on total transmit power, total bandwidth and individual QoS requirements specified in terms of minimum rates. First, users are ordered based on transmit energy per bit requirements to achieve the target received energy per bit to interference power spectral density ratio at the receivers. Based on the initial ordering, we prove that for systems employing multiple codes, the greedy rate scheduling is optimal, and therefore it yields maximum network throughput. For systems employing OVSF codes, the greedy rate scheduling is optimal if the minimum rate requirement of a user is larger than or equal to the minimum rate requirement of any other user with a larger transmit energy per bit requirement. Simulation results show that the greedy algorithm, even when it is suboptimal, is a good heuristic yielding average throughput which is very close to the optimal achievable throughput in OVSF CDMA systems. The simplicity and polynomial time complexity of the greedy algorithms seem to be very attractive from an implementation point of view.

Chapter 3

Joint Power and Rate Control in Multiaccess Systems with Multirate Services

In this chapter, we study joint power and rate control for wireless multiaccess systems providing multirate services in a frequency selective multipath channel. We show that the power control framework [4] can be extended to include rate control as well. Using this framework, we prove that a joint power and rate control algorithm converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient signal to interference plus noise ratio (SINR).

The chapter starts with a brief background on the topic including the relevant literature. In Section 3.2, we describe the system model and define the problem. An iterative algorithm is proposed in Section 3.3, and its convergence properties are analyzed in the same section. In Section 3.4, we apply the proposed algorithm on the downlink of a multirate CDMA wireless network. We conclude the chapter with a brief summary and discussion.

3.1 Introduction

In wireless multiaccess systems, multiple users share a common communication medium. In TDMA systems, the medium is shared via time slots. In CDMA systems, spreading codes provide users access to the communication medium. Similarly, for an OFDM system, a number of subcarriers provide access to the common medium. While multiple user's information bits are transmitted simultaneously in one way or another, each user has to achieve a level of quality of service (QoS) within system constraints such as total transmit power or bandwidth.

Since wireless resources are scarce and expensive, a careful and efficient allocation is vital. For example, in CDMA based IS-95 systems, power control is a useful technique to regulate transmit powers of constant bit rate voice users so as to minimize the effect of multiaccess interference (see [34] for a survey on this topic).

On the other hand, current and future wireless networks such as 3G cellular, WLANs or 4G wireless networks, are based on supporting multirate data services such as multimedia applications, internet access etc., in addition to classical voice service. For data service, users may employ multiple time slots or multiple spreading codes, and may receive variable rates. In this case, efficient resource allocation requires optimization and control of multiple parameters simultaneously, such as joint control of transmit power and rate assignments.

In the context of CDMA systems, combined power and rate control algorithms have been studied in [35–37]. Two algorithms have been proposed in [35], one is based on Lagrangian relaxation techniques and the other, called selective power control, is an extension of a fixed rate power control algorithm. On the other hand, the basic idea in [36, 37] is to adapt (reduce) the rate when the transmit power required to achieve a target QoS exceeds a threshold. For multirate CDMA systems, the uplink throughput maximization problem has been formulated in [13–15]. For the networks with multiple service classes, the authors aim to satisfy different QoS requirements while utilizing the system resources in an efficient way.

Since voice based networks, such as IS-95, provide real-time constant bit rate service with QoS requirement in terms of target SINR at the base station, most of the initial CDMA resource allocation research is focused on power control algorithms in the uplink direction. On the other hand, multimedia oriented wireless networks provide multirate data services, such as web applications, wireless video etc., which require heavy traffic load in the downlink direction. Our focus in this chapter is the downlink power and rate allocation.

In Chapter 2, the downlink throughput maximization problem is considered, where the effect of loss of orthogonality of transmitted waveforms is characterized by the orthogonality factor. Using the orthogonality factor simplifies the problem formulation considerably since, with a single variable, we account for the combined effects of spreading codes employed, receiver structures and instantaneous multipath channel coefficients on the multiaccess interference. The main motivation in this chapter is to exploit the effects of all these parameters on the system performance, which are implicitly neglected by using the orthogonality factor. The rationale is that, unlike uplink where random spreading codes are used, fixed and deterministic Walsh codes[1] are used on the downlink. On the other hand, in frequency selective channels, different waveforms are

[1]Although CDMA networks are emphasized throughout the chapter, the analyses are valid for any multi-access scheme using orthogonal waveforms on the downlink.

filtered in different ways by each user's channel so that the SINR of user j decoding bit i will depend on both the waveform $\mathbf{s}_i$ and the channel $\mathbf{H}_j$. Therefore, it is crucial to determine which orthogonal waveforms provide a given set of rate assignments.

3.2 System Model and Problem Statement

We consider multirate data transmission on the downlink of a single cell multiaccess system. There are K users in the cell. A multiaccess system is represented by a set of N orthogonal unit energy waveforms denoted by $\mathbf{S}(t) = \{\mathbf{s}_1(t), \mathbf{s}_2(t), \ldots, \mathbf{s}_N(t)\}$. Each waveform in $\mathbf{S}(t)$ is zero outside the transmission interval $[0, T]$. For a TDMA system $\{\mathbf{s}_i(t) = \psi(t-(i-1)T/N),\ i = 1, \ldots, N\}$ where $\psi(t)$ is a square pulse on the interval $[0, T/N]$. For a CDMA system $\{\mathbf{s}_i(t) = \sum_{j=1}^{N} s_{ij}\psi(t-(j-1)T/N),\ i = 1, \ldots, N\}$ where $\psi(t)$ is the chip waveform nonzero in the interval $[0, T/N]$ and $s_{ij} = \int_0^T s_i(t)\psi(t-(j-1)T/N)dt$. In the case of an OFDM system, N OFDM tones or subcarriers may be viewed as the waveform set.

Projecting time signals onto an appropriate basis in each multiaccess system, we obtain vector representation of the waveform set, $\mathbf{S} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_N\}$ where $\mathbf{s}_i \in \mathbb{C}^N$. In each tranmission interval $[0, T]$, base station transmits N waveforms in $\mathbf{S}$. Multirate transmission is provided by assigning multiple waveforms to a user. A waveform $\mathbf{s}_i$ is transmitted with power p_i and is assigned to a user j such that user j can reliably decode (achieves target SINR γ) $\mathbf{s}_i$ using filter $\mathbf{c}_{ij}$.

Each bit is denoted by b_i, thus the base station transmits the signal

$$\mathbf{x} = \sum_{i=1}^{N} \sqrt{p_i} b_i \mathbf{s}_i \tag{3.1}$$

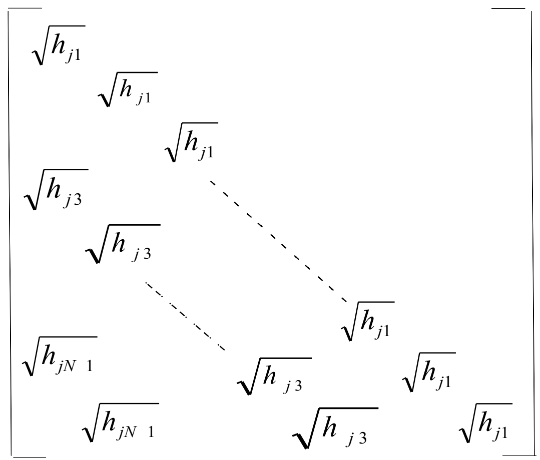

Figure 3.1: Channel Matrix

The channel between the base station and user j is modeled as a frequency-selective multipath channel with impulse response

$$h_j(t) = \sum_{p=1}^{L_j} h_{jp}\delta(t - \tau_{jp}) \tag{3.2}$$

where L_j denotes the number of channel taps, τ_{jp} and h_{jp} denote the delay and the complex gain of pth channel tap of user j respectively. The channel taps are assumed to be chip synchronous so that each tap delay is an integer. In matrix form, $h_j(t)$ corresponds to the matrix $\mathbf{H}_j \in \mathbb{C}^{\mathbb{N}\times\mathbb{N}}$ which is a lower triangular Toeplitz matrix with tap gain h_{jp} on its τ_{jp}th diagonal below the main diagonal. A sample three tap channel is shown in Figure 3.1. We assume that the base station has full knowledge of each user's channel.

Mobile j receives $\mathbf{r}_j$ which is the channel $\mathbf{H}_j$ distorted version of $\mathbf{x}$ plus the white receiver noise $\mathbf{n}_j$ with covariance $\sigma^2\mathbf{I}$

$$\mathbf{r}_j = \mathbf{H}_j\mathbf{x} + \mathbf{n}_j = \sum_{i=1}^{N} \sqrt{p_i} b_i \mathbf{H}_j \mathbf{s}_i + \mathbf{n}_j \tag{3.3}$$

There is no predetermined or fixed assignment of waveforms to users and the base station has to decide which waveform should be assigned to which user. This is a crucial point in our problem formulation. Since the channels are frequency-selective, different waveforms get distorted in a different ways by each user's channel. Therefore the SINR of user j decoding bit i will depend on both the waveform $\mathbf{s}_i$ and the channel $\mathbf{H}_j$.

To decode its own bit or bits, mobile j passes the received signal $\mathbf{r}_j$ through a bank of receiver filters, one for each waveform $\mathbf{s}_i$. Denoting the noise power at the output of a filter by σ^2, the signal to noise plus interference ratio γ_i achieved at the output of the filter $\mathbf{c}_{ij}$ is

$$\gamma_i = \frac{p_i(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2}{\sum_{v \neq i} p_v(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_v)^2 + \sigma^2(\mathbf{c}_{ij}^T \mathbf{c}_{ij})} \tag{3.4}$$

Our problem is to minimize the total power required to transmit N waveforms to K users where N may or may not be equal to K. For each waveform $\mathbf{s}_i$, we will decide on a user j intended to receive $\mathbf{s}_i$, a receiver filter $\mathbf{c}_{ij}$, and a transmit power p_i required to achieve the target SINR γ while user j decodes its transmitted bit on the waveform $\mathbf{s}_i$. Note that the number of waveforms assigned to a user determines the rate assigned to that user.

The optimization problem is as follows

$$\min \quad \sum_{i=1}^{N} p_i \tag{3.5}$$

$$s.t. \quad \max_{j} \max_{\mathbf{c}_{ij}} \left(\frac{p_i(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2}{\sum_{v \neq i} p_v(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_v)^2 + \sigma^2(\mathbf{c}_{ij}^T \mathbf{c}_{ij})} \right) \geq \gamma \tag{3.5b}$$

$$p_i \geq 0, \ \mathbf{c}_{ij} \in \mathbb{C}^N, \ i = 1, \ldots, N$$

The constraint in (3.5b) guarantees that for a given waveform $\mathbf{s}_i$, there is at least one user $j^* \in \{1, \ldots, K\}$ and a receiver filter $\mathbf{c}_{ij^*}$ that can decode $\mathbf{s}_i$ with acceptable quality.

3.3 Solution

Similar power minimization problems have been well studied in literature [4, 38]. Our problem definition adds user selection into the formulation. In each transmission interval, the base station has to determine how many waveforms each user will be assigned to, and accordingly how may bits each user will receive. From this point of view, (3.5) may be viewed as a joint power and rate control problem.

We follow a similar analysis to that in [38]. We rewrite (3.5) in the form of a standard power control problem [4].

$$\begin{aligned}
\min \quad & \sum_{i=1}^{N} p_i \\
s.t. \quad & p_i \geq \min_{j} \min_{\mathbf{c}_{ij}} \left(\frac{\gamma(\sum_{v \neq i} p_v (\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_v)^2 + \sigma^2 (\mathbf{c}_{ij}^T \mathbf{c}_{ij}))}{(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2} \right) \\
& p_i \geq 0,\ \mathbf{c}_{ij} \in \mathbb{C}^N,\ i = 1, \ldots, N
\end{aligned} \tag{3.6}$$

We define

$$\mathbf{p} = [p_1, \ldots, p_N] \tag{3.7}$$

$$I_i(\mathbf{p}, j, \mathbf{c}_{ij}) = \frac{\gamma(\sum_{v \neq i} p_v (\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_v)^2 + \sigma^2 (\mathbf{c}_{ij}^T \mathbf{c}_{ij}))}{(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2} \tag{3.8}$$

In the context of [4], the interference function $\mathbf{I}(\mathbf{p})$ becomes

$$\mathbf{I}(\mathbf{p}) = [I_1(\mathbf{p}), \ldots, I_N(\mathbf{p})] \tag{3.9}$$

where

$$I_i(\mathbf{p}) = \min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij}) \tag{3.10}$$

We propose the following iterative algorithm

$$\mathbf{p}(n+1) \quad = \quad \mathbf{I}(\mathbf{p}(n)) \tag{3.11}$$

The framework of [4] tells us that an iterative algorithm in the form of (3.11) converges to the minimum power solution if the interference function $\mathbf{I}(\mathbf{p})$ is standard [4]. Next, we will show that $\mathbf{I}(\mathbf{p})$ is standard. Therefore, when the algorithm (3.11) converges, we obtain 1) optimum matchings $(\mathbf{s}_i, j)$ between waveforms and the users, 2) optimum receiver filter $\mathbf{c}_{ij}$ that user j will use to decode $\mathbf{s}_i$, 3) optimum power assignments $\bar{\mathbf{p}} = [p_1, \ldots, p_N]$ that minimizes the objective function in (3.5).

Theorem 1. $\mathbf{I}(\mathbf{p}) = [I_1(\mathbf{p}), \ldots, I_N(\mathbf{p})]$ *is a standard interference function.*

Proof. In order $\mathbf{I}(\mathbf{p})$ to be standard, it has to satisfy *Positivity*, *Monotonicity* and *Scalability* properties, see [4] for details about these properties.

The positivity property is straightforward. For any user position j and filter $\mathbf{c}_{ij}$, $I_i(\mathbf{p})$ is always positive for any $\mathbf{p} \geq 0$ since $I_i(\mathbf{p}, j, c_{ij})$ in (3.8) contains only summation, multiplication and division of positive terms.

Monotonicity is satisfied if $\mathbf{p} \geq \mathbf{p}'$ implies $\mathbf{I}(\mathbf{p}) \geq \mathbf{I}(\mathbf{p}')$. From (3.8), for any given j and $\mathbf{c}_{ij}$, $\mathbf{p} \geq \mathbf{p}'$ implies $I_i(\mathbf{p}, j, \mathbf{c}_{ij}) \geq I_i(\mathbf{p}', j, \mathbf{c}_{ij})$. Notice that increasing each power assignment from $p_i^{'}$ to p_i results in a larger sum in the numerator of (3.8). Assume the arguments satisfying the equality $I_i(\mathbf{p}) = \min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ occurs at j^* and $\mathbf{c}_{ij}^*$ then

$$I_i(\mathbf{p}) = \min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij}) \tag{3.12}$$

$$= I_i(\mathbf{p}, j^*, \mathbf{c}_{ij}^*) \tag{3.13}$$

$$\geq I_i(\mathbf{p}^{'}, j^*, \mathbf{c}_{ij}^*) \tag{3.14}$$

$$\geq \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}^{'}, j^*, \mathbf{c}_{ij}) \tag{3.15}$$

$$\geq \min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}^{'}, j, \mathbf{c}_{ij}) = I_i(\mathbf{p}^{'}) \tag{3.16}$$

Scalability is satisfied if $\alpha > 1$ implies $\alpha I(\mathbf{p}) > I(\alpha\mathbf{p})$. From (3.8), for any given j and $\mathbf{c}_{ij}$, $\alpha > 1$ implies $\alpha I_i(\mathbf{p}, j, \mathbf{c}_{ij}) > I_i(\alpha\mathbf{p}, j, \mathbf{c}_{ij})$. Notice that multiplying the whole expression in (3.8) with $\alpha > 1$ results in multiplication of the nonnegative background noise σ^2 with α as well, thus $\alpha I_i(\mathbf{p}, j, \mathbf{c}_{ij}) > I_i(\alpha\mathbf{p}, j, \mathbf{c}_{ij})$ follows. To show scalability

$$\alpha I_i(\mathbf{p}) = \min_j \min_{\mathbf{c}_{ij}} \alpha I_i(\mathbf{p}, j, \mathbf{c}_{ij}) \tag{3.17}$$

$$= \alpha I_i(\mathbf{p}, j^*, \mathbf{c}_{ij}^*) \tag{3.18}$$

$$> I_i(\alpha\mathbf{p}, j^*, \mathbf{c}_{ij}^*) \tag{3.19}$$

$$> \min_{\mathbf{c}_{ij}} I_i(\alpha\mathbf{p}, j^*, \mathbf{c}_{ij}) \tag{3.20}$$

$$> \min_j \min_{\mathbf{c}_{ij}} I_i(\alpha\mathbf{p}, j, \mathbf{c}_{ij}) = I_i(\alpha\mathbf{p}) \tag{3.21}$$

Since $I(\mathbf{p})$ satisfies *positivity*, *monotonicity* and *scalability* properties, $I(\mathbf{p})$ is a standard interference function. $\square$

Theorem 2. *The solution of* $\min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ *occurs at* j^* *and* $\mathbf{c}_{ij^*}$ *where*

$$\mathbf{A}_{i,k} = \sum_{v \neq i}^{N} p_v (\mathbf{H}_k \mathbf{s}_v)(\mathbf{H}_k \mathbf{s}_v)^T + \sigma^2 \mathbf{I} \tag{3.22}$$

$$j^* = \arg \min_{k \in \{1,\ldots,K\}} ((\mathbf{H}_k \mathbf{s}_i)^T \mathbf{A}_{i,k}^{-1} (\mathbf{H}_k \mathbf{s}_i))^{-1} \tag{3.23}$$

$$\mathbf{c}_{ij^*} = \frac{\sqrt{p_i}}{1 + p_i (\mathbf{H}_{j^*} \mathbf{s}_i)^T \mathbf{A}_{ij^*}^{-1} (\mathbf{H}_{j^*} \mathbf{s}_i)} \mathbf{A}_{i,j^*}^{-1} \mathbf{H}_{j^*} \mathbf{s}_i \tag{3.24}$$

Proof. We rewrite (3.8) as

$$I_i(\mathbf{p}, j, \mathbf{c}_{ij}) = \frac{\gamma\ \mathbf{c}_{ij}^T (\sum_{v \neq i}^N p_v (\mathbf{H}_j \mathbf{s}_v)(\mathbf{H}_j \mathbf{s}_v)^T + \sigma^2 \mathbf{I}) \mathbf{c}_{ij}}{(\mathbf{c}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2} \tag{3.25}$$

For a given $\mathbf{p}$ and j, $\mathbf{c}_{ij}$ that minimizes (3.25) maximizes the left side of (3.5b), which is the SINR achieved at the output of $\mathbf{c}_{ij}$. Therefore $\mathbf{c}_{ij}$ must be the SINR maximizing MMSE filter [38, 39] which is given as

$$\mathbf{c}_{ij} = \frac{\sqrt{p_i}}{1 + p_i (\mathbf{H}_j \mathbf{s}_i)^T \mathbf{A}_{ij}^{-1} (\mathbf{H}_j \mathbf{s}_i)} \mathbf{A}_{ij}^{-1} \mathbf{H}_j \mathbf{s}_i \tag{3.26}$$

where

$$\mathbf{A}_{ij} = \sum_{v \neq i}^{N} p_v (\mathbf{H}_j \mathbf{s}_v)(\mathbf{H}_j \mathbf{s}_v)^T + \sigma^2 \mathbf{I} \tag{3.27}$$

For a given user j and its MMSE filter $\mathbf{c}_{ij}$ (3.26), the value of $I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ becomes

$$\gamma\ ((\mathbf{H}_j \mathbf{s}_i)^T \mathbf{A}_{ij}^{-1} \mathbf{H}_j \mathbf{s}_i)^{-1} \tag{3.28}$$

In this case, $j^* \in \{1, \ldots, K\}$ that minimizes (3.28) and its corresponding MMSE filter $\mathbf{c}_{ij^*}$ (3.26) is the solution of $\min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$. $\square$

s_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
s_2	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
s_3	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
s_4	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
s_5	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
s_6	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
s_7	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
s_8	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
s_9	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
s_{10}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
s_{11}	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
s_{12}	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
s_{13}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
s_{14}	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
s_{15}	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1
s_{16}	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1

Table 3.1: The set of Orthogonal Spreading Codes

Since solving $\min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ for $j^* \in \{1, \ldots, K\}$ and $\mathbf{c}_{ij^*} \in \mathbb{C}^{\mathbb{N}}$ is equivalent to finding the user j^* that decodes $\mathbf{s}_i$ with the largest possible SINR, we conclude that the minimum power solution to the problem (3.5) is achieved when, at each iteration of the algorithm (3.11), the base station assigns each signal waveform to the user who can receive that waveform with the best quality.

We observe in the simulations that although the assignment of waveforms to users may change from iteration to iteration, the set of assignments in the unique minimum total power solution is eventually achieved when the algorithm converges.

3.4 Examples

We apply the proposed iterative algorithm (3.11) on the downlink of a multirate CDMA wireless network. There are 8 users in the cell, and the spreading gain is 16. All 16 orthogonal spreading codes used in the simulations are given in Table 3.1.

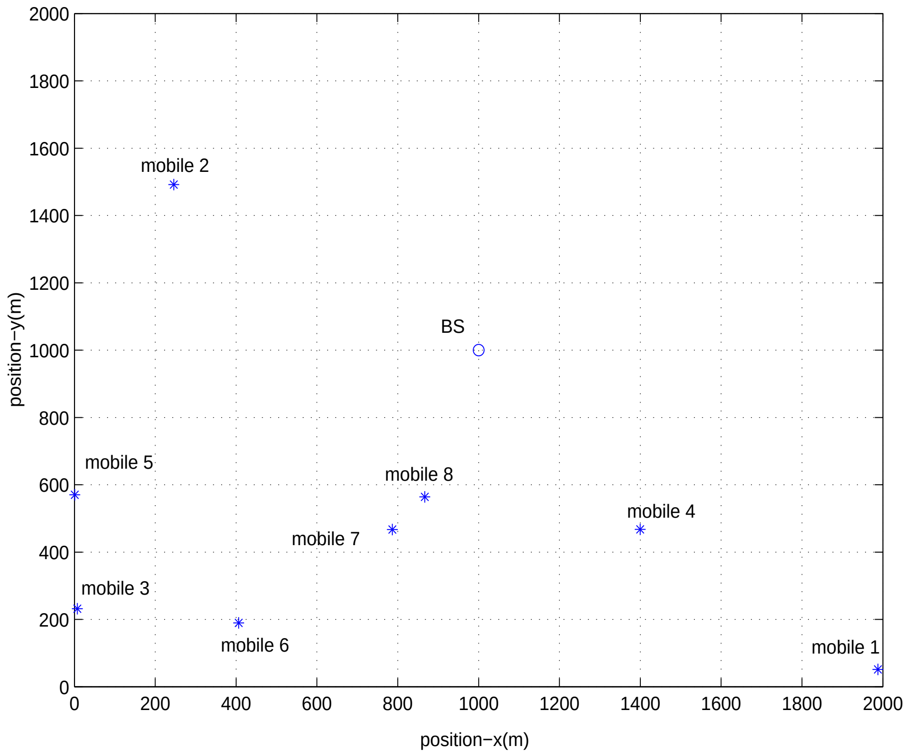

Figure 3.2: Position of Each Mobile Over the Cell

Figure 3.2 shows the locations of the mobiles (stars) and the base station (circle) on a square cell. The x and y coordinates of each mobile location is chosen uniformly on (0-2000m).

The impulse response $h_j(t)$ of the multipath channel between mobile j and the base station is

$$h_j(t) = \sum_{p=1}^{L_j} h_{jp}\delta(t - \tau_{jp}) \tag{3.29}$$

The number of channel taps L_j is chosen uniformly on $\{1, \ldots, 5\}$. The delay of the first path τ_{j1} is set to 0, for all other channel taps, each successive tap is delayed by either 1 or 2 chips, with probability 1/2 each. Therefore the delay spread can be at most 8 chips. From the first arriving channel tap to the last one, the difference in gain, i.e. $20\log(|h_p|/|h_{p+1}|)$, between two successive tap gains is A dB where A ˜

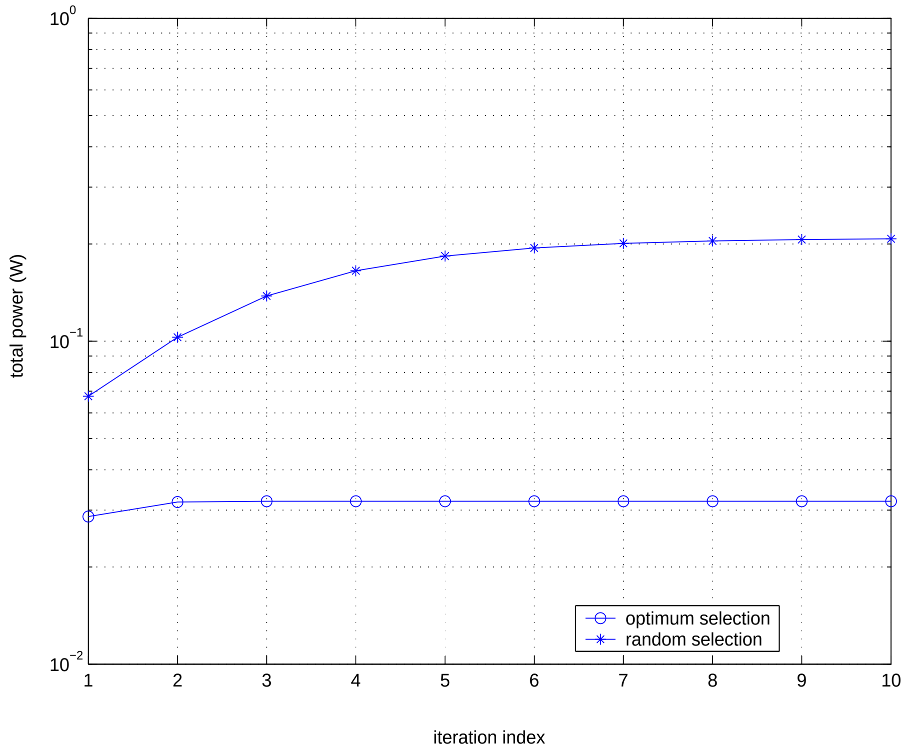

Figure 3.3: Total Power Convergence

N(5,10). Note that, with a small probability, pth channel tap may have worse gain than $(p+1)$th channel tap. The method we used for multipath generation is consistent with the multipath channel model proposed in [40, 41] where the difference in gain between the strongest and the weakest path is around 25dB. The order of magnitudes for tap gains are also consistent with the ARIB Channel B model [42]. A pathloss exponent of 4 is used and channel gains are scaled to the pathloss gain. The target SINR for each spreading code is 5 (7 dB), and $\sigma^2 = 10^{-13}$ Watts.

The circled curve in Figure 3.3 shows the convergence of the iterative algorithm (3.11) to the minimum total power solution. When the algorithm converges, mobile 8 receives $\{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}\}$, mobile 7 receives $\{s_{10}, s_{11}\}$ and mobile 4 receives $\{s_9\}$. The spreading code-user matching generated by the iterative algorithm (3.11) is called "optimum selection" since it minimizes the total power. The

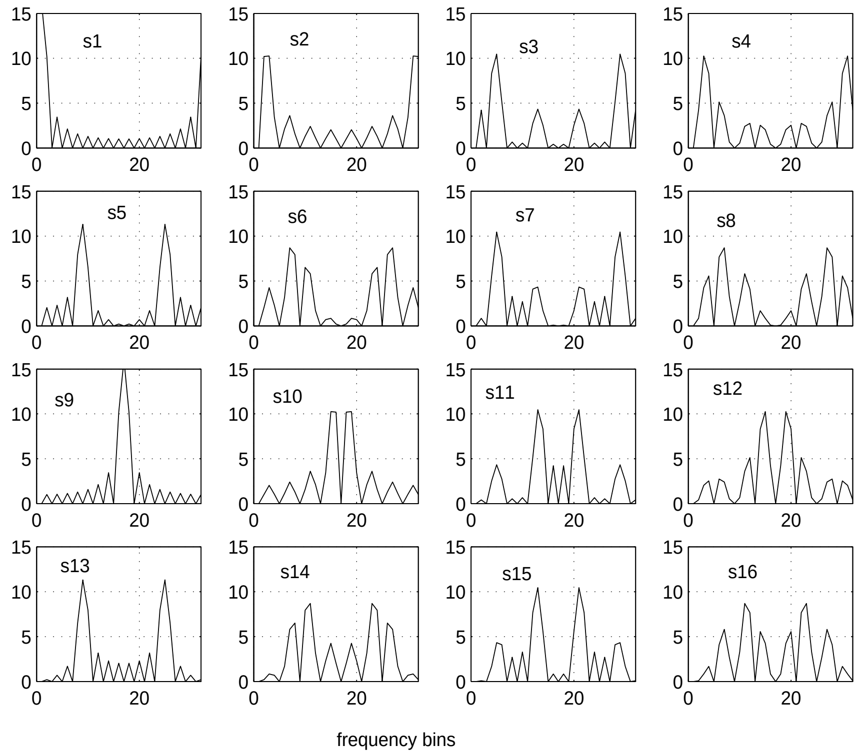

Figure 3.4: The Spreading Codes in Frequency Domain

second curve in Figure 3.3 shows the case of "random selection" where the same set of rate assignments generating the circled curve is provided by random assignments of spreading codes to users (the standard power control iteration is used to minimize the total power once the spreading codes are assigned). Specifically as a result of random selection, mobile 8 receives $\{s_1, s_2, s_3, s_4, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}\}$, mobile 7 receives $\{s_6, s_7\}$ and mobile 4 receives $\{s_5\}$. We observe that there is a considerable gain ($\approx$ 8 dB for the example in the figure) in selecting the appropriate spreading code-user matching on the downlink.

To have an insight into how the algorithm assigns the waveforms, we plot the set of all spreading codes in frequency domain in Figure 3.4. Moreover, Figure 3.5 shows the channel responses of those users who are assigned at least one spreading code (mobile 8,7 and 4) and Figure 3.6 shows the channels of those users who are not assigned any

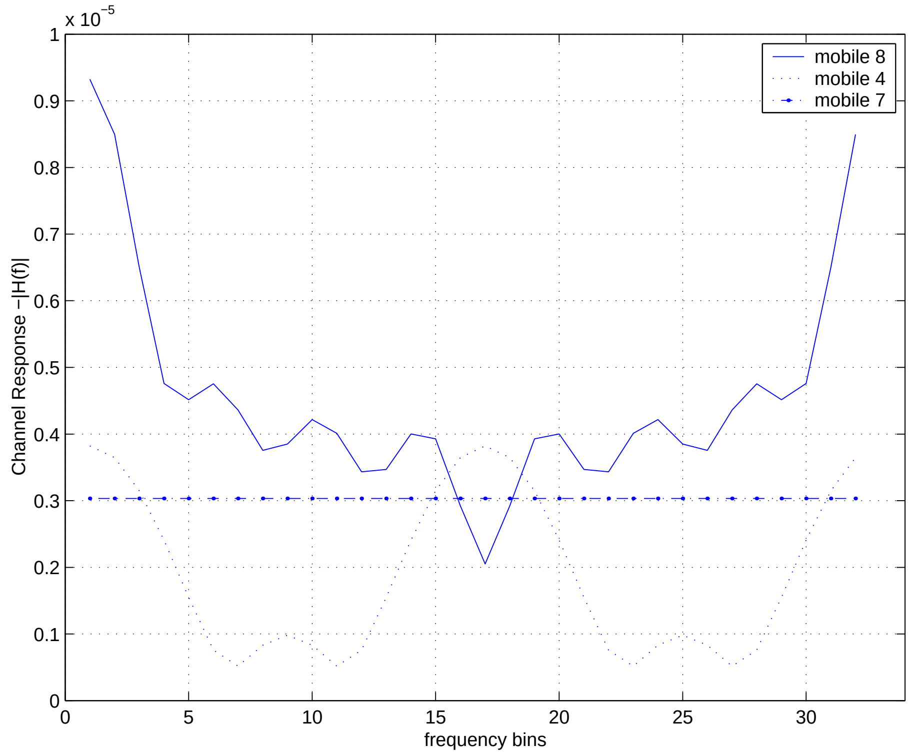

Figure 3.5: Channel Responses of mobile 8,4 and 7

codes (mobile 1,2,3,5 and 6). We see in the figures that those users who are not assigned any codes have relatively deeper channel fades compared to those who receive codes. For example, the channel gains of mobile 1,3,5 and 6 ($\approx< 10^{-6}$) is almost always below the channel gains of mobile 8, 7 and 4.

On the other hand, we observe that assignment of spreading codes to users is done based on frequency domain characteristics of the channels and the waveforms. As an example, mobile 4 has the largest channel gain around the center of the spectrum (15-20th frequency bins in the figures), and 9th Walsh code (s_9) which has a peak over the same frequency range is assigned to mobile 4. The intuition is that s_9, which has a very small gain on the sidelobes, suppresses the interference well from other Walsh codes while the mobile 4's channel scales s_9 well as it has the largest gain among all users' channels over the mainlobe of s_9.

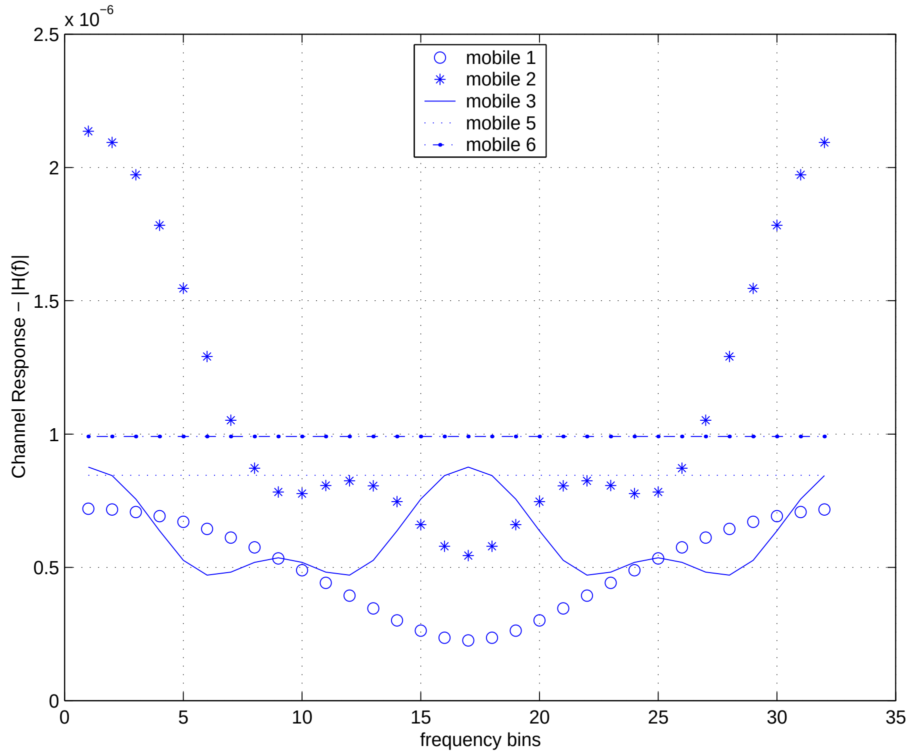

Figure 3.6: Channel Responses of mobile 1,2,3,5,6

As the example points out, multiple users may be served in the minimum total power solution in frequency selective channels. This is because of the fact that different users may have the best channel gain over different frequency ranges in the spectrum; in other words there are multiple "best channel" users. On the other hand even in such cases, the optimum solution of (3.5) may result in an unfair assignment of resources to users. In this case, some users with very bad channels might not be assigned any waveforms, while some applications such as real-time data requires a minimum level of service without delay. In the next chapter we will use minimum cost flow problem formulation and analyze the case with strict minimum rate requirements.

3.5 Chapter Summary and Conclusion

In this chapter, we examined joint power control and orthogonal code selection (rate control) for wireless multiaccess systems providing multirate services in frequency selective multipath channels. We proposed an iterative algorithm converging to optimum assignments of multiaccess resources to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient signal to interference plus noise ratio (SINR). Simulation results for CDMA networks show that there is a considerable gain (≈ 8 dB for the particular example in the text) in selecting the appropriate spreading code-user matching on the downlink.

Chapter 4

Minimum Cost Network Flows and Strict Rate Requirements

Network flows are well-known combinatorial subject with many applications in various fields of computer science, engineering, management and operations research. The minimum cost flow problem, a fundamental problem of network flows, deals with determining a least cost shipment of a commodity through a network in order to satisfy demands at certain nodes from available supplies at other nodes [5]. Here we model the communication through a wireless network as a network flow and we aim to minimize the cost of information flow through the network under constraints on demands (minimum rate) of mobiles and supply (bandwidth) of the base station. This model helps us to improve fairness by enforcing minimum rate constraints on each mobile.

The chapter starts with a brief background on the topic including the relevant literature. In Section 4.2, we explain how we apply minimum cost network flow model on a wireless system. We examine an OFDM system in Section 4.3, and we solve OFDM carrier assignment problem using minimum cost flow technique. In Section 4.4, we illustrate two applications in CDMA systems. We first combine the minimum cost network flow technique with linear receiver processing, and then we propose an iterative minimum cost flow algorithm. We conclude the chapter with a brief summary and discussion.

4.1 Background

Resource allocation problems in wireless systems aim to find the best assignments, or *matchings*, of limited available radio resources, such as power and bandwidth, to mobile users. The objective in such problems can be defined either in terms of maximizing the sum of rates (throughput), or in terms of minimizing the total transmit power. Moreover the objective has to be achieved under constraints on total transmit power, bandwidth and minimum service requirement of each user (fairness).

Such problems get more difficult as the constraint set gets more complicated. As an example, those problems with *discrete* constraints, so-called combinatorial problems, are usually more difficult than those with *continuous* constraint set. On the other hand, practical systems in general assume discrete system parameters, such as discrete rates, discrete spreading gains etc. Although relaxing discrete problem constraints to continuous variables makes the problem more tractable, this relaxation generally leads to suboptimal solutions.

An insightful way to solve resource allocation problems with discrete constraint sets is to use combinatorial models. Here we will use two combinatorial models: *bipartite matchings* and *minimum cost network flows*. These combinatorial problems are closely related in the sense that bipartite matching problems can be solved efficiently by making use of any algorithm that solves the minimum cost network flow problems [5, 30]. The main motivation to adapt these models is to improve fairness by enforcing minimum rate constraints for each mobile in a problem with discrete set of rates. Notice that in Chapter 3, the proposed iterative algorithm achieves the minimum total power

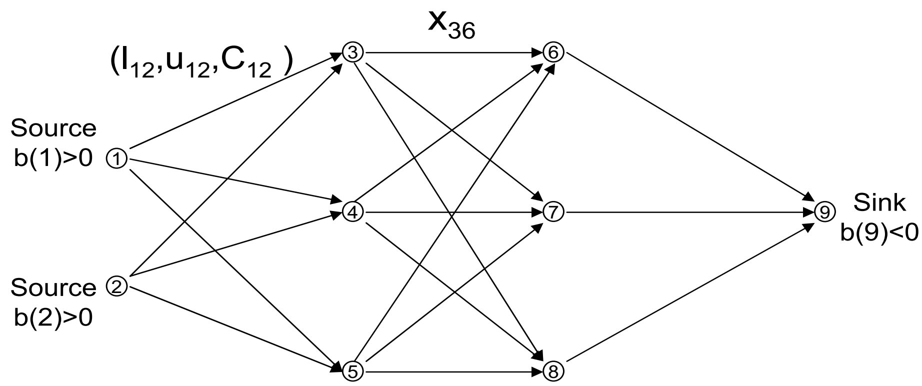

Figure 4.1: A Flow Network

solution. However, since the problem formulation does not include minimum rate constraints, it may result in an unfair assignment of resources to users. The use of bipartite matching and minimum cost network flows greatly helps us to account for minimum rate constraints.

We first define *bipartite matching problem* shortly, since it has the basic intuition behind using these models in resource allocation problems. Later, we will define *minimum cost flow problem* and use it throughout the chapter.

4.1.1 Bipartite Matching and Minimum Cost Network Flows

A *directed graph* $G = (M, A)$ consists of a set M of nodes and a set A of arcs whose elements are ordered pairs of distinct nodes [5]. A *directed network* is a directed graph whose nodes and arcs have associated numerical values such as costs, capacities and supplies/demands. Figure 4.1 shows a sample directed network. In the figure, (l_{ij}, u_{ij}, C_{ij}) denotes the lower bound on the arc flow, the capacity and the cost per unit flow associated with arc $(i, j) \in A$ respectively, x_{ij} denotes the amount of flow on arc (i, j), and $b(i)$ denotes ith node's demand or supply depending on whether $b(i) > 0$ or $b(i) < 0$.

A *matching* in a graph $G = (M, A)$ is a set of arcs with the property that every

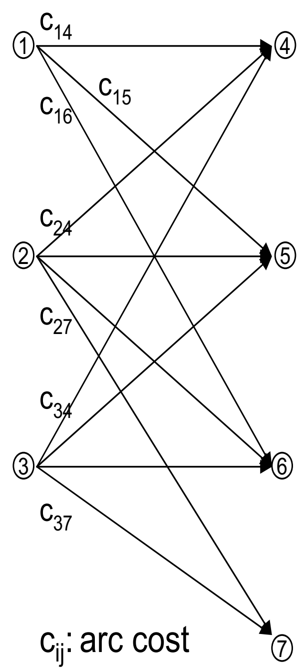

Figure 4.2: A Bipartite Network

node is incident to at most one arc in the set so that the matching results in a pairing of the nodes in the graph using the arcs in A [5]. Matching problems on graphs with two sets of nodes and with arcs that join only nodes between the two sets are called *bipartite matching problems*. Figure 4.2 shows a sample bipartite graph and possible matchings between the two sets of nodes connected by some arcs. Here we are interested in *weighted matching problems* in which there is a *weight* associated with each arc, and the objective is to minimize or maximize the total weight of the matching.

In our problem formulation, we will use the first set of nodes (or objects) on a bipartite graph to represent network resources such as spreading codes, OFDM tones, time slots etc., and the second set of nodes will represent the mobiles. Our target is to find the matching between these two sets that has the minimum total weight (cost or reward).

Although bipartite matching problems underlie the basic idea of resource allocation problems, we will not go into further details of bipartite matchings in this chapter. Instead, we will use the closely related *minimum cost network flows* formulation. There are two reasons to use minimum cost flow model instead of bipartite matchings. First, any bipartite matching problem can be expressed as a minimum cost network flow problem; see [5, 30] for details. In other words, any algorithm that solves minimum cost flow problems solves the bipartite matching problem as well. Second, as we will show in the following sections that it is very easy to define minimum rate constraints in a flow network by assigning lower bounds on the amount of flows on appropriate arcs in the network. The inherent structure of bipartite matching does not have this property, because any node in a set can at most be connected to a single node in the other set. We refer to [5, 30] for further details on the relationship between these two combinatorial models.

The definition of minimum cost flow problem is as follows [5, 30]. Let $G = (M, A)$ be a directed network with a cost C_{ij}, capacity u_{ij}, a lowerbound on the arc flow l_{ij} and flow x_{ij} associated with every arc $(i, j) \in A$. Associated with each node $i \in M$, a number $b(i)$ indicates ith node's demand or supply depending on whether $b(i) > 0$ or $b(i) < 0$. In this case the minimum cost flow problem is [5, 30]:

$$\begin{aligned} \min \quad & \sum_{(i,j)\in A} C_{ij} x_{ij} && (4.1) \\ s.t. \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i) \text{ for all } i \in M \\ & l_{ij} \leq x_{ij} \leq u_{ij} \qquad \text{for all } (i,j) \in A \end{aligned}$$

Thus, we are searching for an optimal flow through the network $G = (M, A)$ with minimum cost while each flow x_{ij} meets lower and upper bound constraints. The following important theorem states that it is possible to achieve integer optimal flows if the bounds on each arc flow and the net flow through each node in the network are integers.

Theorem 1 (Integrality Property, [5]): If all arc capacities and supplies/demands of nodes are integer, the minimum cost flow problem always has an integer minimum cost flow.

In the following section, we will describe how we adapt the minimum cost network flow model on a wireless network.

4.2 System Model and Problem Statement

Figure 4.3 shows how we construct a flow network corresponding to a wireless system. The network consists of $N + K + 2$ nodes and $NK + N + K$ arcs where N denotes the number of available waveforms, K denotes the number of mobiles and 2 additional nodes are the source and the sink nodes. The source node has a supply of N units flow and the sink node has a demand of N units flow; all other nodes have zero net flow.

For each of N arcs connecting the source node to N waveforms, we assign $(0, 1, 0)$. Since the supply of the source node is N unit flow, all of these arcs (and each waveform accordingly) will be included in the minimum cost flow solution with no cost. On the other hand, for each of K arcs connecting the sink node to K users, we assign $(m_k, M_k, 0)$ so that each user k will be connected to at least m_k and at most M_k waveforms in the optimal solution. Note that mobile k requires at least m_k spreading codes to achieve its minimum service rate requirement $R_{k,min}$; i.e $m_k = \lceil R_{k,min}/R_0 \rceil$

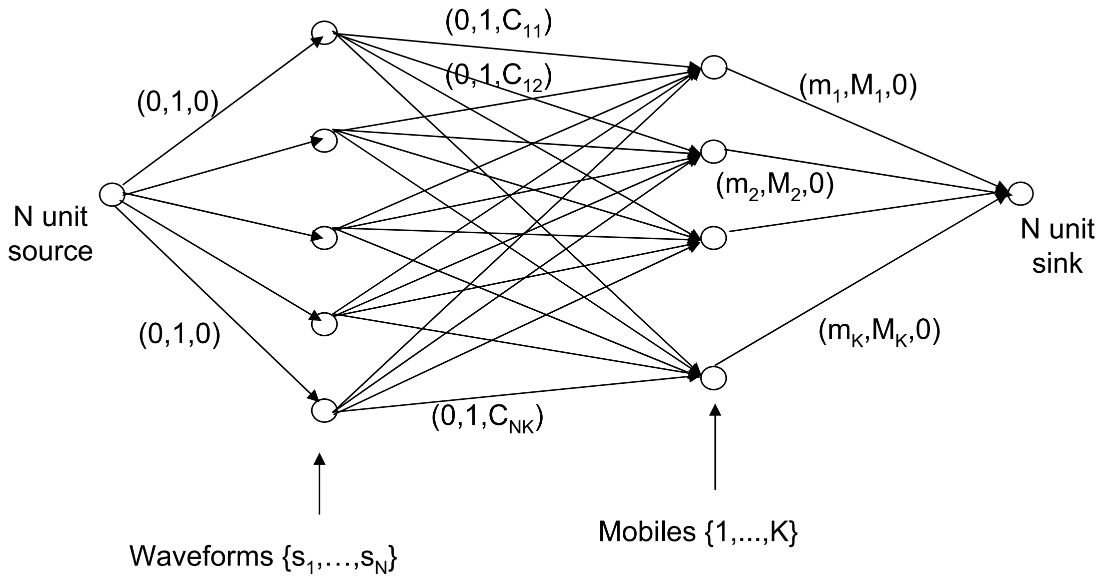

Figure 4.3: Flow network model for a Wireless system

where R_0 denotes the rate corresponding to a single spreading code. Finally, for each of NK arcs connecting waveform $i \in \{1, \dots, N\}$ to user $j \in \{1, \dots, K\}$, we assign $(0, 1, C_{ij})$. Remember that each optimal arc flow will be an integer due to *Theorem 1*. Moreover, since arc flow x_{ij} satisfies $0 \leq x_{ij} \leq 1$, the flow x_{ij} will be either 0 or 1 indicating whether s_i is assigned to user j. In this case if there is a flow on arc (i, j), the waveform $\mathbf{s_i}$ will be assigned to user j.

What is left is to define an appropriate cost function C_{ij} for each of NK arcs connecting the waveforms to users. The cost (or reward) of assigning a given waveform $\mathbf{s}_i$ to mobile j through channel H_j can be defined in various ways. We list some of them here:

- C_{ij}: The transmit power p_{ij} required for user j to decode $\mathbf{s}_i$ reliably. Thus, the minimum cost flow solves the problem of assigning N waveforms to K users with minimum power, under constraints on the number of waveforms each user may receive.

- C_{ij}: The throughput achieved when the waveform $\mathbf{s}_i$ is received by mobile j, i.e. $\log(1 + \text{SINR}_{ij})$, for a given set of transmit power levels. In this case, the maximum cost flow solves throughput maximization problem under constraints on the number of waveforms each user may receive.

- C_{ij}: From a network level point of view, the cost function may depend on both the buffer size (or the queue length) of mobile j and the transmit power levels (or throughput), i.e. $C_{ij} = f(\text{buffersize}_j, p_{ij})$. In this case if mobile j requests too many packets, a penalty may be incurred by increasing the cost of each arc connected to mobile j. By this way, we may have a control over the network level QoS measures.

The minimum cost flow problems are well known combinatorial models and there are numerous algorithms proposed in literature [5, 30]. For the examples we present in the following two sections, we used the source code [43] which uses a network simplex algorithm. We examine an OFDM system in the next section, and we solve OFDM carrier assignment problem using minimum cost flow technique. In Section 4.4, we illustrate examples in CDMA networks.

4.3 Application in an OFDM System

We consider an OFDM system with $N = 16$ subcarriers. There are $K = 8$ mobiles in the cell, each with a different application running, and each with a specific QoS target in terms of minimum service rate, or equivalently in terms of minimum number of subcarriers requested. We assume that mobiles 1, 2 and 3 require at least one OFDM subcarrier, mobiles 4, 5, 6 require at least two subcarriers, mobile 7 and 8 are delay tolerant and may or may not receive any subcarriers. In this case we set $m_k = 1$ for $k \in \{1, \ldots, 3\}$, $m_k = 2$ for $k \in \{4, \ldots, 6\}$ and $m_k = 0$ for $k \in \{7, 8\}$. We set $M_k = 16$ for $k \in \{1, \ldots, 16\}$. Our target is to minimize the total power required to transmit N OFDM tones to K mobiles while providing individual QoS targets.

The channel between each mobile and the base station is modeled as a frequency selective multipath channel. Therefore a mobile experiences different channel fades on different subcarriers. We assume that the channels are flat over each subcarrier. For comparison purposes, the channel impulse response $h_j(t)$ and the location of each mobile is chosen to be the same as in the example in Section 3.4. We refer to Figure 3.5 and Figure 3.6 for frequency domain channel responses.

An OFDM system can be thought of as a set of N independent Gaussian channels, one for each OFDM tone; see [44] for detailed analysis and derivation. Thus at each mobile $j \in \{1, \ldots, K\}$, transmissions using the ith OFDM tone s_i yield

$$y_{ij} = h_{ij} x_i + n_{ij} \tag{4.2}$$

where y_{ij} denotes the received symbol, x_i denotes the transmitted symbol, h_{ij} denotes the channel gain of mobile j over the ith OFDM tone and n_{ij} denotes the Gaussian noise sample. Each channel gain h_{ij} is obtained by N point DFT of jth mobile's channel response $h_j(t)$, i.e. $[h_{1j}, \ldots, h_{Nj}]^T = \mathrm{DFT}_N(h_j(t))$. There is a target SNR γ_{ij} that needs to be achieved by mobile j to decode the symbol x_{ij} reliably, and the transmit power to achieve γ_{ij} is denoted by p_{ij}.

We can express p_{ij} in terms of γ_{ij}, σ^2 and h_{ij}:

$$p_{ij} = \frac{\gamma_{ij}\sigma^2}{|h_{ij}|^2} \tag{4.3}$$

In this case we choose the cost function to be $C_{ij} = p_{ij}$. In the experiment, the SNR target is 5 (7 dB) and $\sigma^2 = 10^{-13}$ W. The optimal solution is as follows. Mobile 1 receives one carrier corresponding to 1th OFDM tone, i.e. s_1, mobile 2 receives one carrier corresponding to 16th OFDM tone, mobile 3 receives one carrier corresponding to 9th OFDM tone, mobile 4 receives two carriers corresponding to 8th and 10th OFDM tones, mobile 5 receives two carriers corresponding to 7th and 13th OFDM tones, mobile 6 receives two carriers corresponding to 5th and 11th OFDM tones, mobile 7 does not receive any OFDM tones, and mobile 8 receives seven subcarriers corresponding to 2nd, 3th, 4th, 6th, 12th, 14th and 15th OFDM tones. The total power required for the optimal set of assignments is 4.44 Watts.

In case all mobiles are delay tolerant, i.e. $m_k = 0$ for $k = \{1, \ldots, K\}$, mobile 4 receives one carrier corresponding to 9th OFDM tone, and mobile 8 receives the rest of the OFDM tones. None of the other mobiles are served in this case. Observe in Figure 3.5 and Figure 3.6 that each OFDM carrier is assigned to the mobile with the largest gain over that subchannel in this case. The total power required for this set of assignments is 0.45 Watts.

To compare the above numerical results with the CDMA example in Section 3.4, notice that in case all mobiles are delay tolerant and minimum number of carrier requirements are all zero, a system using OFDM type multiaccess scheme requires more power than a system using a CDMA type multiaccess scheme ($\approx$ 0.45 Watts for OFDM and

≈ 0.03 Watts for CDMA). This result is counterintuitive since we might expect OFDM type orthogonal transmission schemes to be more efficient than a CDMA scheme with multiaccess interference in the context of power minimization. The reason for larger transmit power in the case of an OFDM system is the requirement of the same SNR on each subchannel. In this case, the transmitter allocates nonzero power even on subchannels with deep fades. In particular, this policy contradicts with the information theoretic optimal water-filling policy where the channels with very deep fades are likely to be allocated zero power [8].

4.4 Application in a CDMA System

Under a frequency selective multipath channel assumption, the problem of minimizing total transmit power on the downlink of a multiaccess system with multirate services has been studied in Chapter 3. In the case of a CDMA system, the proposed iterative algorithm (3.11) converges to the optimal set of spreading code assignments (orthogonal Walsh codes) to mobiles and to the set of optimal transmit power levels. On the other hand, the problem definition in Chapter 3 does not include minimum QoS targets in terms of number of spreading codes requested for each mobile. Thus, not all users are guaranteed to be served, and the optimal algorithm (3.11) may result in an unfair assignment of resources. In this section, we aim to allocate each user at least a number of codes corresponding to its minimum QoS target while minimizing total transmit power in a multipath channel on the downlink of a CDMA system.

We apply the same flow network model shown in Figure 4.3 for a CDMA system. The cost C_{ij} of an arc (i, j) connecting a spreading code $\mathbf{s}_i$ to a mobile j is defined in terms of the transmit power p_i required for mobile j to receive ith spreading code with acceptable quality, i.e. achieving target SINR γ at the output of the receiver filter $\mathbf{f}_{ij}$ (instead of using conventional notation $\mathbf{c}_{ij}$ for the receiver filter, we use $\mathbf{f}_{ij}$ for the **f**ilter in order to avoid confusion with cost C_{ij}). The SINR γ_{ij} achieved at the output of the receiver filter $\mathbf{f}_{ij}$ is

$$\gamma_{ij} = \frac{p_i(\mathbf{f}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2}{\sum_{v \neq i} p_v(\mathbf{f}_{ij}^T \mathbf{H}_j \mathbf{s}_v)^2 + \sigma^2(\mathbf{f}_{ij}^T \mathbf{f}_{ij})} \tag{4.4}$$

where $\mathbf{H}_j$ denotes the channel matrix of mobile j. In this case, the cost of an arc is

$$C_{ij} = p_i|_{\gamma_{ij}=\gamma} = \frac{\gamma(\sum_{v \neq i} p_v(\mathbf{f}_{ij}^T \mathbf{H}_j \mathbf{s}_v)^2 + \sigma^2(\mathbf{f}_{ij}^T \mathbf{f}_{ij}))}{(\mathbf{f}_{ij}^T \mathbf{H}_j \mathbf{s}_i)^2} \tag{4.5}$$

Among all receiver filters, the MMSE filter minimizes (4.5) since it maximizes the inverse of the cost function, i.e. SINR, see Section 3.3 for a similar analysis. Therefore $\mathbf{f}_{ij}$ can be replaced by $\mathbf{f}_{ij}^{MMSE}$ in (4.5). Notice that the cost structure (4.5) in a CDMA system is fundamentally different than the cost structure (4.3) in an OFDM system. While OFDM arc costs (4.3) are not dependent on each others, which is a result of orthogonal transmission and reception, CDMA arc costs are dependent on each others due to multiaccess interference. Notice that C_{ij} (4.5) is a function of p_v which is the cost of another arc.

The following two sections present two approaches to resolve this issue.

4.4.1 CDMA Flow Network Model with Linear Receiver Processing

The fact that the flow network model applies well in case of an OFDM system inspires us to orthogonalize CDMA channels in a way to get rid of mutual dependence of arc costs. To this end, we combine the network flow approach with linear receiver processing in a

CDMA system.

For the sake of clarity, we revisit the transmitted signal vector $\mathbf{x}$, and the received signal vector $\mathbf{r}_j$ by mobile j below. The definitions of each variable used in the expressions are given in detail in Section 3.2.

$$\mathbf{x} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \sqrt{\mathrm{p_i}}\mathrm{b_i}\mathbf{s}_{\mathrm{i}} \tag{4.6}$$

$$\mathbf{r}_j = \sum_{i=1}^{N} \sqrt{p_i} b_i \mathbf{H}_j \mathbf{s}_i + \mathbf{n}_j \tag{4.7}$$

Linear receiver processing multiplies the received signal (4.7) by the inverse of the channel matrix $\mathbf{H}_j$. We assume that each mobile knows its own channel. The processed received signal is denoted by $\tilde{\mathbf{r}}_j$

$$\begin{aligned}\tilde{\mathbf{r}}_j = \mathbf{H}_j^{-1}\mathbf{r}_j &= \mathbf{H}_j^{-1}(\sum_{i=1}^{N} \sqrt{p_i} b_i \mathbf{H}_j \mathbf{s}_i + \mathbf{n}_j) \\ &= \sum_{i=1}^{N} \sqrt{p_i} b_i \mathbf{s}_i + \tilde{\mathbf{n}}_j \end{aligned} \tag{4.8}$$

where $\tilde{\mathbf{n}}_j = \mathbf{H}_j^{-1}\mathbf{n}_j$. By linear receiver processing, we orthogonalize received CDMA waveforms at the expense of coloring the white receiver noise. This operation is reasonable on the downlink of the system since all transmitted waveforms, both the desired one and the interferer waveforms, get distorted by the same channel on the downlink. Thus knowledge of mobile j's channel $\mathbf{H}_j$ is good enough to restore the orthogonality of the transmitted waveforms at mobile j's receiver. Notice that this operation is similar to zero forcing equalization in ISI channels.

The receiver filter $\mathbf{f}_{ij}$, following receiver processing, matches to the spreading code

$\mathbf{s}_i$. In this case the SINR achieved at the output of the receiver filter is

$$\gamma_{ij} = \frac{p_i}{\sigma^2 |\mathbf{s}_i^T \mathbf{H}_j^{-1}|^2} \tag{4.9}$$

Note that while the colored multiaccess interference (before receiver processing) is a function of power levels of interferer spreading codes and correlation between received waveforms, see (4.4), colored receiver noise (after receiver processing) is independent of interferer power levels, and it is only a function of the desired waveform $\mathbf{s}_i$, the channel matrix $\mathbf{H}_j$ and the background noise level σ^2.

The arc cost C_{ij} is defined in terms of the transmit power p_i required for mobile j to achieve target SINR γ at the output of the receiver filter $\mathbf{f}_{ij}$. Thus

$$C_{ij} = p_i|_{\gamma_{ij}=\gamma} = \gamma\sigma^2 |\mathbf{s}_i^T \mathbf{H}_j^{-1}|^2 \tag{4.10}$$

In this case, the cost C_{ij} of an arc (i, j) is determined by the nodes connected to that arc, i.e. $\mathbf{s}_i$ and the channel of mobile j, and therefore costs on different arcs on the flow network are no longer mutually dependent. With this cost definition, the problem is an ordinary well-known minimum cost flow problem.

4.4.2 Iterative Algorithm on a Flow Network

One idea is to start from an initial transmit power vector, solve minimum cost flow problem to determine the set of arc costs in the optimal flow, update the power vector with the selected arc costs and continue iteratively until convergence, assuming the algorithm converges.

Notice that the expression (4.5) of an arc cost is equivalent to the interference function (3.8) defined in Section 3.3 in the context of standard power control algorithms. So the question is whether an iterative algorithm of the form $\mathbf{p}(\mathbf{n}+\mathbf{1}) = \mathbf{C}(\mathbf{p}(\mathbf{n}))$ on a flow network converges to the minimum total power solution. Here $\mathbf{p} = [p_1, \ldots, p_N]$ is the set of transmit power levels, $\mathbf{C} = [C_{1k(1)}, \ldots, C_{Nk(N)}]$ is the set of costs corresponding to the arcs selected by the minimum cost flow algorithm, and the spreading code $\mathbf{s}_i$ is assigned to the mobile $k(i)$ by the flow algorithm.

In case minimum number of spreading code requirements are all zero, i.e. $m_k = 0$ for $k = \{1, \ldots, K\}$, the iterative algorithm $\mathbf{p}(\mathbf{n}+\mathbf{1}) = \mathbf{C}(\mathbf{p}(\mathbf{n}))$ on a flow network converges to the minimum total power solution. To prove this statement, notice that in this case an arc (i, j) is a part of minimum cost flow only if the mobile j is the one requiring the smallest cost to receive $\mathbf{s}_i$ among all mobiles. Since arc costs are identical to interference functions (3.8), $\mathbf{p}(\mathbf{n}+\mathbf{1}) = \mathbf{C}(\mathbf{p}(\mathbf{n}))$ is equivalent to $\mathbf{p}(\mathbf{n}+\mathbf{1}) = \mathbf{I}(\mathbf{p}(\mathbf{n}))$ whose convergence is proven in Chapter 3.

On the other hand, in order for the iterative algorithm to converge to the optimal solution for any instance of the flow network (i.e. for any set of minimum rate requirements), $\mathbf{p}(\mathbf{n}+\mathbf{1}) = \mathbf{C}(\mathbf{p}(\mathbf{n}))$ has to satisfy *Positivity*, *Monotonicity* and *Scalability* properties; see [4] for details. We note that intuitively we expect the performance of the iterative minimum cost flow algorithm to be comparable to, or the same as, the optimal, since each iteration of the minimum cost flow results in the minimum sum of transmit powers based on the updates of the previous iteration, i.e. the algorithm seems to take the right "local" steps. The question is whether these local steps lead to a global optimum solution.

It is hard to prove the *Monotonicity* and *Scalability* properties analytically, since

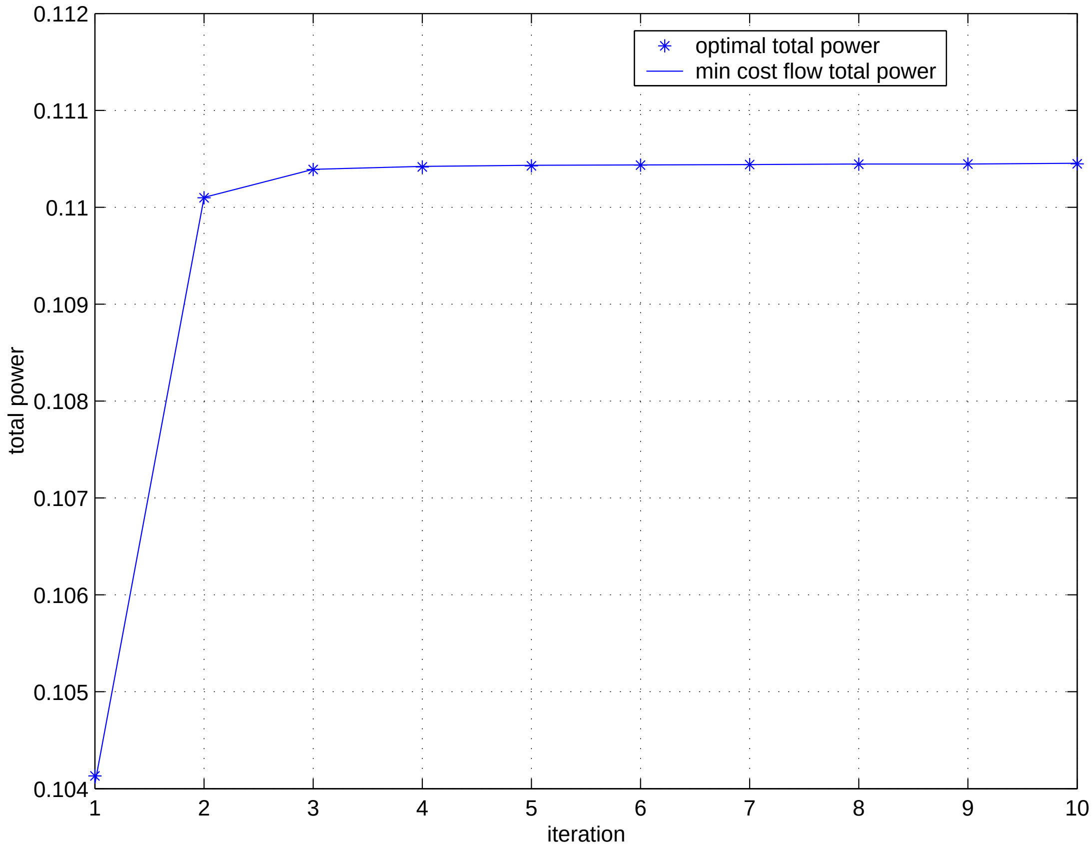

Figure 4.4: Total Power Convergence

they require complicated sensitivity analysis as a function the arc costs, which is hard even for networks with simple cost structures. Instead, we verify the superiority of iterative minimum cost flow algorithm through simulations. In Figure 4.4, we present total power results by the iterative minimum cost flow algorithm and the optimal algorithm. The experiment is designed for a simple CDMA network with processing gain of $N = 8$ (a Walsh set of 8 orthogonal spreading codes). Moreover there are 4 users, 2 of which require at least one spreading code as a QoS measure. The target SINR is 5 (7 dB), and the noise power is 10^{-13} Watts. The average power results by both algorithms are calculated by conducting 1000 successive experiments; in each of them, a different set of user positions and channels are generated. The results in the figure show that the iterative minimum cost flow algorithm achieves the minimum total power solution under strict minimum rate constraints.

4.5 Chapter Summary and Conclusion

This chapter illustrates applications of combinatorial network flow models in wireless resource management problems. The communication through a wireless network is modeled as a network flow, and the target is to minimize the cost of information flow through the network under constraints on demands (minimum rate) of mobiles and supply (bandwidth) of the base station. We present examples in OFDM and CDMA wireless networks to illustrate the use of the minimum cost flow model.

Chapter 5

Conclusion and Future Work

In this thesis, we investigate radio resource management for downlink wireless systems. In the first part of the thesis, we examine how to maximize the network throughput on the downlink of a multirate CDMA wireless network under constraints on total transmit power and minimum QoS (rate) requirement of each user. The optimal algorithm is determined as follows. First, users are ordered based on their transmit energy per bit requirements to achieve the target received energy per bit to interference power spectral density ratio at the receivers. Based on the initial ordering, we prove that for systems employing multiple codes, the greedy rate scheduling is optimal, and therefore it yields maximum network throughput. For systems employing OVSF codes, the greedy rate scheduling is optimal if the minimum rate requirement of a user is larger than or equal to the minimum rate requirement of any other user with a larger transmit energy per bit requirement. Simulation results show that the greedy algorithm, even when it is suboptimal, is a good heuristic yielding average throughput which is very close to the optimal achievable throughput in OVSF-CDMA systems. The simplicity and polynomial time complexity of the greedy algorithms seem to be very attractive from an implementation point of view.

In the second part of the thesis, we investigate joint power control and orthogonal code selection (rate control) in frequency selective multipath channels. We show that the standard power control framework can be extended to include rate control as well. Using this framework, we prove that a joint power and rate control algorithm converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient SINR.

Finally, we show how to apply combinatorial network flow models in wireless resource management problems. We model the communication through a wireless network as a network flow, and we minimize the cost of information flow through the network under constraints on demands (minimum rate) of mobiles and supply (bandwidth) of the base station. This model helps us to improve fairness by enforcing minimum rate constraints on each mobile, and to deal with practical discrete system constraints. We first apply minimum cost network flow idea in the case of an OFDM system where the channel is flat on each subcarrier and frequency selective across the subcarriers. We then combine the minimum cost network flow technique with linear receiver processing. Finally, we propose an iterative minimum cost flow algorithm for CDMA systems.

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