Cooperative Broadcast for Maximum Network Lifetime

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***Abstract*— We consider cooperative data broadcast in a wireless network with the objective to maximize the network lifetime. To increase the energy-efficiency, we allow the nodes that are out of the transmission range of a transmitter to collect the energy of unreliably received overheard signals. As a message is forwarded through the network, nodes will have multiple opportunities to reliably receive the message by collecting energy during each retransmission. We refer to this strategy as *cooperative (accumulative) broadcast*.**

We present the *Maximum Lifetime Accumulative Broadcast (MLAB)* algorithm that specifies the nodes' order of transmission and transmit power levels. We prove that the solution found by MLAB algorithm is optimal but not necessarily unique. The power levels found by the algorithm ensure that the lifetimes of the relays are the same, causing them to fail simultaneously. Therefore, the algorithm performs optimum load balancing. For the same battery levels at all the nodes, the optimum transmit powers become the same.

Cooperative broadcast not only increases the energy-efficiency during the broadcast by allowing for more energy radiated in the network to be collected, but also makes optimum load balancing possible, by relaxing the constraint imposed by the conventional broadcast, that a relay has to transmit with power sufficient to reach its most disadvantaged child. Simulation results demonstrate that cooperative broadcast significantly increased network lifetime compared to conventional broadcast.

***Index Terms*— Cooperative broadcast, maximum network lifetime, optimum transmit powers**

I. Introduction

We consider the problem of energy-efficient broadcasting in a wireless network. Prior work on this subject has been focused on the minimum-energy broadcast problem with the objective of minimizing the total transmitted power in the network. This problem was shown in [1]–[3] to be NP-complete. Several heuristics for constructing energy-efficient broadcast trees have been proposed (see [1], [2], [4]–[6] and references therein).

However, broadcasting data through an energy-efficient tree drains the batteries at the nodes unevenly causing higher drain relays to fail first. The performance objective that addresses this issue is maximizing the network lifetime. The *network lifetime* is defined to be the duration of a data session until the first node battery is fully drained [7]. Finding a broadcast tree that maximizes network lifetime was considered in [8]–[10]. The similar problem of maximizing the network lifetime during a multicast was addressed in [11]. Because the energies of the nodes in a tree are drained unevenly, the optimal tree changes in time and therefore the authors [8]–[11] distinguished between the *static* and *dynamic* maximum lifetime problem. In a static problem, a single tree is used throughout the broadcast session whereas the dynamic problem allows a sequence of trees to be used. Since the latter approach balances the traffic more evenly among the nodes, it generally performs better. For the static problem, an algorithm was proposed that finds the optimum tree [8]. For the special case of identical initial battery energy at the nodes, the optimum tree was shown to be the minimum spanning tree. In a dynamic problem, a series of trees were used that were periodically updated [9] or used with assigned duty cycles [10].

Wireless formulations of the above broadcast problems assume that a node can benefit from a certain transmission only if the received power is above a threshold required for reliable communication. This is a pessimistic assumption. A node for which the received power is below the required threshold, but above the receiver noise floor, can collect energy from the unreliable reception of the transmitted information.

Moreover, it was observed in the relay channel [12] that utilizing unreliable overheard information was essential to achieving capacity. This idea is particularly suited for the broadcast problem, where a node has multiple opportunities to receive a message as the message is forwarded through the network. We borrow this idea and re-examine the broadcast problem under the assumption that nodes accumulate the energy of unreliable receptions. We refer to this particular cooperative strategy as *accumulative broadcast* [13]. For fading channels, the cooperation between the nodes offers the additional benefit as a form of diversity [14]–[16]. In this paper, we address the problem of maximizing the network lifetime by employing the accumulative broadcast. As in the conventional broadcast problem, we impose a *reliable forwarding* constraint that a node can forward a message only after reliably decoding that message.

We show that the maximum lifetime broadcast problem has a simple optimal solution and propose *Maximum Lifetime Accumulative Broadcast (MLAB)* algorithm that finds it. The solution specifies the order of transmissions and transmit power levels at the nodes. The power levels given by the solution ensure that the lifetimes of relay nodes are the same and thus, their batteries die simultaneously. Therefore, the MLAB algorithm performs optimal load balancing, there is no need for dynamic updates of the solution and the algorithm solves both static and dynamic problem at the same time. As shown later, this is due to the accumulative broadcast that naturally allows for load balancing. Moreover, the simplicity

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of the solution allows us to formulate a distributed MLAB algorithm that uses local information at the nodes and is thus better suited for networks with large number of nodes [17].

The paper is organized as follows. In the next section, we give the network model and in Section III, we formulate the problem. In Section IV, we present MLAB algorithm that finds the optimal solution. In Section V, we show the benefit of accumulative broadcast to the network lifetime compared to the conventional broadcast through simulation results. The proofs of the theorems are given in the Appendix.

II. System model

We consider a static wireless network of N nodes such that from each transmitting node k to each receiving node m, there exists an AWGN channel of bandwidth W characterized by a frequency non-selective link gain h_{mk}. In our analysis, we do not consider fading and thus each channel is time-invariant with a constant link gain representing the signal path loss. We further assume large enough bandwidth resources to enable each transmission to occur in an orthogonal channel, thus causing no interference to other transmissions. Each node has both transmitter and receiver capable of operating over all channels.

A receiver node j is said to be in the transmission range of transmitter i if the received power at j is above a threshold that ensures the capacity of the channel from i to j is above the code rate of node i. We assume that each node can use different power levels, which will determine its transmission range. The nodes beyond the transmission range will receive an unreliable copy of a transmitted signal. Those nodes can exploit the fact that a message is sent through multiple hops on its way to all the nodes. Repeated transmissions act as a repetition code for all nodes beyond the transmission range.

After a certain message has been transmitted from a source, labeled node 1, sequence of retransmissions at appropriate power levels will ensure that eventually every node has reliably decoded the broadcast message. Henceforth, we focus on the broadcast of a single message and say that a node is *reliable* once it has reliably decoded the broadcast message. Under the reliable forwarding constraint, a node is permitted to retransmit (forward) only after reliably decoding the message.

The constraint of reliable forwarding imposes an ordering on the network nodes. In particular, a node m will decode a message from the transmissions of a specific set of transmitting nodes that became reliable prior to node m. Starting with node 1, the source, as the first reliable node, a solution to the cooperative broadcast problem will be characterized by a *reliability schedule*, which specifies the order in which the nodes become reliable.

A reliability schedule $[n_1, n_2, n_2, \ldots, n_N]$ is simply a permutation of $[1, 2, \ldots, N]$ that always starts with the source node $n_1 = 1$. Given a reliability schedule, it will be convenient for the following discussion to relabel the nodes such that the schedule is simply $[1, 2, \ldots, N]$. After each node $k \in \{1, \ldots, m-1\}$ transmits with average power p_k, the maximal number of bits per second that can be achieved at node m is [18]

$$r_m = W \log_2 \left(1 + \frac{\sum_{k=1}^{m-1} h_{mk} p_k}{N_0 W} \right) \qquad \text{bits/s}, \qquad (1)$$

where N_0 is the one-sided power spectral density of the noise.

Let the required data rate for broadcasting $\overline{r}$ be given by

$$\overline{r} = W \log_2 \left(1 + \frac{\overline{P}}{N_0 W} \right) \qquad \text{bits/s}. \qquad (2)$$

From (1) and (2), achieving $r_m = \overline{r}$ implies that the total received power at node m has to be above the threshold $\overline{P}$, that is,

$$\sum_{k=1}^{m-1} h_{mk} p_k \geq \overline{P}. \qquad (3)$$

After the data has been successfully broadcasted, all the nodes are reliable and feasibility constraint (3) is satisfied at every node m. When communicating at rate $\overline{r}$, the required signal energy per bit is $E_b = \overline{P}/\overline{r}$ Joules/bit. This energy can be collected at a node m during one transmission interval $[0, T]$ from a transmission of a single node k with power $p_k = \overline{P}/h_{mk}$, as commonly assumed in broadcasting problem [1], [2], [4], [8]–[10] . However, using the accumulative strategy, the required energy E_b is collected from $m-1$ transmissions.

III. Approach

A *lifetime of a node* i transmitting with power p_i is given by $T_i(p_i) = e_i/p_i$ where e_i is initial battery energy at node i. The *network lifetime* is the time until the first node failure

$$T_{net}(\mathbf{p}) = \min_i T_i(p_i) \qquad (4)$$

where $\mathbf{p}$ is a vector of transmitted node powers. The problem is to maximize the network lifetime under the constraints that all nodes become reliable.

In the conventional broadcast problem, the broadcast tree uniquely determines the transmission levels; a relay that is the parent of a group of siblings in the broadcast tree transmits with the power needed to reliably reach the most disadvantaged sibling in the group. Hence, the arcs in the broadcast tree uniquely determine the power levels for each transmission.

In the accumulative broadcast, however, there is no a clear parent-child relationship between nodes because nodes collect energy from the transmissions of many nodes. Furthermore, the optimum solution may require that a relay transmits with a power level different from the level precisely needed to reach a group of nodes reliably; the nodes may collect the rest of the needed energy from the future transmissions of other nodes. In fact, the optimum solution often favors such situations because all nodes beyond the range of a certain transmission are collecting energy while they are unreliable; the more such nodes, the more efficiently the transmitted energy is being used.

The differences from the conventional broadcast problem dictate a new approach. The optimum solution must specify the reliability schedule as well as the transmit power levels used at each node. Given a schedule, we can formulate a linear program (LP) that will find the optimum solution for that schedule. Such a solution will identify those nodes that should transmit and their transmission power levels. A reliability schedule can be represented by a matrix $\mathbf{X}$ where

$$x_{ij} = \begin{cases} 1 & \text{node } i \text{ scheduled to transmit after node } j \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Each x_{ij} is an indicator that a node i collects energy from a transmission by node j. Note that $x_{ii} = 0$, for all i and $x_{ji} = 1 - x_{ij}$. Given a schedule $\mathbf{X}$, we define a gain matrix $\mathbf{H}(\mathbf{X})$ with element (i, j) given by $h_{ij}x_{ij}$. Then, we can define the problem of maximizing the network lifetime for schedule $\mathbf{X}$ in terms of the vector $\mathbf{p}$ of transmitted powers as

$$\min\ \max \frac{p_i}{e_i} \tag{6}$$

$$\text{subject to } \mathbf{H}(\mathbf{X})\mathbf{p} \geq \mathbf{1}\overline{P}, \tag{6a}$$

$$\mathbf{p} \geq \mathbf{0}. \tag{6b}$$

The inequality (6a) contains $N - 1$ constraints as in (3), requiring that the accumulated received power at all the nodes but the source is above the threshold $\overline{P}$. Alternatively, we can define the problem in terms of *normalized* node powers $\overline{p}_i = p_i e_1/e_i$ that account for different battery capacities at the nodes. The lifetime at every node i in terms of the normalized power is as if all the batteries were the same: $T_i = e_i/p_i = e_1/\overline{p}_i$. In terms of normalized node powers, Problem (6) can be defined as

$$\min\ \max \overline{p}_i \tag{7}$$

$$\text{subject to } \overline{\mathbf{H}}(\mathbf{X})\overline{\mathbf{p}} \geq \mathbf{1}\overline{P},$$

$$\overline{\mathbf{p}} \geq \mathbf{0}$$

where each column $\overline{\mathbf{h}}_i$ of the normalized gain matrix $\overline{\mathbf{H}}(\mathbf{X})$ is obtained from the corresponding column $\mathbf{h}_i$ of matrix $\mathbf{H}(\mathbf{X})$ as $\overline{\mathbf{h}}_\mathbf{i} = \mathbf{h}_i e_i/e_1$.

For any schedule $\mathbf{X}$, we can formulate Problem (7) as a LP in terms of transmit power levels $\overline{\mathbf{p}}$

$$\hat{p}^*(\mathbf{X}) = \min\ \hat{p} \tag{8}$$

$$\text{subject to } \overline{\mathbf{H}}(\mathbf{X})\overline{\mathbf{p}} \geq \mathbf{1}\overline{P}, \tag{8a}$$

$$\overline{\mathbf{p}} \leq \mathbf{1}\hat{p} \tag{8b}$$

$$\overline{\mathbf{p}} \geq \mathbf{0}. \tag{8c}$$

If $\hat{p} = \hat{p}^*(\mathbf{X})$, then there exists a power vector $\overline{\mathbf{p}}$ such that (8b) and (8c) are satisfied. It follows that for any $p > \hat{p}$, $\overline{\mathbf{p}} \leq \mathbf{1}p$. Thus, for any power $\hat{p} \geq \hat{p}^*(\mathbf{X})$, we say that power $\hat{p}$ is *feasible* for schedule $\mathbf{X}$. We let p^* denote the optimum power $p^* = \min_\mathbf{X} \hat{p}^*(\mathbf{X})$. Equation (8) is a formal statement of the problem from which finding the best schedule corresponding to p^* is not apparent. We will see that the power p^*, may, in fact, be the solution to (8) for a set of schedules, $\mathcal{X}^*$. Note that, because p^* is the optimum power, schedules in $\mathcal{X}^*$ are the only schedules for which power p^* is feasible.

Rather than identifying $\mathcal{X}^*$, we employ a simple procedure that for any power p, determines the schedules for which power p is feasible. In particular, to distribute a broadcast message, we let each node retransmit with power p *as soon as possible*, namely as soon as it becomes reliable. We refer to such a distribution as the *ASAP*(p) *distribution*. During the ASAP(p) distribution, the message will be resent in a sequence of retransmission stages from sets of nodes $Z_1(p), Z_2(p), \ldots$ with power p where in each stage i, a set Z_i that became reliable during stage $i-1$, transmits and makes Z_{i+1} reliable. Let $S_i(p)$ and $U_i(p)$ denote the reliable nodes and unreliable nodes at the start of stage i. Then, $Z_1(p) = 1$ and $S_i(p) = Z_1(p) \cup \ldots \cup Z_i(p)$. The set $Z_{i+1}(p)$ is given by

$$Z_{i+1}(p) = \{z \in U_i(p) : p \sum_{k \in S_i(p)} h_{zk} \geq \overline{P}\}. \tag{9}$$

Note that if power p is too small, the ASAP(p) distribution can *stall* at stage i with $S_{i+1}(p) = S_i(p)$ and $U_i(p) \neq \emptyset$. In this case, ASAP(p) fails to distribute the message to all nodes. When $U_i(p) = \emptyset$ at any i, the ASAP(p) distribution *terminates successfully*. We will say that ASAP(p) distribution is a *feasible broadcast* if it terminates successfully.

The partial node ordering, $Z_1(p), Z_2(p), \ldots$, specifies the sequence in which nodes became reliable during the ASAP(p) distribution. In particular, any schedule $\mathbf{X}$ that is consistent with this partial ordering is a feasible schedule for power p. Nodes that become reliable during the same stage of ASAP(p) can be scheduled in an arbitrary order among themselves since these nodes do not contribute to each other's received power. The following theorem verifies that in terms of maximizing the network lifetime it is sufficient to consider only schedules consistent with the ASAP(p) distribution.

Theorem 1: If $\tilde{p}$ is a feasible power for schedule $\tilde{\mathbf{X}}$, then the ASAP$(\tilde{p})$ distribution is a feasible broadcast.

In particular, Theorem 1 implies that for optimum power p^*, the ASAP(p^*) distribution is feasible.

We next present the *Maximum Lifetime Accumulative Broadcast (MLAB)* algorithm, that determines the optimum power p^*. Once the power p^* is given, broadcasting with ASAP(p^*) will maximize the network lifetime.

IV. MLAB ALGORITHM

We label node 1 as the source and 2 as its closest neighbor (more precisely, the node with the highest link gain to the source). The idea of the algorithm is the following. In order to broadcast information, node 1 has to make at least one node, its closest neighbor, reliable. Therefore, node 1 has to transmit with power $\overline{P}/h_{21}$. This determines the initial *candidate broadcast power* as $p = \overline{P}/h_{21}$. Once reliable, node 2 can transmit with the same power p without increasing the candidate power. If these two transmissions make a new set of nodes reliable, we can repeat the same procedure: we allow transmissions from new reliable nodes until no new nodes are made reliable and all reliable nodes have transmitted with power p. At this point, if all N nodes are reliable, we are done. Otherwise, at least one reliable node has to increase its transmit power by some power level Δ in order for the information to be broadcast. That, in turn, increases the candidate power p to $p+\Delta$ and therefore all reliable nodes can increase their power by Δ. In fact, the increase Δ is minimized if power $p+\Delta$ is sufficient to make one more unreliable node reliable. This procedure can then be repeated until all nodes are reliable.

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Initialize: p = P̄/h_21
Start: Set S_1(p) = {1}; U_1(p) = S^c
         apply the ASAP(p) distribution;
If ASAP(p) stalls at stage η(p):
    for all j ∈ U_η(p)(p) calculate:
        Δ_j = P̄/∑_{k∈S_η(p)(p)} h_jk − p;
    Set: Δ* = min_{j∈U_η(p)(p)} Δ_j; p ← p + Δ*;
    go to Start;
end
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The cardinality of S is given by $|S|$. S^c denotes the complement.

Fig. 1. MLAB algorithm.

Thus, in the MLAB algorithm we find the optimum power p^* through a series of ASAP(p) distributions, starting with the smallest possible candidate power, $p = \overline{P}/h_{21}$. If the ASAP(p) distribution stalls at some stage $\eta(p)$, we determine the minimum power increase Δ^* for which ASAP$(p+\Delta^*)$ will not stall at stage $\eta(p)$, in the following way. The increase in candidate broadcast power Δ_j needed to make a node $j \in U_{\eta(p)}(p)$ reliable must satisfy

$$\overline{P} = (p+\Delta_j)\sum_{k\in S_{\eta(p)}(p)} h_{jk}. \qquad (10)$$

We choose $\Delta^* = \min_{j\in U_{\eta(p)}(p)} \Delta_j$. Because the ASAP$(p)$ distribution has stalled, we increase p to $p+\Delta^*$ and *restart* the MLAB algorithm.

The pseudocode of the algorithm is given in Figure 1. The MLAB algorithm ends after $L \leq N-1$ restarts. There exists a set of feasible schedules that are consistent with the partial ordering given by the ASAP(p) distribution. The normalized transmit power at all nodes in $S_L(p)$ is p. Note that the last transmitting set $Z_L(p)$ could in fact, transmit with power less than p if it is enough for the last unreliable set $U_L(p)$ to become reliable. Thus, choosing the power level at all nodes to be p is not necessarily a unique solution. While this won't change the network lifetime, the latter solution will reduce the total broadcast power in the network. Next we show that the power found by MLAB is in fact optimum power, that is, $p = p^*$.

Theorem 2: The MLAB algorithm finds the optimum power p^* such that the ASAP(p^*) distribution maximizes the network lifetime.

The full restarts of the MLAB algorithm are used primarily to simplify the proof of Theorem 2. In fact, when MLAB stalls, it is sufficient for the reliable nodes to offer incremental retransmissions at power Δ^*. This observation will be the basis of distributed algorithms proposed in [17].

V. Performance

We now evaluate the benefit of accumulative broadcast to the network lifetime and compare it to the conventional network broadcast that discards overheard data in a network. In particular, networks with randomly positioned nodes in a 10×10 square region were generated. The transmitted power was attenuated with distance d as d^α for different values of propagation exponent $\alpha = 2, 3, 4$. The received power threshold was chosen to be $\overline{P} = 1$. Results were based on the performance of 100 randomly chosen networks.

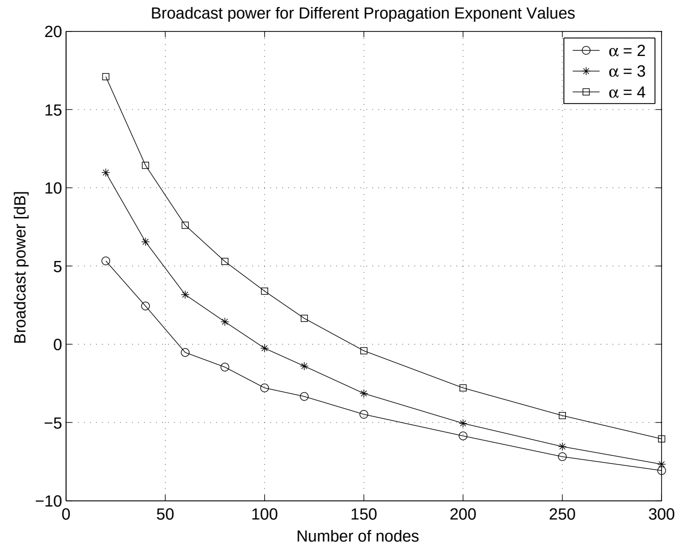

Fig. 2. Broadcast power for different propagation exponent values.

Figure 2 shows the broadcast power p for different values of propagation exponent in networks with different node densities. The observed power decrease is due to shorter hops between nodes in denser networks. For equal battery capacities at the nodes, the corresponding network lifetime is shown in Figure 3.

Figures 4 and 5 show the benefit of accumulative broadcast as compared to conventional broadcast to the network lifetime. For conventional broadcast, the authors in [8], [9] proposed two algorithms, MSNL and MST, that maximize the static network lifetime as well WMSTSW, a greedy algorithm that increases the dynamic lifetime. We compare the performance of these algorithms for three different battery energy distribution as given in [8], [9], to the network lifetime found by the MLAB algorithm. Several other algorithms to increase the dynamic network lifetime were evaluated in [9] with similar performance to WMSTSW. As expected, we see that solution found by MLAB considerably increases network lifetime. Typically, MLAB increased the network lifetime by a factor of 2 or more. The reason is twofold: first, because the broadcast uses the energy of overheard information enabling for more radiated energy to be captured and second, because MLAB finds the optimum solution whereas the solutions given in [8], [9] are generally suboptimal even for conventional broadcast.

VI. Appendix

Proof: Theorem 1

Given a schedule $\tilde{\mathbf{X}}$, it will be convenient to relabel the nodes such that $\overline{\mathbf{H}}(\tilde{\mathbf{X}})$ is lower triangular. Schedule $\tilde{\mathbf{X}}$ is then given by $[1, 2, \ldots N]$. The proof is by induction on k, where k is the index to a sequence of stages during the ASAP$(\tilde{p})$ distribution. We show that at the start of stage k, nodes $\{1, \ldots, k\} \subset S_k(\tilde{p})$.

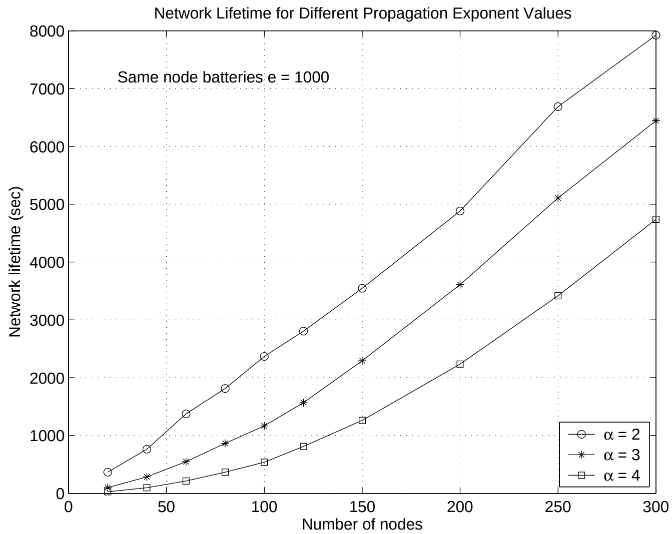

Fig. 3. Network lifetime for different propagation exponent values.

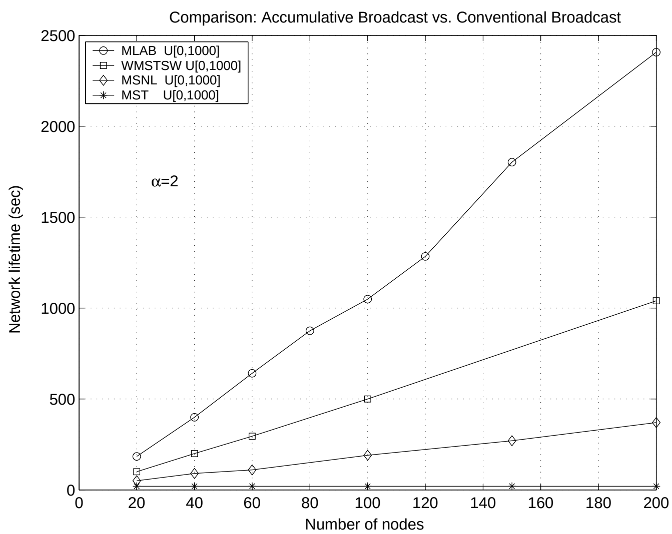

Fig. 4. Network lifetime of accumulative broadcast and conventional broadcast.

This will guarantee that node $k+1$ becomes reliable in stage k since, by schedule $\tilde{\mathbf{X}}$, node $k+1$ is made reliable by nodes $\{1,\ldots,k\}$.

Case $k=1$ is obvious since $S_1(\tilde{p}) = \{1\}$ for any $\tilde{p}$. Next assume that $\{1,\ldots,k\} \subset S_k(\tilde{p})$. This implies

$$\tilde{p} \sum_{j \in S_k(\tilde{p})} h_{k+1,j} \geq \tilde{p} \sum_{j \in \{1,\ldots,k\}} h_{k+1,j} \geq^{(a)} \overline{P} \tag{11}$$

where (a) follows from the feasibility of power $\tilde{p}$ for schedule $\tilde{\mathbf{X}}$. We conclude that $k+1 \in S_{k+1}(\tilde{p})$ and since $\{1,\ldots,k\} \subset S_k(\tilde{p}) \subset S_{k+1}(\tilde{p})$, it follows that $\{1,\ldots,k+1\} \subset S_{k+1}(\tilde{p})$, for any $k < N$. Thus, $\{1,\ldots,N\} \subset S_N(\tilde{p})$, implying the ASAP$(\tilde{p})$ distribution makes all nodes reliable. □

Proof: Theorem 2

Under power p, consider the set $S_i(p)$ of reliable nodes at the start of stage i of the ASAP(p) distribution. Node j belongs

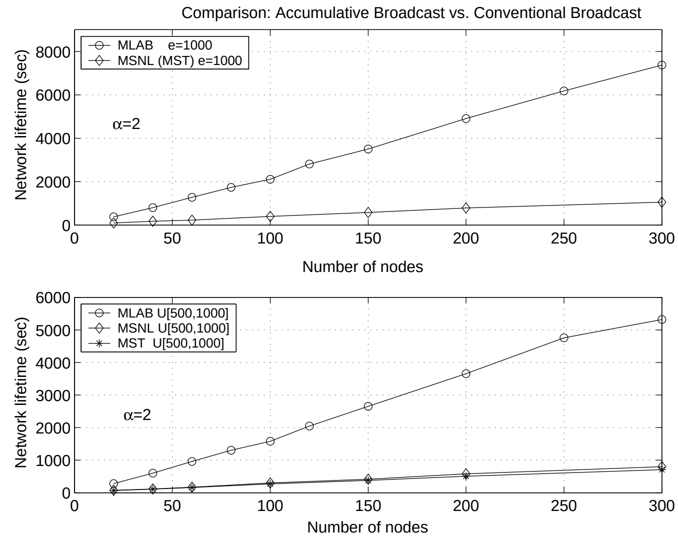

Fig. 5. Network lifetime of accumulative broadcast and conventional broadcast.

to $S_{i+1}(p)$ iff

$$p \sum_{k \in S_i(p), k \neq j} h_{jk} \geq \overline{P}; \tag{12}$$

otherwise, $j \in U_{i+1}(p)$. The ASAP(p) distribution makes node $j \in U_i(p)$ reliable at stage i if $j \in S_{i+1}(p)$. Suppose the last restart of the MLAB algorithm occurs when the power is p and the ASAP(p) distribution stalls at stage τ. This implies

$$p \sum_{k \in S_\tau(p)} h_{jk} < \overline{P}, \qquad j \in U_\tau(p). \tag{13}$$

In this case, we restart MLAB with broadcast power $p + \Delta$ where $\Delta = \min_{j \in U_\tau(p)} \Delta_j$ and Δ_j satisfies

$$(p + \Delta_j) \sum_{k \in S_\tau(p)} h_{jk} = \overline{P}. \tag{14}$$

This implies

$$(p + \Delta) \sum_{k \in S_\tau(p)} h_{jk} \leq \overline{P}, \qquad j \in U_\tau(p). \tag{15}$$

Since this is the last restart of MLAB, the ASAP$(p+\Delta)$ distribution is a feasible broadcast. It follows that $p^* \leq p + \Delta$ since p^* is the optimal broadcast power. To show that $p^* = p + \Delta$ requires the following lemma.

Lemma 1: For any power $p' < p + \Delta$, the ASAP(p') distribution stalls with $S_\tau(p') = S_\tau(p)$.

Lemma 1 implies that if $p^* < p + \Delta$, then the ASAP(p^*) distribution will stall, which is a contradiction of Theorem 1. Thus, at the final restart of the MLAB algorithm, the power is $p + \Delta = p^*$.

Proof: Lemma 1

Let $D = S_\tau(p') \setminus S_\tau(p)$. First, we show by contradiction that D is an empty set. Suppose D is nonempty. Let τ' denote the first stage in which a node $j' \in D$ was made reliable by the ASAP(p') distribution. Thus,

$$\overline{P} \leq p' \sum_{k \in S_{\tau'}(p')} h_{j'k}. \tag{16}$$

Moreover, $S_{\tau'}(p') \subset S_\tau(p)$ since up to stage τ', all nodes that were made reliable by ASAP(p') belong to $S_\tau(p)$. Hence,

$$\overline{P} \leq p' \sum_{k \in S_\tau(p)} h_{j'k} \tag{17}$$

$$<^{(a)} (p+\Delta) \sum_{k \in S_\tau(p)} h_{j'k} \tag{18}$$

$$\leq^{(b)} \overline{P} \tag{19}$$

since (a) follows from $p' < p + \Delta$ and (b) follows from Equation (15). Thus we have the contradiction $\overline{P} < \overline{P}$ and we conclude that D is empty, $S_\tau(p') = S_\tau(p)$, and $U_\tau(p') = U_\tau(p)$. Second, we observe that ASAP(p') stalls at stage τ since for all $j \in U_\tau(p') = U_\tau(p)$,

$$p' \sum_{k \in S_\tau(p')} h_{jk} = p' \sum_{k \in S_\tau(p)} h_{jk} < (p+\Delta) \sum_{k \in S_\tau(p)} h_{jk} \leq \overline{P}. \tag{20}$$

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