# Orthogonal Variable Spreading Factor Codes with Zero-Correlation Zone for TS-UWB

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***Abstract*— We propose an algorithm for construction of ternary orthogonal variable-spreading-factor (OVSF) codes with zero-correlation zone (ZCZ) of particular interest for ternary sequence based UWB (TS-UWB) systems. The conventional approach is to design OVSF codes based on Walsh codes and, in this case, the orthogonality of the corresponding OVSF codes is easily lost when the synchronism is lacking or in a multipath scenario. 2D OVSF codes possess ideal correlation properties which significantly improve the interference-rejection capability of the multicarrier DS-CDMA systems. However, 2D OVSF code schemes, applied over multiple orthogonal channels, suffer from the disadvantages of a high peak-to-average power ratio (PAPR) and high complexities on transceiver design. The proposed OVSF codes derived from two-dimensional (2D) OVSF codes present a ZCZ which allows for significant suppression of multipath and multiuser interference and keep the transceiver of impulse based UWB system simple.**

***Index Terms*— ultra-wide band (UWB), orthogonal variable-spreading-factor (OVSF), zero-correlation zone (ZCZ), ternary sequence**

## I. Introduction

Orthogonal variable-spreading-factor (OVSF) codes are commonly adopted in the forward link of synchronous DS-CDMA systems as channelization codes to accommodate multiple users with different transmission rates [1]. A higher date rate access can be achieved by assigning a lower spreading factor code to the user. In the precious work [2], OVSF codes with a tree structure are constructed based on Walsh codes. In this case, the orthogonality is easily lost when the synchronism is lacking or in a multipath scenario. Thus, the conventional OVSF codes are not suitable for UWB systems which usually suffer in a dense multipath environment [3].

In [4], [5], the authors propose the two-dimensional (2D) OVSF sequence set (matrix) with ideal correlation properties (i.e. zero autocorrelation sidelobes and zero cross-correlation functions), which significantly improve the interference-rejection capability of the multicarrier DS-CDMA or multichannel DS-UWB systems. In such a system, each user applies a unique 2D OVSF sequence set (i.e. an $M$ by $N$ matrix, where $M$ is the number of multichannels and $N$ is the sequence length). The same information bit is spread by different sequences within the set and parallel transmitted over $M$ multiple orthogonal channels. Despreading in the receiver is accomplished on a channel-by-channel basis using a set of $M$ correlators matched to the spreading sequences for respective channels. However, the ideal correlation properties are easily lost when $M$ channels undergo different fadings [5]. On the other hand, the same information bits transmit over $M$ channels , resulting in higher complexities on transceiver design compared with that in a single channel scheme. The multichannel scheme also potentially suffers a high peak-to-average power ratio (PAPR) because the transmitted signal is the sum of the signals from parallel channels. As the power amplifier has a limited peak output power, an increased PAPR reflects that the average radiated power has to be reduced to avoid the nonlinear distortion of transmitted signal [6]. Hence, it is necessary to construct a class of one-dimensional (1D) OVSF codes which has better correlation properties than Walsh code based OVSF codes to support multirate users in DS-UWB systems.

It is known that 1D spreading sequences with ideal correlation properties never exists [7]. However, we can construct zero-correlation zone (ZCZ) sequence sets whose periodic and aperiodic correlation values equal to zero for a contiguous set of delays starting with a single delay [8]. In this paper, based on 2D OVSF codes, we propose a stepwise algorithm for the construction of 1D OVSF codes with both periodic and aperiodic ZCZ. The proposed ternary OVSF codes have the same orthogonality as the conventional Walsh code based OVSF codes and can support the same number of multirate users. In addition, the proposed codes present a ZCZ which allows for significant alleviation of multipath and multiuser interference of particular interests for TS-UWB systems [5], [8], [9].

The remainder of this paper is organized as follows. The basic concepts and definitions are given in Section II, followed by the stepwise algorithm for the construction of OVSF codes with ZCZ. In Section III, we discuss on the parameters relationship of proposed OVSF codes. The DS-UWB system model and numerical results are respectively shown in Section IV and Section V.

## II. Design of OVSF codes with ZCZ

In this section, we first give the definitions of ZCZ sequence set, 2D OVSF code set and mutually orthogonal (MO) complementary sequence. Based on MO complementary sets, we generate 2D OVSF codes and further construct the OVSF codes with both periodic and aperiodic ZCZ.

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*A. Definition of ZCZ sequences*

A set of $M$ sequences $\{\mathbf{a}_i, i = 1, 2\cdots M\}$ each of equal length $N$ is said to be a sequence set $T(N, M, L_{zcz})$ with both periodic and aperiodic ZCZ, if

$$\theta_{ij}(l) = R_{ij}(l) = \begin{cases} E_i, & if \quad l = 0,\, i = j \\ 0, & if \quad l = 0,\, i \neq j \\ 0, & if \quad 1 \leq |l| \leq T \end{cases}$$

where $\Theta_{ij}(l)$ and $R_{ij}(l)$ respectively denotes the aperiodic and periodic correlation function of $\mathbf{a}_i$ and $\mathbf{a}_j$ at time shift $l$. $E_i = R_{ii}(0)$, which is $N$ for binary sequence set. $T$ is a positive integer and its maximum possible value is defined as $L_{zcz}$.

*B. Definition of 2D OVSF codes*

Spreading sequence set $\boldsymbol{T}$, a collection of $M$ by $N$ matrices, is said to be a 2D OVSF code set, if

a) For any matrix $\mathbf{C}^{(i)}_{M,N} \in \boldsymbol{T}$ and an integer $l \in (0, N)$, the out-off phase values of the 2D even and odd autocorrelation of $\mathbf{C}^{(i)}_{M,N}$ are zeros,

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} c^{(i)}_{m,n} c^{(i)}_{m,n\oplus l} = 0$$

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1-l} c^{(i)}_{m,n} c^{(i)}_{m,n\oplus l} - \sum_{m=0}^{M-1}\sum_{n=N-l}^{N-1} c^{(i)}_{m,n} c^{(i)}_{m,n\oplus l} = 0$$

respectively, where"$\oplus$" denotes a modulo-N addition and $c^{(i)}_{m,n}$ is the entry in $m$th row and $n$th column of matrix $\mathbf{C}^{(i)}_{M,N}$ .

b) For any two distinct matrices $\mathbf{C}^{(i)}_{M,N}$ and $\mathbf{C}^{(j)}_{M,N}$ within set $\boldsymbol{T}$ and an integer $l \in [0, N)$, the values of the 2D even and odd cross-correlation functions are zeros,

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} c^{(i)}_{m,n} c^{(j)}_{m,n\oplus l} = 0$$

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1-l} c^{(i)}_{m,n} c^{(j)}_{m,n\oplus l} - \sum_{m=0}^{M-1}\sum_{n=N-l}^{N-1} c^{(i)}_{m,n} c^{(j)}_{m,n\oplus l} = 0$$

respectively, where again $c^{(i)}_{m,n}$ is the entry of matrix $\mathbf{C}^{(i)}_{M,N}$ in $m$th row and $n$th column.

*C. Complementary Sets*

A set of $M$ sequences $\{\mathbf{a}_0, \mathbf{a}_1, \cdots, \mathbf{a}_{M-1}\}$ is said to be a set of complementary sequences, if the sum of the aperiodic autocorrelation functions (ACF) of the $M$ sequences vanishes for any integer shift $l \in (0, N)$,

$$\sum_{i=0}^{M-1} \theta_{\mathbf{a}_i,\mathbf{a}_i}(l) = \sum_{i=0}^{M-1}\sum_{n=0}^{N-1-l} a_{i,n} a_{i,n+l} = 0$$

where $\theta_{\mathbf{a}_i,\mathbf{a}_i}$ denotes the aperiodic ACF of sequences $\mathbf{a}_i$ with length $N$ and $a_{i,n}$ denotes the $n$th element in the sequence $\mathbf{a}_i$. When $M = 2$, $\{\mathbf{a}_0, \mathbf{a}_1\}$ are called complementary pair. Particularly, we call $\{\mathbf{a}_0, \mathbf{a}_1\}$ binary complementary pair (a.k.a Golay pair) if $a_{i,n} \in \{+1, -1\}$ and ternary complementary pair (TCP) if $a_{i,n} \in \{+1, 0, -1\}$.

The complementary sequence set $\{\mathbf{b}_0, \mathbf{b}_1, \cdots, \mathbf{b}_{M-1}\}$ is a mate of complementary set $\{\mathbf{a}_0, \mathbf{a}_1, \cdots, \mathbf{a}_{M-1}\}$ if the length of $\mathbf{b}_i$ is equal to the length of $\mathbf{a}_i$, $0 \leq i \leq M-1$, and for integer shift $l \in [0, N)$,

$$\sum_{i=0}^{M-1} \theta_{\mathbf{a}_i,\mathbf{b}_i}(l) = \sum_{i=0}^{M-1}\sum_{n=0}^{N-1-l} a_{i,n} b_{i,n+l} = 0$$

Mutually orthogonal (MO) complementary set is a collection of complementary sets in which any two of them are mates to each other.

***Theorem 1***: MO complementary sets satisfy the definition of 2D OVSF codes.

MO complementary sets are mutually orthogonal in both periodic and aperiodic sense. Based on the relationship between periodic/aperiodc correlations and odd/even correlations [10], the MO complementary sets satisfy the odd and even correlation properties of 2D OVSF codes.

The following lemmas related to MO complementary sets are used in the following construction algorithm of OVSF codes with ZCZ. We list them here,

***Lemma 1***: Let $\{\mathbf{a}_1, \mathbf{b}_1\}$ be a complementary pair, then $\{\overleftarrow{\mathbf{b}_1}, -\overleftarrow{\mathbf{a}_1}\}$ is its mate, where $\overleftarrow{\mathbf{b}_1}$ denotes the reverse of the sequence $\mathbf{b}_1$ and $-\overleftarrow{\mathbf{a}_1}$ denotes the sequence whose $i$th element is the negation of the $i$th element in sequence $\overleftarrow{\mathbf{a}_1}$.

***Lemma 2***: Let $\{\mathbf{a}_1, \mathbf{a}_2\}$ be a complementary set, then $\{\mathbf{a}_1\mathbf{a}_1, \mathbf{a}_2\mathbf{a}_2,\ \mathbf{a}_1(-\mathbf{a}_1), \mathbf{a}_2(-\mathbf{a}_2)\}$ and $\{\mathbf{a}_1(-\mathbf{a}_1), \mathbf{a}_2(-\mathbf{a}_2), \mathbf{a}_1\mathbf{a}_1, \mathbf{a}_2\mathbf{a}_2\}$ are mates of each other, where $-\mathbf{a}_1$ denotes the sequence whose $i$th element is the negation of the $i$th element of $\mathbf{a}_1$ and $\mathbf{a}_1(-\mathbf{a}_1)$ denotes the concatenation of two sequences $\mathbf{a}_1$ and $-\mathbf{a}_1$.

***Lemma 3***: Let $\{\mathbf{a}_1, \mathbf{a}_2\}$ be a complementary set and $\{\mathbf{b}_1, \mathbf{b}_2\}$ be its mate, then $\{\mathbf{a}_1\mathbf{a}_1, \mathbf{a}_2\mathbf{a}_2, \mathbf{a}_1(-\mathbf{a}_1), \mathbf{a}_2(-\mathbf{a}_2)\}$ and $\{\mathbf{b}_1\mathbf{b}_1, \mathbf{b}_2\mathbf{b}_2,\ \mathbf{b}_1(-\mathbf{b}_1), \mathbf{b}_2(-\mathbf{b}_2)\}$ are also mates.

*D. Design Algorithm*

We illustrate the algorithm with the design of ternary OVSF codes with ZCZ. The construction of binary codes is straightforward by using binary seed in stead of ternary one in the following step.

***Step 1***: Start from any seed TCP $\{\mathbf{c}_1, \mathbf{c}_2\}$ with sequence length $N^{(1)}$ and present them in a matrix form $\mathbf{C}^{(1)}_{2,N^{(1)}}$,

$$\begin{aligned} \mathbf{C}^{(1)}_{2,N^{(1)}} &= \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix}_{2\times N^{(1)}} \\ &= \begin{bmatrix} c_{0,0} & c_{0,1} & \cdots & c_{0,N^{(1)}-1} \\ c_{1,0} & c_{1,1} & \cdots & c_{1,N^{(1)}-1} \end{bmatrix} \end{aligned} \quad (1)$$

From *Lemma* 1, we obtain its mate $\mathbf{C}^{(2)}_{2,N^{(1)}}$,

$$\begin{aligned} \mathbf{C}^{(2)}_{2,N^{(1)}} &= \begin{bmatrix} \overleftarrow{\mathbf{c}_2} \\ -\overleftarrow{\mathbf{c}_1} \end{bmatrix}_{2\times N^{(1)}} \\ &= \begin{bmatrix} c_{1,N^{(1)}-1} & \cdots & c_{1,1} & c_{1,0} \\ -c_{0,N^{(1)}-1} & \cdots & -c_{0,1} & -c_{0,0} \end{bmatrix} \end{aligned} \quad (2)$$

Then, $\mathbf{C}^{(1)}_{2,N^{(1)}}$ and $\mathbf{C}^{(2)}_{2,N^{(1)}}$ are the first layer 2D OVSF codes.

***Step 2***: From $\mathbf{C}^{(1)}_{2,N^{(1)}}$ and $\mathbf{C}^{(2)}_{2,N^{(1)}}$, the recursive procedure generates second layer 2D OVSF codes $\mathbf{C}^{(1)}_{2^2,N^{(2)}}$, $\mathbf{C}^{(2)}_{2^2,N^{(2)}}$, $\mathbf{C}^{(3)}_{2^2,N^{(2)}}$ and $\mathbf{C}^{(4)}_{2^2,N^{(2)}}$ with aid of two 2 by 2 orthogonal matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ below,

$$\mathbf{H}_1 = \begin{bmatrix} + & + \\ + & - \end{bmatrix}$$

and

$$\mathbf{H}_2 = \begin{bmatrix} + & - \\ + & + \end{bmatrix}$$

where "$+$" denotes $+1$ and "$-$" denotes $-1$. Then,

$$\mathbf{C}^{(1)}_{2^2,N^{(2)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(1)}_{2,N^{(1)}} & \mathbf{C}^{(1)}_{2,N^{(1)}} \\ \mathbf{C}^{(1)}_{2,N^{(1)}} & -\mathbf{C}^{(1)}_{2,N^{(1)}} \end{bmatrix} \tag{3}$$

$$\mathbf{C}^{(2)}_{2^2,N^{(2)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(1)}_{2,N^{(1)}} & -\mathbf{C}^{(1)}_{2,N^{(1)}} \\ \mathbf{C}^{(1)}_{2,N^{(1)}} & \mathbf{C}^{(1)}_{2,N^{(1)}} \end{bmatrix} \tag{4}$$

$$\mathbf{C}^{(3)}_{2^2,N^{(2)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(2)}_{2,N^{(1)}} & \mathbf{C}^{(2)}_{2,N^{(1)}} \\ \mathbf{C}^{(2)}_{2,N^{(1)}} & -\mathbf{C}^{(2)}_{2,N^{(1)}} \end{bmatrix} \tag{5}$$

$$\mathbf{C}^{(4)}_{2^2,N^{(2)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{C}^{(2)}_{2,N^{(1)}} & -\mathbf{C}^{(2)}_{2,N^{(1)}} \\ \mathbf{C}^{(2)}_{2,N^{(1)}} & \mathbf{C}^{(2)}_{2,N^{(1)}} \end{bmatrix} \tag{6}$$

where $\otimes$ denotes Kronecker product of two matrices and the second layer 2D OVSF codes with dimension $2^2$ by $N^{(2)}$ and $N^{(2)} = 2N^{(1)}$.

***Step 3***: We numerate $2^i$ 2D OVSF codes at $i$th layer by index $k$, thus $k \in [1, 2^i]$. Then, $(i+1)$th layer 2D OVSF codes can be constructed from $i$th layer 2D OVSF codes using the following general formula,

$$\begin{aligned} \mathbf{C}^{(2k-1)}_{2^{i+1},N^{(i+1)}} &= \mathbf{H}_1 \otimes \mathbf{C}^{(k)}_{2^i,N^{(i)}} \\ &= \begin{bmatrix} \mathbf{C}^{(k)}_{2^i,N^{(i)}} & \mathbf{C}^{(k)}_{2^i,N^{(i)}} \\ \mathbf{C}^{(k)}_{2^i,N^{(i)}} & -\mathbf{C}^{(k)}_{2^i,N^{(i)}} \end{bmatrix} \end{aligned} \tag{7}$$

and

$$\begin{aligned} \mathbf{C}^{(2k)}_{2^{i+1},N^{(i+1)}} &= \mathbf{H}_2 \otimes \mathbf{C}^{(k)}_{2^{i-1},N^{(i-1)}} \\ &= \begin{bmatrix} \mathbf{C}^{(k)}_{2^i,N^{(i)}} & -\mathbf{C}^{(k)}_{2^i,N^{(i)}} \\ \mathbf{C}^{(k)}_{2^i,N^{(i)}} & \mathbf{C}^{(k)}_{2^i,N^{(i)}} \end{bmatrix} \end{aligned} \tag{8}$$

where $N^{(i+1)} = 2N^{(i)} = 2^i N^{(1)}$.

***Theorem 2***: At any step $i$, the constructed matrices $\mathbf{C}^{(k)}_{2^i,N^{(i)}}$, $k \in [1, 2^i]$, are MO complementary set. Thus, they are 2D OVSF codes at $i$th layer of code tree.

*Proof*: $Lemma$ 2 and $Lemma$ 3 guarantee that constructed matrices $\mathbf{C}^{(k)}_{2^i,N^{(i)}}$, $k \in [1, 2^i]$ are MO complementary sets. Thus, they meet the autocorrelation and cross-correlation requirements of 2D OVSF codes.

***Example 1***: Based on *Lemma* 1, from seed TCP

$$\mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix} = \begin{bmatrix} + & + & - \\ + & 0 & + \end{bmatrix}_{2\times 3} \tag{9}$$

we obtain its mate $\mathbf{C}^{(2)}_{2,N^{(1)}}$,

$$\mathbf{C}^{(2)}_{2,N^{(1)}} = \begin{bmatrix} \overleftarrow{\mathbf{c}_2} \\ -\overleftarrow{\mathbf{c}_1} \end{bmatrix} = \begin{bmatrix} + & 0 & + \\ + & - & - \end{bmatrix}_{2\times 3} \tag{10}$$

Then, we use recursive procedure to generate second layer 2D OVSF codes $\mathbf{C}^{(1)}_{2^2,N^{(2)}}$, $\mathbf{C}^{(2)}_{2^2,N^{(2)}}$, $\mathbf{C}^{(3)}_{2^2,N^{(2)}}$ and $\mathbf{C}^{(4)}_{2^2,N^{(2)}}$,

$$\mathbf{C}^{(1)}_{2^2,2N^{(1)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} ++- & ++- \\ +0+ & +0+ \\ ++- & --+ \\ +0+ & -0- \end{bmatrix}_{4\times 6} \tag{11}$$

$$\mathbf{C}^{(2)}_{2^2,2N^{(1)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} ++- & --+ \\ +0+ & -0- \\ ++- & ++- \\ +0+ & +0+ \end{bmatrix}_{4\times 6} \tag{12}$$

$$\mathbf{C}^{(3)}_{2^2,2N^{(1)}} = \mathbf{H}_1 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} +0+ & +0+ \\ +-- & +-- \\ +0+ & -0- \\ +-- & -++ \end{bmatrix}_{4\times 6} \tag{13}$$

$$\mathbf{C}^{(4)}_{2^2,2N^{(1)}} = \mathbf{H}_2 \otimes \mathbf{C}^{(1)}_{2,N^{(1)}} = \begin{bmatrix} +0+ & -0- \\ +-- & -++ \\ +0+ & +0+ \\ +-- & +-- \end{bmatrix}_{4\times 6} \tag{14}$$

We can go further to generate the $3$rd layer 2D OVSF codes using general formula $(7)$ and $(8)$.

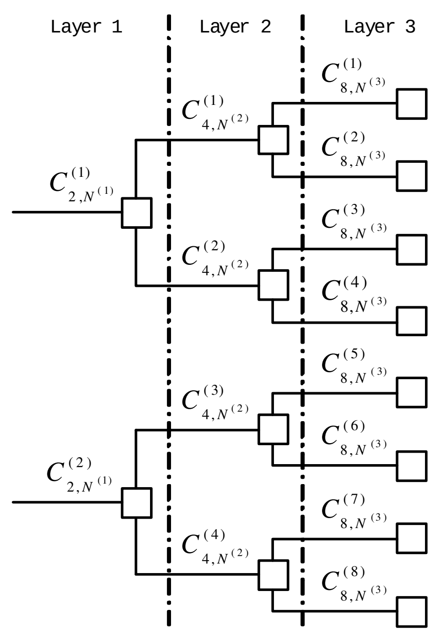


Fig. 1. The tree structure of 2D OVSF codes with first 3 layers

The tree structure of the first 3 layer 2D OVSF codes is illustrated in Figure 1. From Theorem 2, any two 2D OVSF codes in the same layer are orthogonal. Furthermore, any two codes of different layers are also orthogonal except for the case that one of the two codes is a mother code of the other.

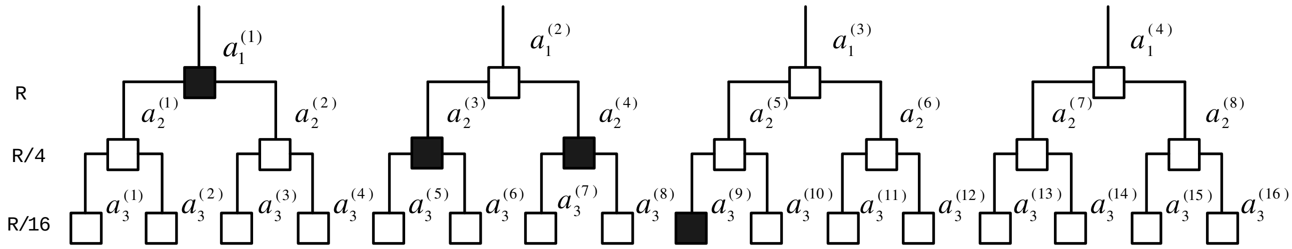


Fig. 2. The first 3 layers of OVSF codes with ZCZ generated based on second layer of 2D OVSF codes

The mother codes are defined as all high layer codes linking this particular code to the first layer code [2]. For example, both $\mathbf{C}^{(1)}_{2,N^{(1)}}$ and $\mathbf{C}^{(2)}_{4,N^{(2)}}$ are mother codes of $\mathbf{C}^{(3)}_{8,N^{(3)}}$.

***Step 4***: The construction of OVSF codes with ZCZ can be initiated from any layer of 2D OVSF code tree, resulting in different $L_{zcz}$. Let us assume we start from the layer $p$ of the 2D OVSF code tree.

For any $i \geq p$ and $k \in [1, 2^i]$,

$$\mathbf{C}^{(k)}_{2^i,N^{(i)}} = \begin{bmatrix} \mathbf{c}^{(k)}_1 \\ \mathbf{c}^{(k)}_2 \\ \vdots \\ \mathbf{c}^{(k)}_{2^i} \end{bmatrix} = \begin{bmatrix} c_{1,1}c_{1,2}\cdots c_{1,N^{(i)}} \\ c_{2,1}c_{2,2}\cdots c_{2,N^{(i)}} \\ \vdots \\ c_{2^i,1}c_{2^i,2}\cdots c_{2^i,N^{(i)}} \end{bmatrix}_{2^i \times N^{(i)}} \tag{15}$$

The length of sequence $\mathbf{c}^{(k)}_j$, $j \in [1, 2^i]$, is $N^{(i)}$. We partition each of them into subsequences with equal length $N^{(p)}$, thus the sequence $\mathbf{c}^{(k)}_j$ is divided into $N^{(i)}/N^{(p)}$ subsequences. Let us denote the resulting subsequences as $\mathbf{b}^{(k)}_{j,r}$, $j \in [1, 2^i]$ and $r \in [1, t]$, where $t = N^{(i)}/N^{(p)}$. That is,

$$\mathbf{C}^{(k)}_{2^i,N^{(i)}} = \begin{bmatrix} \mathbf{c}^{(k)}_1 \\ \mathbf{c}^{(k)}_2 \\ \vdots \\ \mathbf{c}^{(k)}_{2^i} \end{bmatrix} = \begin{bmatrix} \mathbf{b}^{(k)}_{1,1}\mathbf{b}^{(k)}_{1,2}\cdots\mathbf{b}^{(k)}_{1,t} \\ \mathbf{b}^{(k)}_{2,1}\mathbf{b}^{(k)}_{2,2}\cdots\mathbf{b}^{(k)}_{2,t} \\ \vdots \\ \mathbf{b}^{(k)}_{2^i,1}\mathbf{b}^{(k)}_{2^i,2}\cdots\mathbf{b}^{(k)}_{2^i,t} \end{bmatrix}_{2^i \times N^{(i)}} \tag{16}$$

We reshape $\mathbf{C}^{(k)}_{2^i,N^{(i)}}$ by concatenating the corresponding subsequences $\mathbf{b}^{(k)}_{j,r}$ columnwise with $N^p$ zeros padded between them. Thus,

$$\mathbf{d}^{(k)}_i = \mathbf{b}^{(k)}_{1,1}\mathbf{z}_p\mathbf{b}^{(k)}_{2,1}\mathbf{z}_p\cdots\mathbf{b}^{(k)}_{2^i,1}\mathbf{z}_p\mathbf{b}^{(k)}_{1,2}\mathbf{z}_p\cdots\mathbf{b}^{(k)}_{2^i,t}\mathbf{z}_p \tag{17}$$

where $\mathbf{z}_p$ denotes the sequence with $N^{(p)}$ continuous zero elements. Finally, the OVSF codes $\mathbf{a}^{(k)}_i$ with $L_{zcz} = N^{(p)}$ are obtained by,

$$\mathbf{a}^{(k)}_i = \mathbf{d}^{(k)}_{p+i-1} \tag{18}$$

where the subscripts $i$ of $\mathbf{a}^{(k)}_i$ is the index of layer in OVSF code tree and $i = 1, 2, ....$

***Example 2***: Let us start from the second layer of 2D OVSF code tree, that is $p = 2$. Reshaping the second layer 2D OVSF codes, we obtain the first layer OVSF codes with ZCZ. Base on Example 1, we have

$$\mathbf{C}^{(1)}_{2^2,N^{(2)}} = \begin{bmatrix} \mathbf{b}^{(1)}_{1,1} \\ \mathbf{b}^{(1)}_{2,1} \\ \mathbf{b}^{(1)}_{3,1} \\ \mathbf{b}^{(1)}_{4,1} \end{bmatrix} = \begin{bmatrix} ++-++- \\ +\,0\;++\,0\,+ \\ ++---+ \\ +\,0\;+-\,0\,- \end{bmatrix}_{4\times 6} \tag{19}$$

According to Eq.16, here we have $i = 2, k = 1$ and $t = \frac{N^{(i)}}{N^{(p)}} = \frac{N^2}{N^2} = 1$. Then,

$$\mathbf{d}^{(1)}_2 = \mathbf{b}^{(1)}_{1,1}\mathbf{z}_2\mathbf{b}^{(1)}_{2,1}\mathbf{z}_2\mathbf{b}^{(1)}_{3,1}\mathbf{z}_2\mathbf{b}^{(1)}_{4,1}\mathbf{z}_2 \tag{20}$$

Further,

$$\begin{aligned} \mathbf{a}^{(1)}_1 &= \mathbf{d}^{(1)}_{p+1-1} = \mathbf{d}^{(1)}_2 \\ &= (++-++-000000+0++0+000000 \\ &\quad ++---+000000+0+-0-000000) \end{aligned} \tag{21}$$

$$\begin{aligned} \mathbf{a}^{(2)}_1 &= \mathbf{d}^{(2)}_{p+1-1} = \mathbf{d}^{(2)}_2 \\ &= (++---+000000+0+-0-000000 \\ &\quad ++-++-000000+0++0+000000) \end{aligned} \tag{22}$$

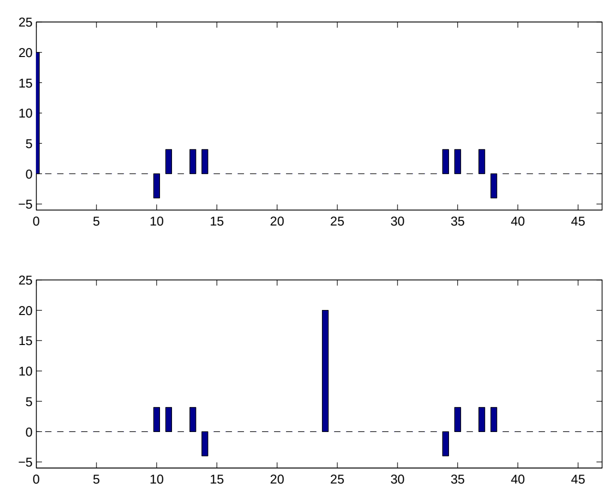


Fig. 3. upper plot: periodic autocorrelation of $\mathbf{a}^{(1)}_1$ ; lower plot: periodic cross-correlation between $\mathbf{a}^{(1)}_1$ and $\mathbf{a}^{(2)}_1$

Using the same procedure, we can construct other OVSF codes with ZCZ. Fig 2 depicts the first 3 layers of OVSF code tree with p=2. The length of ZCZ between any two codes on this OVSF code tree is $L_{zcz} = N^{(2)} = 6$, as long as one code is not the mother code of the other one. The mother codes are defined as all high layer codes linking this particular code to the first layer code [2]. E.g., In Fig. 2, $\mathbf{a}_1^{(1)}$ and $\mathbf{a}_2^{(1)}$ are mother codes of $\mathbf{a}_3^{(1)}$ and $\mathbf{a}_3^{(2)}$. Thus, to maintain the orthogonality and zero-correlation duration between any two OVSF codes, one code can not be assigned to a user when its mother codes are used by others.

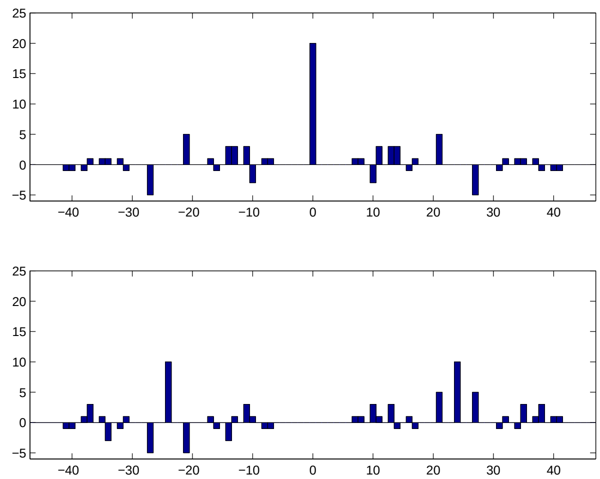

Fig. 4. upper plot: aperiodic autocorrelation of $\mathbf{a}_1^{(1)}$ ; lower plot: aperiodic cross-correlation between $\mathbf{a}_1^{(1)}$ and $\mathbf{a}_1^{(2)}$

Fig. 3 and Fig. 4 respectively show the periodic and aperiodic auto/cross-correlation of OVSF codes $\mathbf{a}_1^{(1)}$ and $\mathbf{a}_1^{(2)}$. The $L_{zcz}$ in either case is 6.

## III. Discussions on the proposed OVSF Codes

In the preceding section, we generate the OVSF code tree from 2D OVSF codes. Starting from different layer of 2D OVSF code tree, we obtain the OVSF codes with different $L_{zcz}$. Let us assume we start from the $p$ layer of 2D OVSF code tree. The parameters of proposed OVSF codes are listed in Table 1.

The most left column indicates the layer index of OVSF codes. In the same layer, all codes are with equal length. For example, in the third layer, $2^{p+2}$ OVSF codes with equal length $2^{p+2i-1}N^{(p)}$ are available to be assigned. The length of ZCZ between any two codes in this OVSF code tree is $N^{(p)}$, as long as one code is not the mother code of the other one. If we assume the highest data rate is $R$, then the most right column of Table 1 shows the achievable rate of the OVSF codes on different layer.

The parameter $N^{(p)} = 2^{p-1}N^{(1)}$ is the sequence length of $p$ layer 2D OVSF codes, where $N^{(1)}$ is the sequence length of seed complementary pair. For binary complementary pair (a.k.a Golay pair), $N^{(1)} = 2^{\alpha}10^{\beta}26^{\gamma}$, where $\alpha$,$\beta$ and $\gamma$ are non-negative integers. For TCP, $N^{(1)}$ can be any positive integers. Hence, we have two basic parameters $N^{(1)}$ and $p$ which can be changed to construct OVSF codes with different sequence length and different $L_{zcz}$.

TABLE I

The relationship of parameters in proposed OVSF codes with ZCZ.

| Layer Index | Number of Codes | Code Length | ZCZ Length | Data Rate |
|---|---|---|---|---|
| 1 | $2^p$ | $2^{p+1}N^{(p)}$ | $N^{(p)}$ | R |
| 2 | $2^{p+1}$ | $2^{p+3}N^{(p)}$ | $N^{(p)}$ | $R/4$ |
| 3 | $2^{p+2}$ | $2^{p+5}N^{(p)}$ | $N^{(p)}$ | $R/16$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $i$ | $2^{p+i-1}$ | $2^{p+2i-1}N^{(p)}$ | $N^{(p)}$ | $R/4^{i-1}$ |

## IV. System Model

The impulse response of the UWB channel with $L$ resolvable paths is

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \tag{23}$$

where $\alpha_l$ and $\tau_l$ denote the channel gain and the propagation delay of the $l_{th}$ path, respectively. When sufficient multipath resolution is available, small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients $\alpha_l$ can be modelled as independent real valued random variables whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [11].

The transmitted signal for user $k$ is given by

$$s^{(k)}(t) = \sum_r b_r^{(k)} \sum_{n=0}^{N-1} c_n^{(k)} \psi(t - rT_p - nT_c) \tag{24}$$

where $r$ is the index of the information symbols and $N$ is the length of spreading sequences. The spreading sequence $c_n$ is the OVSF code chosen from the OVSF tree. $b_r$ are binary antipodal symbols , $T_c$ is the chip duration time and $T_p = NT_c$ is the symbol period. $\psi_m(t)$ is the unit energy signaling pulse and assumed known to the receiver.

For a synchronous UWB system with $K$ users, the corresponding received signal model is:

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} \alpha_l s^{(k)}(t) + n(t) \tag{25}$$

where $n(t)$ is a white Gaussian noise with zero mean and two-sided power spectral density $\frac{N_0}{2}$.

## V. Numerical Results

We study correlator receiver performance of DS-UWB systems employing different type of OVSF codes. The mean power of multipath components are chosen to be equal to average value given in [12], which is based on the in-door line of sight (LOS) measurements performed in 23 homes. The sign of the reflected path coefficient is modelled as a uniformly distributed random variable. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration $T_c$. We assume that each bin contains exactly one multipath component (emulating a dense multipath environment) and the delay spread was restricted to be 4 nanosecond. The effects of interchip interference has been assumed negligible.

In a single user DS-UWB system, we compare the BER performances of proposed ZCZ sequence with that of Walsh code $[++----++++----++++----++++----++]$ and Gold sequence $[----+--+-++--+++++---++-+++-+-+]$. For comparison purpose, since both the sequence lengths of Walsh code and Gold sequence are the power of 2. The ZCZ sequence that we employ here is $[++++0000++--0000-+-+0000-++-0000]$ which constructed from binary complementary pair [+ +] and [- -]. Fig. 5 shows that the OVSF code with $L_{zcz} = 4$, which may help suppress the multipath interference from the first 4 indirect paths, has the same BER performance of Gold sequence. However, the user employ Walsh code has the worst BER performance.

In Fig. 6, we compare the correlator receiver performance of DS-UWB systems employing different type of OVSF codes. For OVSF codes with ZCZ, we use the tree structure in Example 2. The four codes assigned to four users are marked in black so that none of the above codes is the mother code of the others. We also construct the OVSF codes based on Gold sequence using the method in [2] for comparison purpose. The first layer OVSF codes are chosen from the Gold sequence set with sequence length 31 (i.e. $\mathbf{a}_1^{(1)} = [----+--+-++--+++++---++-+++-+-+]$, $\mathbf{a}_2^{(1)} = [---+-+-++-+----++--+--+++++-+++]$, $\mathbf{a}_3^{(1)} = [---+++--++---++--+-+-+-+-----+-]$ and $\mathbf{a}_4^{(1)} = [+-+--+---++-+-++-+-++--++--++-+]$). We observe that, in multiuser scenario, proposed OVSF codes have much better BER performance than Gold sequence based OVSF codes.

## REFERENCES


[1] E. Dahlman, B. Gudmundson, M. Nilsson, and J. Skold, “UMTS/IMT-2000 based on wideband CDMA,” *IEEE commun. Mag.*, vol. 36, pp. 70–80, Sep. 1998.

[2] F. Adachi, M. Sawahashi, and K. Okawa, “Tree-structured generation of orthogonal spreading codes with differnt lengths for forward link of DS-CDMA mobile radio,” *Electronics Letters*, vol. 33, pp. 27 –28, Jan. 1997.

[3] M. Win and R. Scholtz, “On the energy capture of ultrawide bandwidth signals in dense multipath environments,” *IEEE Communications Letters*, vol. 2, pp. 245–247, Sept. 1998.

[4] C. M. Yang, P. H. Lin, G. C. Yang, and W. C. Kwong, “2D orthogonal spreading codes for multicarrier DS-CDMA systems,” *IEEE International Conference on Communications, ICC’03*, vol. 5, pp. 3277 – 3281, May 11-15 2003.

[5] D. Wu, P. Spasojević, and I. Seskar, “Ternary complementary sets for orthogonal pulse based UWB,” *Conference Record of the Thirty-Seventh Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1776–1780, Nov. 2003.

[6] B. M. Popovic, “Spreading sequences for multicarrier CDMA systems,” *IEEE Transactions on Communications*, vol. 47, pp. 918–926, June 1999.

[7] X. Deng and P. Fan, “Spreading sequence sets with zero correlation zone,” *Electronics Letters*, vol. 36, pp. 993–994, May 2000.

[8] D. Wu, P. Spasojević, and I. Seskar, “Ternary zero-correlation zone sequences for multiple code UWB,” *Proc. of 38th Conference on Information Sciences and Systems, Princeton, NJ*, pp. 939 – 943, March 17-19 2004.

[9] D. Wu, P. Spasojević, and I. Seskar, “Multipath beamforming UWB signal design based on ternary sequences,” *Proc. of 40th Allerton conference on Communication, Control, and Computing, Allerton, IL*, Oct. 2002.

[10] D. V. Sarwate and M. B. Pursley, “Cross correlation properties of pseudo-random and related sequences,” *IEEE Proc.*, vol. 68, pp. 593–619, May 1980.

[11] T. S. Rappaport, *Wireless Communications Principles and Practice*. Prentice Hall, 1997.

[12] S. Ghassemzadeh, R. Jana, V. Tarokh, C. Rice, and W. Turin, “A statistical path loss model for in-home UWB channels,” *Ultra Wideband Systems and Technologies, 2002. Digest of Papers. 2002 IEEE Conference on*, pp. 59 –64, May 2002.


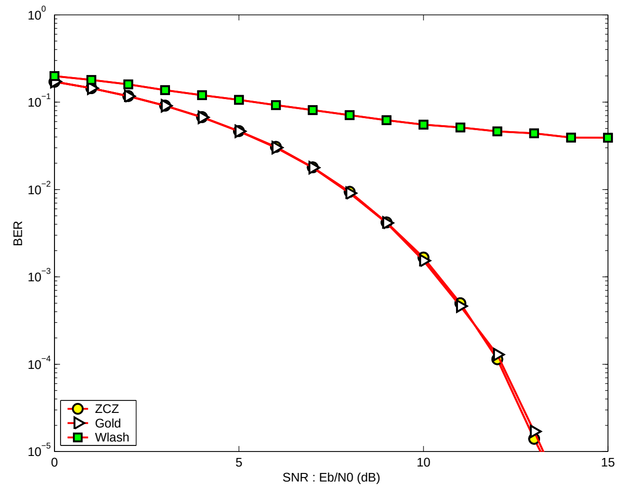


Fig. 5. Comparison on the BER performances of ZCZ sequence, Gold sequence and Walsh code in a single user DS-UWB sysetm

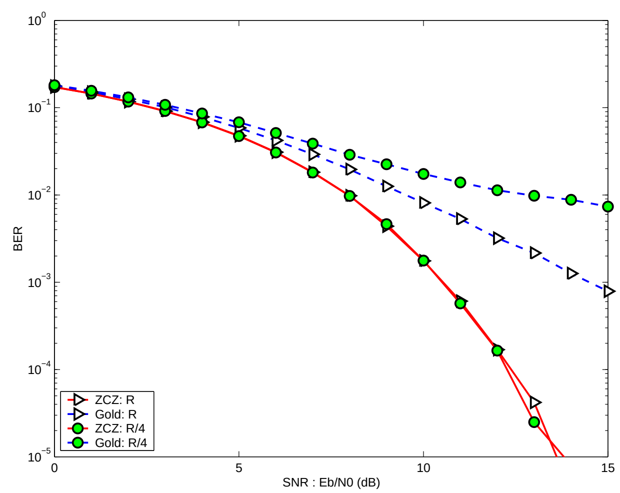


Fig. 6. Comparison on the BER performances of OVSF code with ZCZ and OVSF codes based on Gold sequence in a four user DS-UWB sysetm