Highly Mobile Users and Paging: optimal polling strategies

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Abstract— **We consider the problem of minimizing average paging cost subject to delay constraints in a wireless system. Previous work assumed the unit to be found did not move during the paging process whereas here the unit may change location during polling events. We show that the conditionally most probable locations given that the unit has not yet been found should be searched first. We find the optimal sequential paging strategies for given maximum delay constraints and compute both paging and delay costs as a function of the time between polling events. The results show that sequential paging strategies are beneficial in all but the extremely high mobility cases where polling failures provide little information about the unit location. It is observed that optimal sequential paging strategies substantially lower the paging cost compared to the classical *blanket polling* at the expense of a small degradation in the average delay performance.**

I. INTRODUCTION

Timely localization of users is important for service quality in mobile communication networks. To establish location, users can regularly inform the network of their locations (registration), the system can search for the user after a call arrival (paging) or some combination of the two. Considerable research has been done on registration strategies and in identifying the tradeoff between registration and paging, see for example [1–6]. Various paging strategies have been investigated in [7,8]. Paging strategies that are based on user location probabilities are constructed in [9].

The problem of minimizing the average cost of locating a mobile unit given the probability distribution on unit location was considered in [8]. The same problem was later modeled in [10] and [11] using Search Theory [12]. The cost in [8] was formulated in terms of delay and number of locations searched. However, units were assumed to move slowly relative to the total duration of the paging process; i.e., it was assumed the unit did not change location during or between polling events. This assumption was used also in all previous work on paging mentioned above. In this paper, we remove this assumption and derive optimal paging strategies for the case where the unit may change location during the paging process.

We find that the most probable locations at each time step, conditioned on the mobile having not yet been found, should be searched first. We then compute the optimal paging and delay costs and the associated paging strategies in the presence of a maximum delay constraint. We observe that, unlike the stationary case where the optimal strategies may be found easily via Dynamic Programming [13], the state space of the problem grows geometrically at each paging step. Therefore, we resort to exhaustive search to find the best strategy on how many locations should be searched at each paging step, and thence identify trends in the polling patterns as a function of user mobility.

II. PROBLEM FORMULATION

Let $p_{X_1}(x_1)$ be the probability distribution on unit location at some initial polling time t_1. We assume that unit location evolves from a known location x_n according to some mobility process with known transition probability distribution; i.e., $p_{X_{n+1}|X_n,X_{n-1},\ldots,X_1}(x_{n+1}|x_n,x_{n-1},...,x_1)$ where X_{n+1} is the unit location random variable at polling time t_{n+1} given previous location-time pairs $\{(x_i,t_i)\}$ $i \leq n$.[1]

We define *polling group* A_n to be a group of locations $\{x_i\}$ polled at time t_n with k_n the number of locations in A_n. Polling groups are polled in sequence, $\mathcal{A} = (A_1, A_2, \ldots)$ until the unit is found. Thus, the total number of locations searched if the unit is found in polling step n is $s_n = \sum_{i=1}^{n} k_i$. Finally, we define q_n as the probability that the unit is found on the n^{th} polling step.

Our cost functions are then the expected number of locations searched $E[P]$ (a surrogate for signaling cost),

$$E[P] = \sum_{n=1}^{\infty} q_n s_n \tag{1}$$

and the mean number of polling events $E[D]$ (a surrogate for polling delay),

$$E[D] = \sum_{n=1}^{\infty} n q_n \tag{2}$$

We seek paging sequences $\mathcal{A}$ which minimize the weighted sum of $E[P]$ and $E[D]$,

$$G = \sum_{n=1}^{\infty} (s_n + \alpha n) q_n \tag{3}$$

where $\alpha \geq 0$ is defined as the delay weighting factor. Furthermore, we note that $E[P]$, $E[D]$ and G can also be written as [14,15]

$$E[P] = \sum_{n=1}^{\infty} k_n \bar{F}_q(n-1) \tag{4}$$

$$E[D] = \sum_{n=1}^{\infty} \bar{F}_q(n-1) \tag{5}$$

$$G = \sum_{n=1}^{\infty} \bar{F}_q(n-1)(k_n + \alpha) \tag{6}$$

[1] Although the notation seems to imply both time and space invariance, the results to be derived are applicable to the time/space varying cases. Notational simplicity was chosen over exactness.

where $\bar{F}_q(n)$ is the complementary CDF of q_n;

$$\bar{F}_q(n) = 1 - \sum_{i=1}^{n} q_i \tag{7}$$

III. ANALYSIS

A. Preview

The polling problem requires that we decide which locations to poll when. To simplify the problem we will first show that given sizes of the polling groups $\{k_n\}$, all costs ($E[P]$, $E[D]$ and G) are minimized by composing each A_n of the k_n most likely locations given that the unit was not found up to step n. Once it is established that the structure of the optimum strategy is such that the conditionally most probable places must be searched at each polling instant, the problem of determining the optimum strategy simplifies to determining *how many* locations must be searched at each polling instant. This simplification reduces the problem of polling group construction to that of finding the $\{k_n\}$ which minimize equations (4), (5), (6).

B. Conditional Distributions and q_n

The joint distribution on unit location at times t_1 through t_n is

$$p_{X_1,\dots X_n}(x_1,\dots x_n) = \prod_{i=1}^{n} p_{X_i|X_{i-1}\dots X_1}(x_i|x_{i-1}\dots x_1) \tag{8}$$

Given polling groups A_i, $i = 1, 2, \dots$ we can write down the probability of finding the user in the n^{th} step as

$$q_n = \sum_{x_n \in A_n} \sum_{x_{n-1} \notin A_{n-1}} \dots \sum_{x_1 \notin A_1} p_{X_1,\dots X_n}(x_1,\dots x_n) \tag{9}$$

Now notice that equation (9) can be rewritten as

$$\begin{aligned} q_n = & \sum_{x_1 \notin A_1} p_{X_1}(x_1) \sum_{x_2 \notin A_2} p_{X_2|X_1}(x_2|x_1)\dots \\ & \sum_{x_n \in A_n} p_{X_n|X_{n-1},\dots,X_1}(x_n|x_{n-1},\dots,x_1) \end{aligned} \tag{10}$$

and thence

$$\begin{aligned} q_n = & \sum_{x_1 \notin A_1} p_{X_1}(x_1) \sum_{x_2 \notin A_2} p_{X_2|X_1}(x_2|x_1)\dots \\ & \left(1 - \sum_{x_n \notin A_n} p_{X_n|X_{n-1},\dots,X_1}(x_n|x_{n-1},\dots,x_1)\right) \end{aligned} \tag{11}$$

Enumerating the first few q_n we have

$$q_1 = 1 - \sum_{x_1 \notin A_1} p_{X_1}(x_1) \tag{12}$$

$$q_2 = \sum_{x_1 \notin A_1} p_{X_1}(x_1) \left(1 - \sum_{x_2 \notin A_2} p_{X_2|X_1}(x_2|x_1)\right) \tag{13}$$

$$\begin{aligned} q_3 = & \sum_{x_1 \notin A_1} p_{X_1}(x_1) \sum_{x_2 \notin A_2} p_{X_2|X_1}(x_2|x_1) \cdot \\ & \left(1 - \sum_{x_3 \notin A_3} p_{X_3|X_2,X_1}(x_3|x_2,x_1)\right) \end{aligned} \tag{14}$$

We then note that

$$\begin{aligned} \sum_{i=1}^{n} q_i = & 1 - \sum_{x_1 \notin A_1} p_{X_1}(x_1) \sum_{x_2 \notin A_2} p_{X_2|X_1}(x_2|x_1)\dots \\ & \sum_{x_n \notin A_n} p_{X_n|X_{n-1},\dots,X_1}(x_n|x_{n-1},\dots x_1) \end{aligned} \tag{15}$$

which means

$$\begin{aligned} \bar{F}_q(n) = & \sum_{x_1 \notin A_1} p_{X_1}(x_1) \sum_{x_2 \notin A_2} p_{X_2|X_1}(x_2|x_1)\dots \\ & \sum_{x_n \notin A_n} p_{X_n|X_{n-1},\dots,X_1}(x_n|x_{n-1},\dots,x_1) \end{aligned} \tag{16}$$

C. Optimal Polling Strategies

Assume that exactly N polling steps are allowed. That is, after $N-1$ unsuccessful polling steps we poll all locations in step N. We may then rewrite equations (4), (5) and (6) as

$$E[P] = \sum_{n=1}^{N} k_n \bar{F}_q(n-1) \tag{17}$$

$$E[D] = \sum_{n=1}^{N} \bar{F}_q(n-1) \tag{18}$$

$$G = \sum_{n=1}^{N} \bar{F}_q(n-1)(k_n + \alpha) \tag{19}$$

Rewriting Equation (16) yields

$$\begin{aligned} \bar{F}_q(n) = & \sum_{x_n \notin A_n} \sum_{x_{n-1} \notin A_{n-1}} \dots \\ & \sum_{x_1 \notin A_1} p_{X_n|X_{n-1},\dots,X_1}(x_n|x_{n-1},\dots,x_1)\dots p_{X_1}(x_1) \end{aligned} \tag{20}$$

which by Equation 8 is equal to

$$\begin{aligned} \bar{F}_q(n) = & \sum_{x_n \notin A_n} \sum_{x_{n-1} \notin A_{n-1}} \dots \\ & \sum_{x_1 \notin A_1} p_{X_n,X_{n-1},\dots,X_1}(x_n,x_{n-1},\dots,x_1) \end{aligned} \tag{21}$$

Thus, we can write

$$\bar{F}_q(n) = \sum_{x_n \notin A_n} p_{X_n \bar{B}_{n-1}\dots\bar{B}_1}(x_n \bar{B}_{n-1}\dots\bar{B}_1) \tag{22}$$

where $\bar{B}_i$ is the event that the unit is not found by polling step i. We then notice that since $\text{Prob}(\bar{B}_{n-1}\dots\bar{B}_1) = \bar{F}_q(n-1)$, we must have

$$\begin{aligned} p_{X_n \bar{B}_{n-1}\dots\bar{B}_1}(x_n \bar{B}_{n-1}\dots\bar{B}_1) = & \, p_{X_n|\bar{B}_{n-1}\dots\bar{B}_1}(x_n|\bar{B}_{n-1}\dots\bar{B}_1) \cdot \\ & \bar{F}_q(n-1) \end{aligned} \tag{23}$$

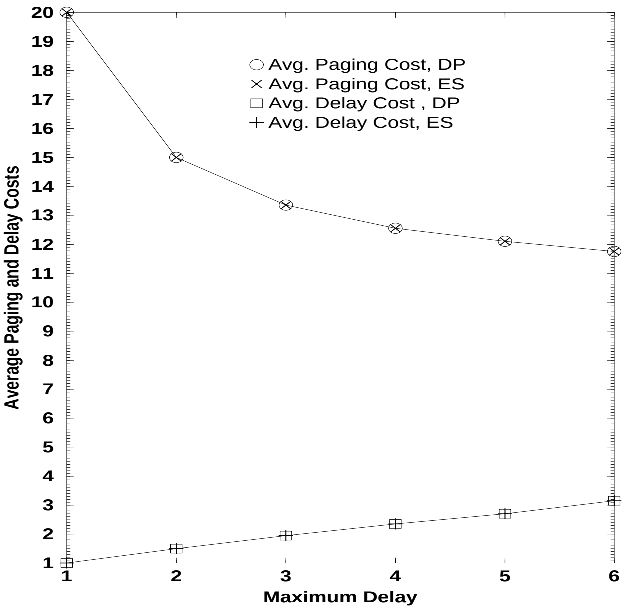

Fig. 1. Optimum paging and delay costs corresponding to the optimum strategy with $\alpha = 0.5$ vs maximum delay for a stationary unit, 20 locations, uniform initial location distribution. DP=Dynamic Programming, ES=Exhaustive Search.

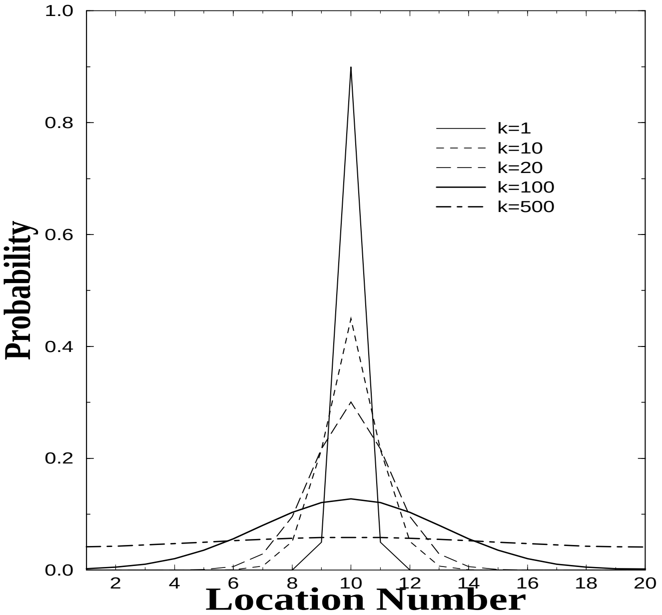

Fig. 2. Probability distribution "bloom" from a known location as a function of motion process steps (inter-polling interval) k.

by Bayes Rule. Thus,

$$\bar{F}_q(n) = \bar{F}_q(n-1) \sum_{x_n \notin A_n} p_{X_n|\bar{B}_{n-1}...\bar{B}_1}(x_n|\bar{B}_{n-1}...\bar{B}_1) \quad (24)$$

Theorem 1: The polling sequence which minimizes Equation (17), (18) or (19) polls the most probable k_{i+1} locations after the i^{th} polling failure.

Proof: If polling groups $A_1...A_{N-1}$ are given, then $\bar{F}_q(i)$ is specified for $i = 1, 2, ..., N-1$. Since $k_N = L$, the total number of locations, all locations have to be polled. If polling groups $A_1...A_{N-2}$ are given, then $\bar{F}_q(i)$ is specified for $i = 1, 2, ..., N-2$. Given k_{N-1}, minimizing any of the cost functions (17) through (19) requires us to choose the elements of A_{N-1} such that $\bar{F}_q(N-1)$ is minimized. However, $\bar{F}_q(N-1)$ is minimized by choosing the most likely elements of $p_{X_{N-1}|\bar{B}_{N-2}...\bar{B}_1}(x_{N-1}|\bar{B}_{N-2}...\bar{B}_1)$ for A_{N-1}.

Proceeding recursively, we see that for given polling group sizes k_n, the optimal polling group A_n contains the k_n most likely elements of the distribution $p_{X_n|\bar{B}_{n-1}...\bar{B}_1}(x_n|\bar{B}_{n-1}...\bar{B}_1)$. ■

Thus, as explained in Section III-A, the problem of finding N optimal polling groups reduces to one of finding optimal polling group sizes k_n. In the following section, we will adopt a system model and a mobility model for the unit to be found, present the optimum polling strategies by calculating the polling group sizes and investigate the effect of delay constraints and user mobility on these strategies.

IV. Results

We consider the polling problem with a maximum delay constraint. That is, if the mobile is not found in the first $N-1$ paging steps then we poll the entire set of locations in step N.

Unfortunately, unlike the case where the unit is assumed to be stationary [8, 10, 11], finding the polling sizes at each polling step is not easily solvable via Dynamic Programming. This complication is due to the fact that because the optimum locations to be polled are obtained via the probability distributions conditioned on all previous failures, the state space of the problem grows geometrically with number of polling steps. Thus, we resort to determining the optimum polling sizes via exhaustive search. For the rest of this section, the various cost values we plot are the costs that correspond to the optimum polling sizes calculated.

A network of 20 locations is assumed with an initially uniform distribution on unit location probability. As a check against previous work [8], the algorithm is first applied to the case where the unit to be found remains stationary during the paging process. Using $\alpha = 0.5$ in equation (19), we compare previous results with the results of exhaustive search in Figure 1 and find they agree.

We then allow the unit to move between the paging events. As a simple example we consider a "race track" model where a unit moves according to a one dimensional discrete isotropic diffusion process, with probability transition matrix $\mathbf{S} = \{s_{ij}\}$, on a circular track with L locations [16]. Specifically,

$$\begin{aligned} s_{ij} &= p_{X_m|X_{m-1}}(x_m = i|x_{m-1} = j) \\ &= \begin{cases} \gamma & \text{if } i = \text{mod}_L(j-1) \text{ or } \text{mod}_L(j+1) \\ 1-2\gamma & \text{if } j = i \\ 0 & \text{else} \end{cases} \end{aligned}$$

where γ is chosen to be 0.05. That is, the unit may move to the right or the left neighboring location with probability 0.05, or remain stationary with probability 0.9 at each discrete time instant. The number of motion process steps between polling events is defined as the *inter-polling interval*, k ($k \geq 1$). Given polling failures up to the n^{th} polling step, the location distribution at the $n+1^{\text{st}}$ polling step becomes[2]:

$$\mathbf{p}_{X_{n+1}|\bar{B}_n \ldots \bar{B}_1} = \mathbf{S}^k \mathbf{p}_{X_n|\bar{B}_n \ldots \bar{B}_1} \tag{25}$$

As such, k is a surrogate for unit mobility with increased k implying higher mobility. For k sufficiently large, the probability distribution on location is essentially uniform just prior to a polling event.

Representative plots of the location distribution after different numbers of steps k are provided in Figure 2 for $L = 20$ assuming the unit starts at location 10. These plots provide some indication of location probability "bloom" into previously searched locations as a function of inter-polling interval duration k.

The effect of mobility on paging cost and delay was investigated for different delay weights (α). The variation of the optimum average paging cost ($E[P]$) and average delay ($E[D]$) are shown in Figure 3 and Figure 4 for $\alpha = 0$ and $\alpha = 0.5$ for a number of maximum delay constraints. Paging cost increases with increased inter-polling interval (increased mobility) and decreases with increased maximum delay tolerance. These results are intuitively pleasing, since when the inter-polling time increases, the user location before each polling event is less certain and one would expect paging cost to increase. Likewise, if greater maximum delay may be tolerated, the paging cost can be significantly reduced since more latitude is available to employ sequential search methods [8,9]. By comparing the delay performances of Figure 3 and Figure 4, we see that the average delay performance is improved for $\alpha = 0.5$ (Figure 4). This is due to the fact that for Figure 3, the cost function consists only of the average paging cost and thus the strategy has no control over the average delay performance whereas for Figure 4, the average delay is incorporated into the cost by the delay weight factor $\alpha = 0.5$. Note also that, because the solution space of the optimization problem is discrete, the optimal polling sizes found that minimize the paging cost may not yield monotonic delay performance (Figure 3).

In Figure 5 we plot the optimum $E[P]$ versus $E[D]$ where each point is parameterized in the maximum delay; i.e. $(E[D]_n, E[P]_n)$ with $n = 1, 2 \ldots 6$. The inter-polling interval is $k = 20$. Note that large decreases in paging cost are achievable with only modest increases in average delay.

Experiments were performed using a maximum delay tolerance of 6 for a large range of inter-polling intervals (2 to 801) to investigate the limiting behavior of the strategies and cost function values. The cost function values are shown in Figure 6. The corresponding optimum strategies, i.e. how many most likely locations should be polled at each polling instant for a given inter-polling interval k, $k = 2, \ldots, 801$, are tabulated in Table I. Notice that as the inter-polling interval becomes very large the optimum strategy approaches the classical "blanket polling" strategy where all locations are polled simultaneously in the first step. This result stems from the observation that when the inter-polling interval is very large, the unit location

[2] $\mathbf{p}$ is the L-vector that contains the conditional probabilities.

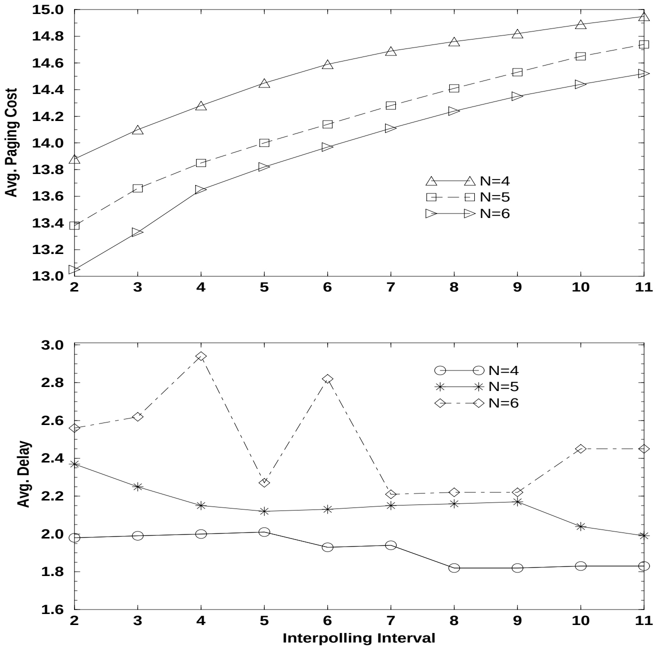

Fig. 3. Optimum paging cost and the corresponding average delay versus inter-polling interval, 20 locations, $\alpha = 0$, uniform initial location distribution, race track location model.

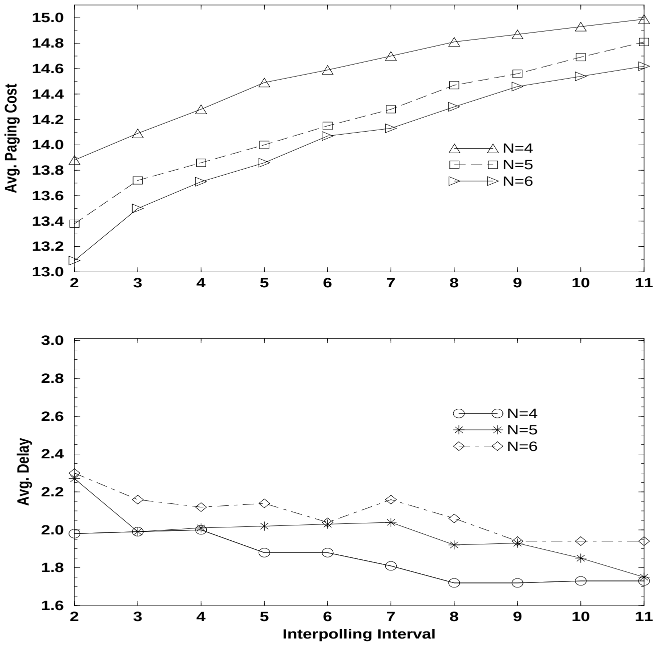

Fig. 4. Paging and delay costs corresponding to the optimum total cost with $\alpha = 0.5$ versus inter-polling interval, 20 locations, uniform initial location distribution, race track location model.

probability distribution is uniform just prior to a polling event. To show how the paging cost can be expressed in this case, let us define r_n to be the probability that the unit is found in step n given that it was not found in the first $n-1$ steps. r_n can be easily expressed in terms of the unconditional probability q_n of Section II as:

$$r_n = \frac{q_n}{1 - \sum_{i=1}^{n-1} q_i} = \frac{q_n}{\bar{F}_q(n-1)} \tag{26}$$

Now, observe that when the unit location distribution prior to a polling event is uniform, finding the unit at any loca-

IPI	*St. 1*	*St. 2*	*St. 3*	*St. 4*	*St. 5*	*St. 6*
2	6	6	5	3	7	20
3	7	6	5	4	7	20
4	7	6	6	4	8	20
5	7	6	6	4	8	20
6	7	7	6	6	8	20
7	7	6	6	5	9	20
8	8	7	3	4	9	20
9	8	7	6	8	8	20
10	8	7	6	8	9	20
11	8	7	6	9	11	20
101	8	10	10	11	10	20
251	8	10	10	10	10	20
401	10	10	10	11	11	20
451	11	11	11	11	11	20
501	12	10	11	11	11	20
601	16	12	12	12	12	20
651	20	0	0	0	0	0
701	20	0	0	0	0	0
801	20	0	0	0	0	0

TABLE I

BEST POLLING STRATEGIES FOR MAXIMUM DELAY=6. THE TABLE SHOWS HOW MANY LOCATIONS SHOULD BE POLLED AT EACH TIME STEP FOR VARIOUS INTER-POLLING INTERVAL VALUES (IPI).

tion is equally probable. Thus,

$$r_n = \frac{k_n}{L} \tag{27}$$

where k_n is the number of places searched at step n and L is the total number of locations. Combining equations (26) and (27), we see that:

$$k_n \bar{F}_q(n-1) = L q_n \tag{28}$$

Inserting equation (28) into equation (17), we see that equation (19) becomes:

$$G = \sum_{n=1}^{N} k_n \bar{F}_q(n-1) + \alpha \sum_{n=1}^{N} \bar{F}_q(n-1) \tag{29}$$

$$G = L \sum_{n=1}^{N} q_n + \alpha \sum_{n=1}^{N} \bar{F}_q(n-1) \tag{30}$$

$$G = L + \alpha E[D] \tag{31}$$

The paging cost is therefore independent of the polling pattern and always equal to the total number of locations[3] L. Thus, minimizing G corresponds to minimizing the average delay and the delay is minimized when all locations are polled on the first step: the classical paging strategy.

For smaller inter-polling intervals we observe a characteristic decrease then increase in the number of locations

[3]The above analysis implicitly assumes that the number of places the unit can reside is finite.

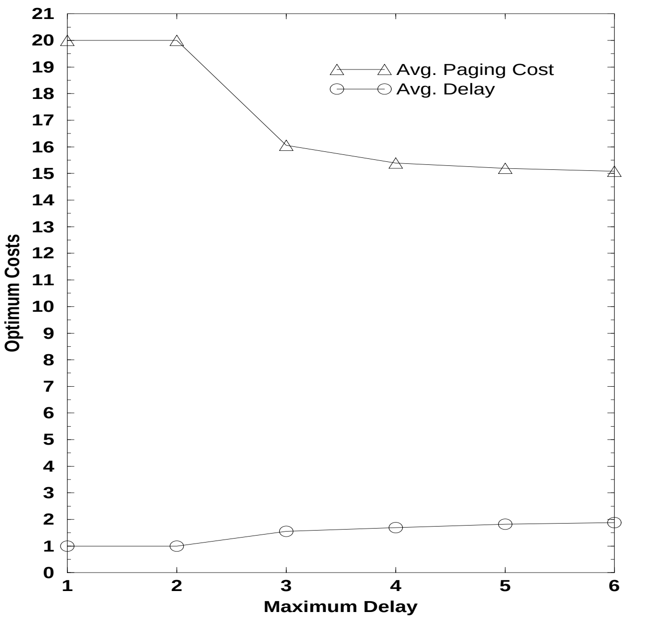

Fig. 5. Optimum paging cost and the corresponding average delay versus maximum delay, $\alpha = 0.5$, $k = 20$, 20 locations, uniform initial location distribution, race track location model.

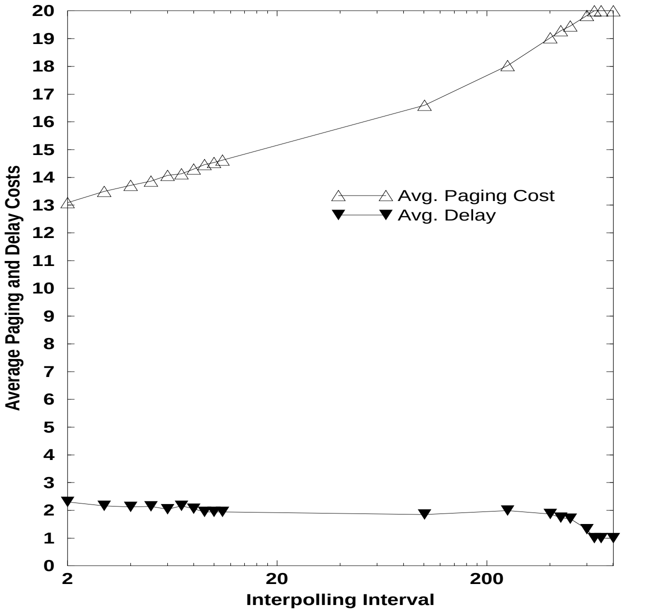

Fig. 6. Optimum paging and delay costs with $\alpha = 0.5$ versus inter-polling interval (2-801), 20 locations, uniform initial location distribution, race track location model.

polled. This is as expected since small inter-polling intervals imply that a polling failure in group A_i greatly reduces the immediately subsequent probability of finding the unit at any location covered by A_i. However, as the number of polling failures increases, more locations are searched to avoid the penalty of searching all locations in the final step.

Finally, we have studied the effect of delay factor α on the cost and the optimal strategies. The values of $E[P]$ and $E[D]$ are given in Figure 7. The values show that the paging cost is lowered substantially with very little increase in the average delay compared to the classical polling strategy. The best polling strategies for various α are provided in Table II. For increasing α a premium is placed on finding the unit early during the paging process. Thus, the

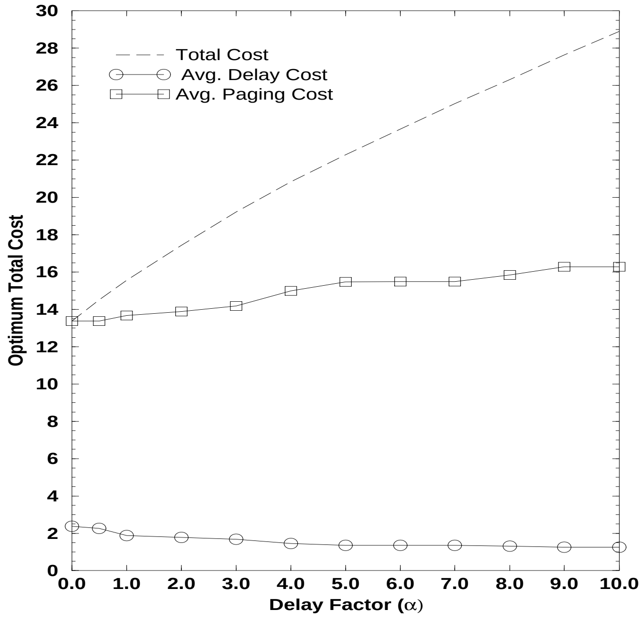

Fig. 7. Optimum costs versus delay factor α for delay tolerance 5, 20 locations, inter-polling interval=2, uniform initial location distribution, race track location model.

α	*St. 1*	*St. 2*	*St. 3*	*St. 4*	*St. 5*
0.0	5	6	6	6	20
0.5	6	6	5	6	20
1.0	8	7	5	4	20
2.0	9	7	4	6	20
3.0	10	7	3	6	20
4.0	12	7	3	4	20
5.0	13	7	2	6	20
6.0	13	7	3	5	20
7.0	13	7	3	5	20
8.0	14	6	3	5	20
9.0	15	5	4	6	20
10.0	15	5	4	6	20
100.0	20	0	0	0	0

TABLE II

Best polling strategies for maximum delay=5. The table shows how many locations should be polled at each time step as the delay factor (α) in the cost function increases.

size of the first group increases monotonically with α until all locations are searched on the first step. This strategy achieves an absolute minimum delay of one polling step.

V. Summary and Conclusion

We have investigated optimum paging strategies for a mobile communications network where units are allowed to move during the paging process. It is shown that the strategy which minimizes the average number of paging events, the average paging delay or a linear combination of the two must search the conditionally most likely locations after each polling failure. Therefore, the problem of finding the best strategies is reduced to finding how many locations to include in each polling group.

The problem of finding the optimal polling group sizes is, unfortunately, not amenable to standard methods such as Dynamic Programming. Thus, we have used exhaustive search to determine the best strategy and derive heuristic/approximate principles from experiment.

We observed that as the unit mobility (time interval between polling events) increases, the number of locations searched early-on in the paging process must increase until it becomes optimal to poll all locations during the first step; the classical "blanket polling" strategy for mobile communications systems. However, this effect occurs only at relatively extreme levels of mobility where a failed polling event provides little information about unit location just prior to the next scheduled polling event. Thus, there should usually be some benefit to sequential planned polling of location groups in all but the most extreme cases.

The effect of increasing the importance of delay reduction had the expected effect of lowering delay at the expense of increased paging cost. However, the optimal strategies lowered the paging cost substantially as compared to the classical strategy but with little increase in the average delay. Thus, it may often be more efficient to sacrifice a small amount of delay performance and thereby gain a substantial reduction in paging cost.

Finally, we note that our observations came from the application of our theory to a one dimensional network where the unit to be found moved according to a discrete diffusion process. The extension of the model to two dimensions is straightforward and we believe that the general observations given in this paper would not change drastically by this extension. Qualitatively, one can argue that as the inter-polling interval (k) gets larger, the effective user mobility will be higher than that of the one-dimensional case since diffusion would be in two dimensions. Thus, the convergence to the limiting case, i.e. the *blanket polling* strategy would be faster than that in one-dimensional mobility model. However, for a reasonable range of mobility indices sequential paging should still be useful in minimizing signaling cost.

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