Interference Avoidance and Collaborative Multibase Systems, Part I: greedy SINR maximization and sum capacity

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Abstract

We consider distributed iterative interference avoidance algorithms for uplink wireless systems where base stations share information (collaborate). Though the structure of the multiple base collaborative problem is significantly different from single base and single user multiple antenna systems, we show that if users are allowed to greedily optimize their own performance (SINR or rate), then maximum sum capacity solutions result. After providing interference avoidance algorithm variants, we numerically study the improvement afforded by interference avoidance over random codewords, the speed of convergence, and how closely sum capacity bounds are approached when each user is allowed exactly one signature – a currently open problem.

Contents

1 Introduction

We consider the uplink of a wireless communication system with multiple receivers as depicted in Figure 1 and assume that the receivers are allowed to share information as was considered by Hanly [2] and indirectly by [17]. Assuming such collaboration might seem odd in the usual context of multiple base cellular communication systems where, in general, different bases don't share information. However, the availability of relatively low cost high speed terrestrial links moves the concept into the realm of possibility. In addition, the alternative of disallowing sharing of information between bases places us squarely in a murky region of network information theory (e.g. the interference channel). Thus, since collaboration is possibly practicable and also provides an upper bound of sorts (one can do no better than to jointly decode) and collaboration is often assumed [2, 15], we also make that simplifying assumption.

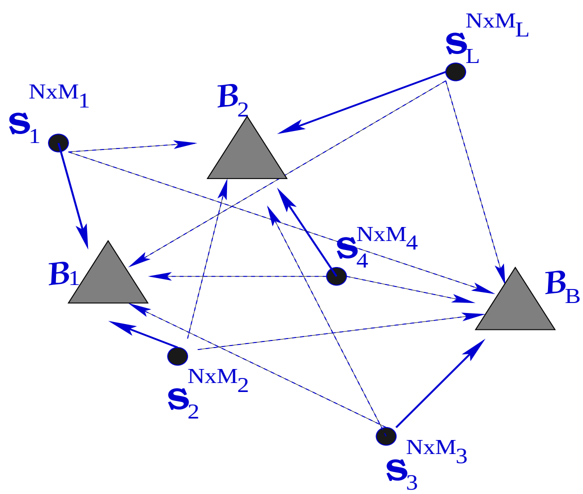

Figure 1: A multibase system with B receiving bases and L transmitting locations, each location k using M_k signatures. Triangles denote receivers and circles denote transmitters/users

We show how interference avoidance [9,13,14] which greedily improves the signal to noise/interference ratio of individual signatures employed by each user, leads to a globally optimum solution which maximizes sum capacity.

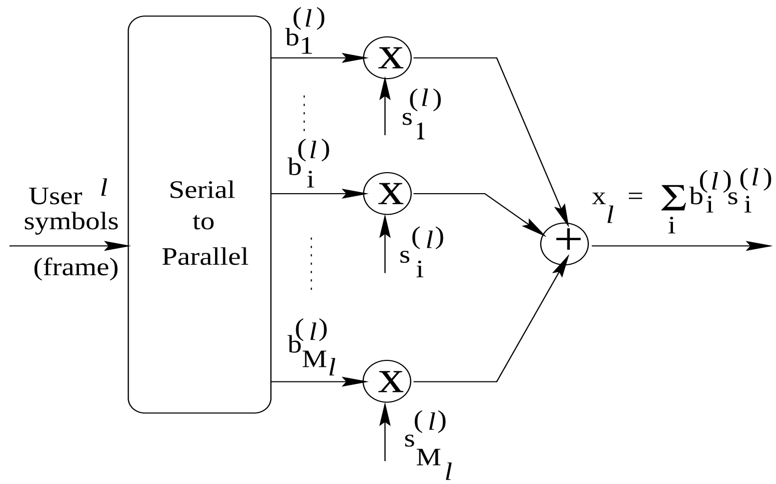

Figure 2: Multicode CDMA approach for sending frames of information. Each symbol in user k frame is assigned a distinct signature sequence (codeword) and the transmitted signal is a superposition of all codewords scaled by their corresponding information symbols. Note that the term CDMA *does not necessarily imply chip-based signatures*, but rather the spirit of using different vectors in a signal space for transmission of information.

1.1 System Description

Let B be the number of basestations and L the number of users transmitting from various locations. We assume that each user sends a "frame" of data in a given time interval using a multiple signatures for each user. That is, each symbol is transmitted using a distinct signature sequence (codeword) which spans the frame as depicted in Figure 2. We emphasize that the signature can be represented in any convenient signal space and *does not necessarily imply the usual chip-based CDMA signaling.*

Thus, associated with each user $\ell = 1, \ldots, L$ at a given transmit location is a codeword matrix

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \cdots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \tag{1}$$

of dimension $N \times M_\ell$ whose columns represent the M_ℓ codewords of user ℓ. We often assume that $M_\ell \geq N$, thus allowing each user's covariance to span the entire signal space, but this condition can be relaxed. For simplicity but with no loss of generality, we assume unit energy signature sequences. In addition, we will assume that a gain matrix $\mathbf{G}_{\ell j}$ characterizes the vector channel between transmitter ℓ and basestation j.

An M_ℓ-dimensional vector of information symbols $\mathbf{b}_\ell$ is transmitted from each location ℓ Thus, the received vector $\mathbf{r}_j$ at basestation j is the superposition of transmitted vectors by all users from all transmit locations $\ell = 1, \ldots, L$ operated on by the gain matrices $\mathbf{G}_{\ell j}$ plus additive noise $\mathbf{w}_j$ with covariance matrix $\mathbf{W}_j$

$$\mathbf{r}_j = \sum_{\ell=1}^{L} \mathbf{G}_{\ell j} \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{w}_j \tag{2}$$

Pooling the information received at all basestations we form the BN-dimensional received vector

$$\mathbf{r}^\top = \begin{bmatrix} \mathbf{r}_1^\top & \mathbf{r}_2^\top & \ldots & \mathbf{r}_B^\top \end{bmatrix} \tag{3}$$

with correlation matrix

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \sum_{\ell=1}^{L} \mathbf{R}(\ell) + \mathbf{W} \tag{4}$$

where $\mathbf{R}(\ell)$ represents the user ℓ contribution – written in terms of its codeword matrix $\mathbf{S}_\ell$ and gain matrix $\mathbf{G}_\ell$ as

$$\mathbf{R}(\ell) = \mathbf{G}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{G}_\ell^\top \tag{5}$$

with

$$\mathbf{G}_\ell = \begin{bmatrix} \mathbf{G}_{\ell 1} \\ \vdots \\ \mathbf{G}_{\ell B} \end{bmatrix} \tag{6}$$

and $\mathbf{W}$ the overall noise covariance. We therefore have

$$\mathbf{R} = \sum_{\ell=1}^{L} \mathbf{G}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{G}_\ell^\top + \mathbf{W} \tag{7}$$

We will consider two potential sources of noise: 1) independent thermal receiver noise with covariances $\{\boldsymbol{\mathcal{V}}_i\}$, and 2) noise from random emitters (possibly associated with other systems) at discrete geographic locations. If each emitter has covariance $\boldsymbol{\mathcal{W}}_n$, $n = 1, 2, \cdots, \mu$, then the total noise covariance is

$$\mathbf{W} = \begin{bmatrix} \boldsymbol{\mathcal{V}}_1 & & & \\ & \boldsymbol{\mathcal{V}}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{\mathcal{V}}_B \end{bmatrix} + \sum_{n=1}^{\mu} \mathbf{H}_n \boldsymbol{\mathcal{W}}_n \mathbf{H}_n^\top \tag{8}$$

where $\mathbf{H}_n$ is the gain between emitter n and the bases, exactly as defined for users.

1.2 Sum Capacity

The capacity is defined as the maximum of the mutual information between the transmitted and received signals. The capacity region for such wireless systems has been completely characterized in [17]. Here we make the simplest assumptions, more akin to [2] than to [17]. Specifically, we do not assume scheduling over fading states (because we do not consider fading), nor do we seek to obtain optimum power assignments for each user – these are assumed fixed and given. However unlike [2], we allow for vector channels with possibly non-flat frequency responses as well as fixed colored noise (from geographically dispersed users of other systems, for instance). Regardless, we will not derive new capacity results, but rather, show that a simple distributed greedy algorithm attains the sum capacity bound.

For the covariance matrix defined in equation (7), assuming Gaussian noise and therefore Gaussian signaling by users, the sum capacity is

$$C_{\text{sum}} = \frac{1}{2} \log |\mathbf{R}| - \frac{1}{2} \log |\mathbf{W}| \tag{9}$$

as shown in [2]. Our objective is to

$$\max_{\mathbf{S}_\ell} C_{\text{sum}} \tag{10}$$

with $\mathbf{S}_\ell$ as defined in equation (1).

It is worth noting that the optimization of equation (10) is different from that pursued in [21,22]. Specifically, optimization considered in [21, 22] is

$$\max_{\mathbf{X}_\ell} \sum_\ell \mathbf{G}_\ell \mathbf{X}_\ell \mathbf{G}_\ell^\top + \mathbf{W} \tag{11}$$

with a trace constraint on $\mathbf{X}$ corresponding to the power available to user ℓ. Here we fix the modulation method for user ℓ to be a superposition of Gaussian-modulated, unit energy codewords and directly seek those codewords which will maximize sum capacity.

In purusing the optimization of equation (10) we will first develop an interference avoidance algorithm which maximizes signature SINRs at each step. We will then show that its application also increases sum capacity and in fact leads to sum capacity optimal ensembles.

2 Interference Avoidance

Interference avoidance was envisioned as a distributed method for unlicensed bands whereby users independently adjust their modulation schemes in response to ambient interference as opposed to a centralized procedure done by an omnescient receiver. Of course, the mathematics of the algorithm also lends itself to central application, so the distinction is really only important for practical application. Here we will assume distributed application which implicitly suggests that user i knows its associated channel $\mathbf{G}_i$ and in addition that each user has access to the system covariance $\mathbf{R}$ through a side-channel beacon. The receiver can adaptively track codeword variation in a manner reminiscent of adaptive equalization. Since communication is two-way and physical channels are reciprocal, it is not unreasonable to assume that both the user and the system can know the channel. More important is the rate at which the channel varies. We will assume that channel variation is slow relative the frame rate [8, 11], or if the channel variation rate is rapid that the average channel varies slowly enough for interference avoidance to be applied [10].

2.1 Greedy SINR Maximization

The basis for interference avoidance is greedy codeword adaptation by each user. As such, a natural metric for adaptation is codeword signal to noise/interference ratio (SINR). Thus, suppose we adjust codewords one at a time so that their SINR is maximized assuming minimum mean square error (MMSE) filtering at the receiver [4]. For notational simplicity we will express the covariance in terms of each codeword $\mathbf{s}_i$ and its associated gain matrix $\mathbf{G}_i$ while recognizing that some of the $\mathbf{G}_i$ will be equal under the multicode assumption. Thus, let $\mathbf{c}_i$ be the NB-dimensional filter associated with the received vector

$$\mathbf{y}_i = \mathbf{G}_i \mathbf{s}_i \tag{12}$$

The system covariance is then

$$\mathbf{R} = \sum_{i=1}^{M} \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top + \mathbf{W} = \sum_{i=1}^{M} \mathbf{y}_i \mathbf{y}_i^\top + \mathbf{W} \tag{13}$$

where M is the total number of codewords. The SINR, γ_i, for codeword $\mathbf{s}_i$ is then

$$\gamma_i = \frac{(\mathbf{c}_i^\top \mathbf{y}_i)^2}{\sum_{k\neq i}^{M} (\mathbf{c}_i^\top \mathbf{y}_k)^2 + E[(\mathbf{c}_i^\top \mathbf{n})^2]} = \frac{\mathbf{c}_i^\top \mathbf{y}_i \mathbf{y}_i^\top \mathbf{c}_i}{\mathbf{c}_i^\top \left[\mathbf{R} - \mathbf{y}_i \mathbf{y}_i^\top\right] \mathbf{c}_i} = \frac{\mathbf{c}_i^\top \mathbf{y}_i \mathbf{y}_i^\top \mathbf{c}_i}{\mathbf{c}_i^\top \mathbf{R}_i \mathbf{c}_i} \tag{14}$$

where $\mathbf{R}_i = \mathbf{R} - \mathbf{y}_i \mathbf{y}_i^\top$.

Since $\mathbf{R}_i$ is positive definite[1] we can consider an eigen-decomposition $\mathbf{R}_i = \mathbf{\Phi}_i \mathbf{\Lambda}_i \mathbf{\Phi}_i^\top$, and define a new vector $\mathbf{z}_i = \mathbf{\Lambda}_i^{1/2} \mathbf{\Phi}_i^\top \mathbf{c}_i$, such that $\mathbf{c}_i = \mathbf{\Phi} \mathbf{\Lambda}_i^{-1/2} \mathbf{z}_i$. We then have

$$\gamma_i = \frac{\mathbf{z}_i^\top \mathbf{\Lambda}_i^{-1/2} \mathbf{\Phi}_i^\top \mathbf{y}_i \mathbf{y}_i^\top \mathbf{\Phi}_i \mathbf{\Lambda}_i^{-1/2} \mathbf{z}_i}{\mathbf{z}_i^\top \mathbf{z}_i} \tag{15}$$

This is the Rayleigh quotient of a rank one matrix and it is maximized when $\mathbf{z}_i = \mathbf{\Lambda}_i^{-1/2} \mathbf{\Phi}_i^\top \mathbf{y}_i$. Thus, the SINR maximizing filter $\mathbf{c}_i$ is

$$\mathbf{c}_i = \mathbf{R}_i^{-1} \mathbf{y}_i = \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \tag{16}$$

and the associated value of the SINR is

$$\gamma_i^0 = \mathbf{y}_i^\top \mathbf{R}_i^{-1} \mathbf{y}_i = \mathbf{s}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \tag{17}$$

which is in turn maximized when $\mathbf{s}_i$ is a maximum eigenvalue eigenvector (maximum eigenvector for short) of $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$.

Now, the SINR associated with codeword $\mathbf{s}_i$ before replacement is

$$\gamma_i^0 = \mathbf{s}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \tag{18}$$

and after replacement we have

$$\gamma_i = \mathbf{x}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_i = \gamma_i^{max} \geq \gamma_i^0 \tag{19}$$

thereby increasing the SINR of codeword $\mathbf{s}_i$. Now suppose we repeat this process iteratively for each codeword. Does it converge? To answer this question, we first seek a Lyapunov function [16]

[1] $\mathbf{W}$ is assumed positive definite, otherwise capacity is infinite.

for the iteration. That is, if we can show that each iteration monotonically increases/decreases some global and bounded metric associated with the codeword ensemble, then the procedure must converge – though convergence to a unique fixed point is not necessarily assured.

To this end, consider the determinant of $\mathbf{R}$ which we write in terms of codeword $\mathbf{s}_i$ as

$$|\mathbf{R}| = \left|\sum_{k\neq i}^{M} \mathbf{G}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{G}_k^\top + \mathbf{W} + \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top\right| = \left|\mathbf{R}_i + \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top\right| \tag{20}$$

$\mathbf{R}_i$ is invertible so that

$$\mathbf{R} = \mathbf{R}_i \left[\mathbf{I}_{BN} + \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top\right] \tag{21}$$

Thus,

$$|\mathbf{R}| = |\mathbf{R}_i| \left|\mathbf{I}_{BN} + \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i \mathbf{s}_i^\top \mathbf{G}_i^\top\right| = |\mathbf{R}_i| \left(1 + \mathbf{s}_i^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i\right) \tag{22}$$

where the last equality follows from

$$|\mathbf{I}_k + \mathbf{AB}| = |\mathbf{I}_m + \mathbf{BA}|\ , \qquad \mathbf{A} \in M_{k\times m} \qquad \mathbf{B} \in M_{m\times k} \tag{23}$$

From equation (19) we obtain

$$|\mathbf{R}| = |\mathbf{R}_i| \left(1 + \gamma_i\right) \tag{24}$$

so that each iteration increases $|\mathbf{R}|$. We then note that the determinant of any matrix with bounded elements (such as $\mathbf{R}$) is bounded in magnitude. Thus, since $\mathbf{R}_i$ does not depend on $\mathbf{s}_i$, the determinant is increased after each codeword replacement and the determinant is bounded from above, the procedure is guaranteed to converge. That is, under interference avoidance $|\mathbf{R}|$ must converge to some value.

However, we have not shown that the codewords converge, nor illuminated properties of fixed points from a communications perspective other than that they comprise a Nash [5,18] equilibrium where greedy adjustment of codewords cannot obtain better performance for that codeword. We explore these convergence and fixed point issues the following sections.

2.2 "In Class" Convergence of Codeword Ensembles

Codeword convergence proofs for distributed interference avoidance [1,8,13] have proven strangely difficult in light of the numerical robustness of the algorithm in practice [7–9, 11, 14]. Here we modify the approach taken in [13] and show that a slightly different greedy algorithm (**Greedy+**) causes codewords to converge to eigenvectors of their respective interference plus noise covariance matrice $\mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i$. Note that we do not prove convergence to particular codewords, but to a *class* of codeword ensemble.

We first define an informal version of **Greedy+** interference avoidance:

Greedy+ MMSE Algorithm

1. Find the codeword over all users whose replacement SINR less its original SINR is maximized.
2. Replace this codeword with the maximum eigenvector of its associated $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$
3. Repeat

We then state the main theorem:

Theorem 1 *Codewords must converge to ensembles where each codeword* $\mathbf{s}_i$ *is an eigenvector of the matrix* $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$ *under the* **Greedy+** *MMSE algorithm.*

Proof: Theorem 1 First we define an *iteration* κ as a greedy interference avoidance step where a codeword $\mathbf{s}_i(\kappa)$ is replaced and note that the determinant of $\mathbf{R}$ after replacement of codeword $\mathbf{s}_i(\kappa)$ is

$$|\mathbf{R}_{\kappa+1}| = |\mathbf{R}_\kappa| \frac{1+\gamma_i(\kappa+1)}{1+\gamma_i(\kappa)} \tag{25}$$

where

$$\gamma_i(\kappa) = \mathbf{s}_i^\top(\kappa) \mathbf{G}_i^\top \mathbf{R}_i^{-1}(\kappa) \mathbf{G}_i \mathbf{s}_i(\kappa) \tag{26}$$

and

$$\gamma_i(\kappa+1) = \mathbf{x}_i^\top(\kappa) \mathbf{G}_i^\top \mathbf{R}_i^{-1}(\kappa) \mathbf{G}_i \mathbf{x}_i(\kappa) \tag{27}$$

with $\mathbf{x}_i^\top(\kappa)$ the maximum eigenvector of $\mathbf{G}_i^\top \mathbf{R}_i^{-1}(\kappa) \mathbf{G}_i$.

We then define

$$\Delta_i(\kappa) = \gamma_i(\kappa+1) - \gamma_i(\kappa) \geq 0 \tag{28}$$

and then

$$\Delta_{i_\kappa}(\kappa) = \max_i \left\{ \mathbf{x}_i^\top(\kappa) \mathbf{G}_i^\top \mathbf{R}_i^{-1}(\kappa) \mathbf{G}_i \mathbf{x}_i(\kappa) - \mathbf{s}_i^\top(\kappa) \mathbf{G}_i^\top \mathbf{R}_i^{-1}(\kappa) \mathbf{G}_i \mathbf{s}_i(\kappa) \right\} \tag{29}$$

with

$$i_\kappa = \text{argmax}_i \left\{ \mathbf{x}_i^\top(\kappa)\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i\mathbf{x}_i(\kappa) - \mathbf{s}_i^\top(\kappa)\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa) \right\} \tag{30}$$

That is, i_κ is the index of the codeword at iteration κ which will result in the maximum SINR difference before and after replacement.

Owing to the convergence of $|\mathbf{R}_\kappa|$ with interference avoidance, we must have

$$\lim_{\kappa\to\infty} \frac{1+\gamma_{i_\kappa}(\kappa+1)}{1+\gamma_{i_\kappa}(\kappa)} = 1 \tag{31}$$

which in turn implies that

$$\lim_{\kappa\to\infty} \frac{\gamma_{i_\kappa}(\kappa+1)-\gamma_i(\kappa_\kappa)}{1+\gamma_{i_\kappa}(\kappa)} = 0 \tag{32}$$

Since the eigenvalues of any matrix with bounded elements are bounded and the γ_i are strictly positive, we must have $0 < \gamma_{i_\kappa}(\kappa) \leq \Gamma\ \forall i_\kappa, \kappa$ where Γ is some suitably large constant. Therefore we also have

$$\lim_{\kappa\to\infty} \gamma_{i_\kappa}(\kappa+1) - \gamma_{i_\kappa}(\kappa) = \lim_{\kappa\to\infty} \Delta_{i_\kappa}(\kappa) = 0 \tag{33}$$

And since $\Delta_{i_\kappa}(\kappa) \geq \Delta_i(\kappa)$ we must also have

$$\lim_{\kappa\to\infty} \gamma_i(\kappa+1) - \gamma_i(\kappa) = \lim_{\kappa\to\infty} \Delta_i(\kappa) = 0 \tag{34}$$

for any *potential* replacement of codeword i at iteration κ.

Now consider that the SINR difference $\Delta_i(\kappa)$ before and after the *potential* replacement of *any* codeword $\mathbf{s}_i$ at iteration κ can be written as

$$\Delta_{i_\kappa}(\kappa) \geq \Delta_i(\kappa) = \mathbf{x}_i(\kappa)^\top\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i\mathbf{x}_i(\kappa) - \mathbf{s}_i^\top(\kappa)\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa) \geq 0 \tag{35}$$

We define the eigenvalues of $\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i$ as $\{\lambda_{ij}(\kappa)\}$, $j = 1, 2, \cdots, N$ and assume that they are ordered from largest to smallest. If we further define the corresponding eigenvectors as $\boldsymbol{\phi}_{ij}(\kappa)$, $j = 1, 2, \cdots, N$ we can rewrite $\mathbf{s}_i(\kappa)$ as

$$\mathbf{s}_i(\kappa) = \sum_{j=1}^{N} \alpha_{ij}(\kappa)\boldsymbol{\phi}_{ij}(\kappa) \tag{36}$$

where we assume

$$\sum_{j=1}^{N} \alpha_{ij}^2(\kappa) = |\mathbf{s}_i(\kappa)|^2 = |\mathbf{x}_i(\kappa)|^2 = 1 \tag{37}$$

This leads to

$$\Delta_i(\kappa) = \sum_{j=1}^{N} \alpha_{ij}^2(\kappa)(\lambda_{i1}(\kappa) - \lambda_{ij}(\kappa)) \tag{38}$$

Since all terms in the sum are non-negative we must have

$$\Delta_i(\kappa) \geq \alpha_{ij}^2(\kappa)(\lambda_{i1}(\kappa) - \lambda_{ij}(\kappa)) \tag{39}$$

for $j = 1, 2, \cdots, N$. Now suppose via equation (39) we define $\epsilon_{ij}(\kappa) \leq \Delta_i(\kappa)$ as

$$\epsilon_{ij}(\kappa) = \alpha_{ij}^2(\kappa)(\lambda_{i1}(\kappa) - \lambda_{ij}(\kappa)) \tag{40}$$

Dividing by nonzero $\alpha_{ij}(\kappa)$ results in

$$\frac{\epsilon_{ij}(\kappa)}{\alpha_{ij}(\kappa)} = \alpha_{ij}(\kappa)\lambda_{i1}(\kappa) - \alpha_{ij}(\kappa)\lambda_{ij}(\kappa) \tag{41}$$

We wish to see how closely each $\mathbf{s}_i(\kappa)$ approximates an eigenvector of

$$\mathbf{G}_i^\top \mathbf{R}^{-1}(\kappa)\mathbf{G}_i = \mathbf{G}_i^\top \left[\mathbf{R}_i(\kappa) + \mathbf{G}_i\mathbf{s}_i(\kappa)\mathbf{s}_i^\top(\kappa)\mathbf{G}_i^\top\right]^{-1} \mathbf{G}_i \tag{42}$$

To this end we rewrite

$$\mathbf{R}^{-1}(\kappa) = \mathbf{R}_i^{-1}(\kappa) - \frac{\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa)\mathbf{s}_i^\top(\kappa)\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)}{1 + \mathbf{s}_i(\kappa)^\top\mathbf{G}_i^\top\mathbf{R}_i^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa)} \tag{43}$$

and at iteration κ, we form the product

$$\begin{aligned} \mathbf{G}_i^\top \mathbf{R}^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa) &= \sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}(\kappa)\lambda_{ij}(\kappa)\boldsymbol{\phi}_{ij}(\kappa) - \frac{\sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}^2(\kappa)\lambda_{ij}(\kappa)}{1+\sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}^2(\kappa)\lambda_{ij}(\kappa)} \sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}(\kappa)\lambda_{ij}(\kappa)\boldsymbol{\phi}_{ij}(\kappa) \\ &= \frac{1}{1+\sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}^2(\kappa)\lambda_{ij}(\kappa)} \sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}(\kappa)\lambda_{ij}(\kappa)\boldsymbol{\phi}_{ij}(\kappa) \end{aligned} \tag{44}$$

where $\mathbf{J}_i(\kappa)$ is the set of all j such that $\alpha_{ij}(\kappa) \neq 0$. Using equation (41) in equation (44) yields

$$\mathbf{G}_i^\top \mathbf{R}^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa) = \frac{1}{1 + \sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}^2(\kappa)\lambda_{ij}(\kappa)} \sum_{j\in\mathbf{J}_i(\kappa)} \left(\lambda_{i1}(\kappa)\alpha_{ij}(\kappa) - \frac{\epsilon_{ij}(\kappa)}{\alpha_{ij}(\kappa)}\right) \boldsymbol{\phi}_{ij}(\kappa) \tag{45}$$

Regrouping we have

$$\mathbf{G}_i^\top \mathbf{R}^{-1}(\kappa)\mathbf{G}_i\mathbf{s}_i(\kappa) = \frac{1}{1 + \sum_{j\in\mathbf{J}_i(\kappa)} \alpha_{ij}^2(\kappa)\lambda_{ij}(\kappa)} \left[\lambda_{i1}(\kappa)\mathbf{s}_i(\kappa) - \sum_{j\in\mathbf{J}_i(\kappa)} \frac{\epsilon_{ij}(\kappa)}{\alpha_{ij}(\kappa)}\boldsymbol{\phi}_{ij}(\kappa)\right] \tag{46}$$

However, since $\lim_{\kappa\to\infty}\Delta_i(\kappa) = 0$ and $\alpha_{ij}^2(\kappa) < 1$, then for any $\lim_{\kappa\to\infty}\alpha_{ij}(\kappa) \neq 0$ we must have by equation (40)

$$\lim_{\kappa\to\infty}\lambda_{i1}(\kappa) - \lambda_{ij}(\kappa) = 0 \tag{47}$$

Therefore, we have

$$\lim_{\kappa\to\infty}\frac{\epsilon_{ij}(\kappa)}{\alpha_{ij}(\kappa)} = \lim_{\kappa\to\infty}\alpha_{ij}(\kappa)\left(\lambda_{i1}(\kappa) - \lambda_{ij}(\kappa)\right) = 0 \tag{48}$$

for any $\alpha_{ij}(\kappa)$ which does not approach zero. Thus, $\exists\kappa$ such that the terms $\epsilon_{ij}(\kappa)/\alpha_{ij}(\kappa)$ are arbitrarily small. Since $\Delta_{i_\kappa}(\kappa) \geq \Delta_i(\kappa)$, this implies that for suitably large κ, all codewords $\mathbf{s}_i(\kappa)$ are *simultaneously* arbitrarily close to being eigenvectors of $\mathbf{G}_i^\top\mathbf{R}^{-1}(\kappa)\mathbf{G}_i$, thus completing the proof.

We define such codeword convergence as *convergence in class*. •

We can now formalize the greedy interference avoidance algorithm as

Formal Greedy+ Algorithm:

1. Find the codeword over all users whose replacement SINR less its original SINR is maximized.

2. Replace this codeword with the maximum eigenvector of its associated $\mathbf{G}_i^\top\mathbf{R}_i^{-1}\mathbf{G}_i$

3. Repeat steps 1 and 2 until within some suitable tolerance of an "in class" fixed point

4. If trapped at suboptimal fixed point (see section 2.4), then apply escape procedures [13] and go to step 1.

We must note that in numerical studies, the **Greedy+** method of replacement was not necessary. Convergence occurred regardless of codeword replacement order as well as with "lagged" algorithms where codewords, instead of being replaced, were adjusted in the direction of the optimal codeword. So though the **Greedy+** conditions are needed to *prove* convergence, in practice interference avoidance appears robust.

2.3 General Fixed Point Properties

At a fixed point, let $\{\lambda_j^i\}$ be the set of eigenvalues for matrix $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$ in decreasing order, $\gamma_i = \lambda_1^i \geq \lambda_2^i \geq \ldots \geq \lambda_N^i$, with $\{\mathbf{x}_j^i\}$ the corresponding eigenvectors. By definition we have

$$\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_j^i = \lambda_j^i \mathbf{x}_j^i \tag{49}$$

and specifically

$$\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i = \lambda_1^i \mathbf{s}_i = \gamma_i \mathbf{s}_i \tag{50}$$

We now show that any eigenvector of $\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i$ is also an eigenvector of $\mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i$. Consider that

$$\mathbf{R} = \mathbf{R}_i + \mathbf{G}_i \mathbf{x}_1^i (\mathbf{x}_1^i)^\top \mathbf{G}_i^\top$$

The matrix inversion lemma produces

$$\mathbf{R}^{-1} = \mathbf{R}_i^{-1} - \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_1^i \left(1 + (\mathbf{x}_1^i)^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{x}_1^i\right)^{-1} (\mathbf{x}_1^i)^\top \mathbf{G}_i^\top \mathbf{R}_i^{-1}$$

We then have via equation (49)

$$\mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i \mathbf{x}_j^i = \lambda_j^i \mathbf{x}_j^i - \frac{\gamma_i^2}{1+\gamma_i} \mathbf{x}_1^i (\mathbf{x}_1^i)^\top \mathbf{x}_j = \left(\lambda_j^i - \frac{(\lambda_j^i)^2}{1+\lambda_j^i} \delta_{1j}\right) \mathbf{x}_j^i$$

Thus at equilibrium, each pair of matrices $\left(\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i, \mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i\right)$ share the same set of eigenvectors. They also share the same eigenvalues, except that corresponding to $\mathbf{s}_i$:

$$\mathbf{G}_i^\top \mathbf{R}_i^{-1} \mathbf{G}_i \mathbf{s}_i = \gamma_i \tag{51}$$

and

$$\mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i \mathbf{s}_i = \frac{\gamma_i}{1+\gamma_i} \tag{52}$$

Now suppose some of the codewords share the same gain matrix as would be expected in a system employing multicode for each user. We first note that at equilibrium, such codewords must be orthogonal if they have different SINRs. Likewise we must have

$$\gamma_i \geq \frac{\gamma_j}{1+\gamma_j} \tag{53}$$

since otherwise a codeword $\mathbf{s}_i$ could increase its SINR – thereby violating the assumption of a fixed point. Thus, a single oversized SINR implies that all other codewords have SINRs close to 1.

For an ensemble of small SINRs (large numbers of codewords sharing the same gain, or high background noise), equation (53) suggests

$$\gamma_i \geq \gamma_j - \epsilon_j \tag{54}$$

where $\epsilon_j \approx \gamma_j^2$ and implies near equal SINRs. In simulations, when the number of codewords is at least as large as the signal space dimension N, we find that the algorithm produces codeword ensembles with equal SINR for codewords with common gain matrices. Interestingly, this empirical result is a necessary condition for ensemble waterfilling over some signal space and resultant capacity maximization. We extend this observation in the following section.

2.4 A Connection to Sum Capacity

We note that maximizing sum capacity implies maximizing $|\mathbf{R}|$ by equation (9). We also see that interference avoidance using an MMSE criterion maximizes the increase in sum capacity for a given codeword replacement via equation (24). So, we might suspect that interference avoidance is also a capacity optimization algorithm as well.

Thus, consider that

$$|\mathbf{R}| = \left|\mathbf{Q}_i + \mathbf{G}_i\mathbf{S}_i\mathbf{S}_i^\top\mathbf{G}_i^\top\right| \tag{55}$$

where $\mathbf{S}_i$ is a matrix of codewords with identical gain matrices $\mathbf{G}_i$ and $\mathbf{Q}_i$ is the remaining noise plus interference covariance.[2] We then have

$$|\mathbf{R}| = |\mathbf{Q}_i|\left|\mathbf{I} + \mathbf{Q}_i^{-1/2}\mathbf{G}_i\mathbf{S}_i\mathbf{S}_i^\top\mathbf{G}_i^\top\mathbf{Q}^{-1/2}\right| = |\mathbf{Q}_i|\left|\mathbf{I} + \mathbf{H}_i\mathbf{S}_i\mathbf{S}_i^\top\mathbf{H}_i^\top\right| \tag{56}$$

where $\mathbf{H} = \mathbf{Q}_i^{-1/2}\mathbf{G}_i$. We then use the usual SVD to write

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^\top = \mathbf{U}\begin{bmatrix} \mathcal{D} \\ \mathbf{0} \end{bmatrix}\mathbf{V}^\top \tag{57}$$

and

$$\bar{\mathbf{H}} = \mathbf{U}\begin{bmatrix} \mathcal{D}^{-1} \\ \mathbf{0} \end{bmatrix}\mathbf{V}^\top \tag{58}$$

[2]Note that $\mathbf{R}_i$ and $\mathbf{Q}_i$ are different matrices. $\mathbf{R}_i$ is the covariance less the contribution of *codeword* i while $\mathbf{Q}_i$ is the covariance less the contribution of *user* i.

We may then write

$$|\mathbf{R}| = |\mathbf{Q}_i| \left|\mathbf{H}^\top \mathbf{H}\right| \left|\bar{\mathbf{H}}^\top \bar{\mathbf{H}} + \mathbf{S}_i \mathbf{S}_i^\top\right| \tag{59}$$

which reduces to

$$|\mathbf{R}| = |\mathbf{Q}_i| \left|\mathbf{G}_i^\top \mathbf{Q}_i^{-1} \mathbf{G}_i\right| \left|\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1} \mathbf{G}_i\right)^{-1} + \mathbf{S}_i \mathbf{S}_i^\top\right| \tag{60}$$

The maximum determinant is obtained when $\mathbf{S}_i$ has at least N columns and $\mathbf{S}_i\mathbf{S}_i^\top$ can thereby waterfill the covariance $\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1} \mathbf{G}_i\right)^{-1}$. The requisite unit norm codewords which comprise the columns of $\mathbf{S}$ can be obtained using interference avoidance [13, 14] or with exact finite step algorithms [19]. If $\mathbf{S}_i$ has $K < N$ columns, then the determinant is maximized when $\mathbf{S}_i\mathbf{S}_i^\top$ waterfills the K-dimensional eigenspace of $\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1} \mathbf{G}_i\right)^{-1}$ having the K smallest eigenvalues. Regardless, independent of K or how maximizing codewords are obtained, the resulting fixed point has each column of $\mathbf{S}_i$ an eigenvector of $\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1} \mathbf{G}_i\right)^{-1} + \mathbf{S}_i\mathbf{S}_i^\top$, or for water level c,

$$\left[\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1} \mathbf{G}_i\right)^{-1} + \mathbf{S}_i \mathbf{S}_i^\top\right] \mathbf{S}_i = c\mathbf{S}_i \tag{61}$$

Now, note that for the MMSE algorithm at an equilibrium where codewords sharing the same gain matrix $\mathbf{G}_i$ share the same SINR we must have

$$\mathbf{G}_i^\top \mathbf{R}^{-1} \mathbf{G}_i \mathbf{S}_i = \frac{\gamma_i}{1+\gamma_i} \mathbf{S}_i \tag{62}$$

We rewrite $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{Q}_i + \mathbf{G}_i \mathbf{S}_i \mathbf{S}_i^\top \mathbf{G}_i^\top \tag{63}$$

and then

$$\mathbf{R} = \mathbf{Q}_i^{1/2} \left(\mathbf{I} + \mathbf{Q}_i^{-1/2} \mathbf{G}_i \mathbf{S}_i \mathbf{S}_i^\top \mathbf{G}_i^\top \mathbf{Q}_i^{-1/2}\right) \mathbf{Q}_i^{1/2} \tag{64}$$

With $\mathbf{H}$ defined as in equation (57) we then have

$$\mathbf{R} = \mathbf{Q}_i^{1/2} \mathbf{U} \begin{bmatrix} \mathbf{I} + \mathcal{D}\mathbf{V}^\top \mathbf{S}_i \mathbf{S}_i^\top \mathbf{V}_i \mathcal{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{U}^\top \mathbf{Q}_i^{1/2} \tag{65}$$

which implies that

$$\mathbf{R}^{-1} = \mathbf{Q}_i^{-1/2} \mathbf{U} \begin{bmatrix} \mathcal{D}^{-1} \mathbf{V}^\top \left[\left(\mathbf{V}\mathcal{D}^{-2}\mathbf{V}^\top\right) + \mathbf{S}_i \mathbf{S}_i^\top\right]^{-1} \mathbf{V}\mathcal{D}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{U}^\top \mathbf{Q}_i^{-1/2} \tag{66}$$

Again using the definition of equation (57) and applying it to equation (66) we obtain

$$\mathbf{G}_i^\top \mathbf{R}^{-1}\mathbf{G}_i = \left[\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1}\mathbf{G}_i\right)^{-1} + \mathbf{S}_i\mathbf{S}_i^\top\right]^{-1} \tag{67}$$

Combining equation (67) and equation (62) yields

$$\left[\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1}\mathbf{G}_i\right)^{-1} + \mathbf{S}_i\mathbf{S}_i^\top\right]^{-1}\mathbf{S}_i = \frac{\gamma_i}{1+\gamma_i}\mathbf{S}_i \tag{68}$$

which implies

$$\left[\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1}\mathbf{G}_i\right)^{-1} + \mathbf{S}_i\mathbf{S}_i^\top\right]\mathbf{S}_i = \frac{1+\gamma_i}{\gamma_i}\mathbf{S}_i \tag{69}$$

So, $\frac{1+\gamma_i}{\gamma_i}$ is analogous to the water level c. However, waterfilling also requires that any other eigenvalues of $\left(\mathbf{G}_i^\top \mathbf{Q}_i^{-1}\mathbf{G}_i\right)^{-1} + \mathbf{S}_i\mathbf{S}_i^\top$ be larger than c.

We then note that any set of simultaneously waterfilling covariances $\{\mathbf{S}_i\mathbf{S}_i^\top\}$, $i = 1, 2, ..., L$ in equation (61) maximizes sum capacity [21, 22]. Thus, since any set of codeword ensembles such that codewords within each ensemble i share the same SINR γ_i and any remaining eigenvalues of the corresponding $\mathbf{G}_i^\top \mathbf{R}^{-1}\mathbf{G}_i$ are less than $\frac{\gamma_i}{1+\gamma_i}$ is exactly a simultaneously waterfilled solution, any such set must also maximize sum capacity. Therefore, **Greedy+** interference avoidance, suitably augmented by suboptimal fixed point escape methods [13] when necessary, is guaranteed to produce codeword ensembles which maximize sum capacity.

Of course, when there are fewer than N_ℓ codewords for each gain matrix $\mathbf{G}_\ell$, the implied constrained optimization is currently an open problem [20]. Numerical experiments which address optimality are discussed in Section 3.

2.5 IA Via Subspace Projection

As suggested in the previous section, there are a number of ways interference avoidance could be applied. We began with a MMSE and SINR approach which was shown to be equivalent to a more direct approach using the definition of sum capacity. Here we provide a variant procedure where interference avoidance is performed in the subspace spanned by the gain matrix of the user codewords. As usual, we start with

$$\mathbf{R} = \mathbf{G}_i\mathbf{S}_i\mathbf{S}_i^\top\mathbf{G}_i^\top + \sum_{\ell\neq i}\mathbf{G}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{G}_\ell^\top + \mathbf{W} = \mathbf{G}_i\mathbf{S}_i\mathbf{S}_i^\top\mathbf{G}_i^\top + \mathbf{Q}_i \tag{70}$$

We then note that

$$\mathbf{G}_i = \mathbf{U}_i \begin{bmatrix} \mathcal{D}_i \\ 0 \end{bmatrix} \mathbf{V}_i^\top \tag{71}$$

the usual SVD and isolate the user i covariance in the subspace spanned by $\mathbf{G}_i$

$$\begin{bmatrix} \mathbf{V}_i\mathcal{D}_i^{-1}\mathbf{V}_i^\top & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{U}_i^\top \mathbf{R} \mathbf{U}_i \begin{bmatrix} \mathbf{V}_i\mathcal{D}_i^{-1}\mathbf{V}_i^\top & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \tilde{\mathbf{R}} = \begin{bmatrix} \mathbf{S}_i\mathbf{S}_i^\top & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \tilde{\mathbf{Q}}_i \tag{72}$$

We then rewrite equation (72) as

$$\tilde{\mathbf{R}} = \begin{bmatrix} \mathbf{S}_i\mathbf{S}_i^\top + \mathbf{A} & \mathbf{B}^\top \\ \mathbf{B} & \mathbf{C} \end{bmatrix} \tag{73}$$

where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are the appropriate sub-blocks of $\tilde{\mathbf{Q}}_i$. Thus, we have via Schur factoring [3]

$$|\mathbf{R}| = \frac{|\mathbf{C}|}{\left|\mathbf{V}_i\mathcal{D}_i^{-1}\mathbf{V}_i^\top\right|^2} \left|\mathbf{S}_i\mathbf{S}_i^\top + \mathbf{A} - \mathbf{B}^\top\mathbf{C}^{-1}\mathbf{B}\right| \tag{74}$$

and we can apply the usual greedy interference avoidance methods to the covariance $\mathbf{S}_i\mathbf{S}_i^\top + \mathbf{A} - \mathbf{B}^\top\mathbf{C}^{-1}\mathbf{B}$ and thereby maximize $|\mathbf{R}|$ in $\mathbf{S}_i$.

This form of interference avoidance might be a bit less complex in that it requires the inversion of the $N(B-1) \times N(B-1)$ matrix $\mathbf{C}$ instead of the $NB \times NB$ matrices $\mathbf{Q}_i$ or $\mathbf{R}_i$ required previously. Of course, the complexity (matrix multiplcations) involved in deriving $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ might offset the inversion complexity reduction.

3 Numerical Results and Discussion

3.1 Assumptions

For simplicity we assumed an OFDM-like signal space where each tone was comprised of an in phase and quadrature component. Thus, each signal space dimension corresponds to sine or cosine at some frequency. This is not the only possible decomposition and was used primarily to enable the application of results from standard propagation models.

Gain matrices $\mathbf{G}_k$ were derived by random placement of users and bases within some fixed area. The propagation envelope was perturbed by additional gain factors chosen from a zero mean

Gaussian distribution with variance σ^2. That is, the gain g between a given base and a user on a given signal space dimension was modeled as

$$g = \mathcal{G} \left(\frac{d_0}{d} \right)^{\alpha} \tag{75}$$

where the propagation constant $\alpha > 2$, d is the distance between user and base, d_0 is the minimum allowable distance (comparable to base antenna height in most formulations) and $\mathcal{G} \sim N(0, \sigma^2)$ is the usual Rayleigh fading coefficient. For our simulations we used $\sigma^2 = 2$. The $\mathcal{G}$ are assumed i.i.d.

3.2 Full Codeword Complement

We first assume as many codewords per user as there are signal space dimensions. Without the use of escape procedures [13], we have been unable to prove convergence to optimal fixed points using greedy interference avoidance. However, numerical studies with signal space dimensions ranging from 2 to 20 and different numbers of users indicate that greedy interference avoidance seems to reach the optimum from random initial codeword ensembles, without special assistance. That is, we have invariably seen that greedy interference avoidance produces ensembles where all the codewords of any given user k are eigenvectors of $\mathbf{G}_k\mathbf{R}^{-1}\mathbf{G}_k$ with identical eigenvalues $\gamma_k/(1+\gamma_k)$. Furthermore, at such fixed points, the remaining eigenvalues of $\mathbf{G}_k\mathbf{R}^{-1}\mathbf{G}_k$ are smaller than $\gamma_k/(1+\gamma_k)$ which ensures a simultaneously waterfilled solution.

Also of note, the codewords converged to within tight norm difference tolerances ($|\mathbf{s}_i(\kappa+1) - \mathbf{s}_i(\kappa)| \leq 10^{-10}$) though this is not formally predicted by the "in class" convergence proof. As might be expected, the convergence of the global metric, C_{sum}, was much more rapid (3 iteration cycles) than codeword convergence (> 10 iteration cycles).

We found that the sum capacity improvement afforded by optimal codeword ensembles can be considerable. This result is consonant with those obtained in [9] for multiuser multiple antenna systems. For $N = 6$ signal space dimensions, $B = 4$ bases and a different numbers of users with average capacity improvements from 30% to 40%. We defer discussion of results for flat channels to the companion paper [12].

3.3 One Codeword Per User

We also investigated the effect of using fewer codewords on the attainable sum capacity. That is, sum capacity optimization is an open problem when there are fewer codewords, M, than signal space dimensions, N [21,22]. We therefore performed trials where each user's power budget could be applied to only a single codeword and calculated the sum capacity achieved after application of interference avoidance. In all cases, the sum capacity value obtained for given gain matrices were identical even when starting from different randomly chosen codeword ensembles. This might indicate that interference avoidance attains the sum capacity maximum and provide an analytic path for solution of the reduced rank problem.

We also compared the sum capacity obtained using single codewords to that obtained using a full complement of codewords (under the same total power constraint). Here, an interesting trend was noted. For few users, the average penalty associated with using a single codeword was pronounced – 50% for 1 user per base with $N = 6$ dimensions and $B = 4$ bases. However, as the number of users was increased to 40 (10 users per base), the difference between attainable sum capacity and that achieved using single codewords was only 7% on average. Thus, the addition of users seems to enable the ensemble of user codewords to more exactly approximate a simultaneously waterfilled solution.

This result might not be too unexpected owing to the collective/emergent waterfilling properties of interference avoidance shown previously [8,13,14]. However, for single base dispersive systems with diagonalized gain matrices [8], users tend to segregate in the signal space and overlap with one another only in single dimensions under loose assumptions on differences between channel gain matrices. Thus, as the number of users increases, each user tends to be forced into fewer dimensions [6, 8]. Therefore, the results noted here could indicate some generalization of this rule to different gain matrix structures so that even were each user afforded a full complement of codewords, the codeword covariance obtained by interference avoidance might limit its energy to one or two dimensions.

To see which effect predominated, the energy distribution of optimal user codeword sets was examined. We found that even with a single user, the codewords naturally constrained themselves to one or two dimensions[3]. Thus, some concentration seems to be a natural consequence of the

[3] 90% of the total received energy appeared in one or two dimensions

random gain matrix structure. In fact, when more users were added, the codeword energy distribution spread out. This results suggests that the collective/emergent waterfilling associated with interference avoidance is a more likely reason for the close agreement between the sum capacity achievable with full and reduced rank codeword covariance matrices.

4 Conclusions

We have extended application of interference avoidance to systems with multiple bases which pool information and jointly decode all users. Each user iteratively and greedily adjusts its codewords to maximize SINR, and when no further improvement is possible, the codeword ensemble has attained maximum sum capacity under user transmit power constraints. In this paper we have developed the algorithm, proven convergence and shown the equivalence of optimal fixed points to simultaneously waterfilled codeword covariance matrices known to maximize sum capacity [21,22]. In addition to providing a slightly less complex realization of the algorithm we also empirically investigated convergence and invariably found that application of interference avoidance resulted in sum capacity optimal codeword ensembles for random starting ensembles. We found that significant improvement can be afforded by using optimal codeword ensembles over Rayleigh channels.

Perhaps most interesting, we also found that when users are restricted to a single codeword (so that the rank of the codeword covariance is 1), a significant penalty is paid when the number of users per base is small. However, increasing the number of users per base seems to allow the "full-codeword-complement" sum capacity bound to be closely approached. This suggests that for heavily loaded systems, at least under the Rayleigh gain assumptions used here, each user need not carry a full complement of codewords, thus reducing modulation/demodulation complexity.

In the companion paper [12] we consider the effects of channel gain matrix simplifications afforded by flat gain profiles between users and bases.

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