# Write or Radiate? Inscribed Mass vs. Electromagnetic Channels

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#### Abstract

We consider information flow via physical transport of inscribed mass through space and compare it to information flow via electromagnetic radiation. Counterintuitively, for point to point links physical transport of inscribed mass is often more energy efficient than electromagnetic broadcast by many orders of magnitude. Perhaps more surprising, in a broadcast setting inscribed mass transport may still be more energy efficient. We discuss the implications of these results for terrestrial telecommunications networks as well as point to point and broadcast communication over great distances with loose delay constraints.

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# **1** Introduction

At one time or another, every communication theorist has had the following epiphany:

Driving a truck filled with storage media (books, cd's, tapes, etc.) across town constitutes a very reliable channel with an extremely large bit rate.

Tannenbaum mentions the possibility in his communications textbook [1] and so does Rolf Landauer regarding inscription and physical transport in the context of reversible communication [2]. Similarly, Gray *et al* [3] considers the viability of information carriage through transport of mass storage media from an economic perspective.

Our epiphanies have occurred a few times over many years, but most recently with the study of short range high data rate channels [4–13] and mobility assisted wireless networks [14, 15] where communications nodes only transfer data to one another when the channel is good – typically at close range. One natural extension of this work is to not radiate electromagnetic energy at all, but rather, to have nodes physically exchange "letters" inscribed on some medium. And from such imaginings comes a simple question: when is it better to write than to radiate?

To begin, consider that one could easily pack ten 60 GByte laptop disk drives in a small box and push it across a table — with a correspondingly impressive data rate of about 4.8 Tb s<sup>-1</sup>. Without much imagination, the idea can be extended to more exotic storage media. Consider a 1 mm<sup>3</sup> "bouillon cube" containing information coded as single stranded RNA (such as the polio virus). At about 1 base per nm<sup>3</sup> [16–20], each cube could store about 1000 petabits (10<sup>18</sup> bits) of information. A 10 cm<sup>3</sup> volume of such material, if driven from New York to Boston in an automobile would constitute a rate of about 90,000 petabits s<sup>-1</sup> (9 × 10<sup>19</sup> bits s<sup>-1</sup>) — dwarfing by about six orders of magnitude the 100 terabit s<sup>-1</sup> theoretical maximum information rate over an optical fiber [21].

Next consider the mass of 1000 petabits since mass will determine the amount of energy necessary for transport. Again using the virus analogy, single stranded RNA has an average mass of about 330 kDa per kilobase. A Dalton (Da) is the molecular weight of hydrogen and is about  $1.67 \times 10^{-24}$  g [22]. The  $10^{15}$  kilobases implied by 1000 petabits would weigh 330,  $000 \times 10^{15}$  Da. Conversion to more familiar units shows the total mass of our hypothetical 1000 petabit message would be 551 µg. The *mass information density* would be

$$\tilde{\rho} = 1.82 \times 10^{24} \text{ bits kg}^{-1}$$
 (1)

which we will later see is about two orders of magnitude better than rough extrapolations based on the current best micropatterning technology [23].

This impressive figure, however, may leave *some* room at the bottom. That is, there is no published theoretical limit to the amount of information that can be reliably stored as ordinary ordered mass. Thus, although Feynman argued a conservative bound of  $5 \times 5 \times 5$  atoms per

bit [18,24], and RNA molecules achieve densities on the order of 32 atoms per bit [20], our  $\approx 2000$  petabit g<sup>-1</sup> biological "existence proof" could be pessimistic by an order of magnitude. Regardless, the point is that it is not hard to imagine large amounts of information being stored reliably and compactly using very little mass.

So, why hasn't inscribed mass transport been exploited in modern telecommunications networks? There are a number of reasons, but two seemingly obvious answers spring to mind. First, the key problem in telecommunications is *energy efficient* transport of information and delivering inscribed mass from New York to Boston would seem to consume a great deal of energy. Second, modern networks require *rapid* transport of information while the NY-Boston trip requires approximately 3 hours by car – or a few hundred seconds ballistically. These "answers" illustrate the key tensions which concern all telecommunications theorists:

> tolerable delay vs. tolerable energy vs. tolerable throughput

In quantifying these tensions for what we will call *inscribed mass channels*, we will find that under a surprising variety of circumstances they are, bit for bit, much more energy efficient than methods based on electromagnetic radiation. Moreover, from a theoretical perspective, the cost of writing the information into some medium can be made infinitessimally small at arbitrary rate [2, 25, 26], so the energy savings are not necessarily diminished by adding the inscription or readout costs. Thus, something seemingly so primitive as hurling carved pebbles through space can require many orders of magnitude less energy and support dramatically more users than isotropically broadcasting the same information.

Perhaps even more surprising, it is exactly that image which leads to another interesting point. For very large distances and very loose delay requirements, we will find that mass transport can be many more orders of magnitude more efficient than isotropic radiation. This can remain true even when radiation is carefully focussed using enormous antennas. That is, inscribed mass channels might be a *preferred* way to carry information between specks of matter separated by the vastness of interstellar or intergalactic space.

Though such a conclusion may seem directly at odds with previous work by Cocconi and Morrison [27] which proposed microwave interstellar communications, it is exactly the assumption of loose delay constraints which tips the balance strongly in favor of inscribed mass transport.<sup>1</sup> So, perhaps in addition to scouring the heavens for radio communications from other worlds, we might

<sup>&</sup>lt;sup>1</sup>Others (notably Bracewell [28]) have touched upon the issue of physical information transport via interstellar probes of varying complexity, but did not carefully consider the energetics of delivery relative radiation.

also wish to more closely examine the seeming detritus which is passing, falling, or has already fallen to earth.

# 2 Preliminaries

# 2.1 Definitions and Problem Statement

- *B*: message size (bits).
- $\tilde{\rho}$ : mass information density for inscribed information (bits kg<sup>-1</sup>).
- W: bandwidth available for radiated communication (Hz).
- $A = \pi R^2$ : effective receiver aperture (m<sup>2</sup>).
- *D*: distance to receiver (m).
- $N_0$ : background noise energy (W Hz<sup>-1</sup>).
- $\tau$ : time allowed for the message to arrive; the "deadline" for delivery (s).
- T: time to transmit a radio message of length B (s).
- $\delta = \tau c/D$ : ratio of  $\tau$  to the light travel time.
- $\eta = Bc/DW$ : the ratio of the message size to the time-bandwidth product of the radio channel.
- $\sigma$ : the volume density of message recipients (m<sup>-3</sup>).
- $\tilde{\sigma}$ : the areal density of message recipients distributed in a plane (m<sup>-2</sup>).

We compare the energy required to transport B bits over distance D under delay constraint  $\tau$  using electromagnetic radiation with bandwidth W, receiver aperture area A and receiver noise  $N_0$  to that required using inscribed mass with information density  $\tilde{\rho}$ .

### 2.2 Empirical Values for Mass Information Density

Detailed consideration of the practicalities of rendering information as inscribed mass and hardening it for transport is provided in separate work [29]. However, it is still useful to examine a few different possible methods of storage to get an empirical feel for "practical" values of mass information density,  $\tilde{\rho}$ , based on current technology. At present, RNA base pair storage seems to be the most compact method for which we have an existence proof with a mass information density as stated in the introduction of

$$\tilde{\rho}_{\rm RNA} = 1.8 \times 10^{24} \text{ bits kg}^{-1} \tag{2}$$

In comparison, as of this writing a scanning tunneling microscope (STM) can place an equivalent of about  $10^{15}$  bits per square inch using individual xenon atoms on a nickel substrate [23]. The per bit dimension is then 8 Å on a side. By somewhat arbitrarily assuming a 100 Å nickel buffer between layers we obtain a bit density of  $1.55 \times 10^{20}$  bits cm<sup>-3</sup>. The density of nickel (8.9 g cm<sup>-3</sup>) will predominate owing to the relatively thick layering so that we have

$$\tilde{\rho}_{\rm STM} = 1.74 \times 10^{22} \, {\rm bits \, kg^{-1}}$$
 (3)

or about two orders of magnitude smaller than RNA storage.

E-beam lithography on silicon can achieve feature sizes of 5 nm which implies a bit density of  $4 \times 10^{12}$  bits cm<sup>-2</sup>. Assuming 100 Å substrate layers we then have  $4 \times 10^{18}$  bits cm<sup>-3</sup>. Given silicon's density of 2.6 g cm<sup>-3</sup> [22] the mass information density is

$$\tilde{\rho}_{\text{e-beam}} = 1.54 \times 10^{21} \text{ bits kg}^{-1}$$
 , (4)

about three orders of magnitude smaller than RNA.

Current optical lithographic techniques routinely achieve 0.1  $\mu$ m feature sizes. Assuming a substrate thickness of 100 Å, this corresponds to a density of 10000 bits  $\mu$ m<sup>-3</sup> or 10<sup>13</sup> bits mm<sup>-3</sup>. Again assuming that silicon is the substrate material gives

$$\tilde{\rho}_{\text{optical}} = 3.85 \times 10^{18} \text{ bits kg}^{-1} \tag{5}$$

or approximately six orders of magnitude smaller than RNA.

Magnetic storage density is on the order of 10 Gb cm<sup>-2</sup> so that again, each bit is about 0.1  $\mu$ m on a side. Assuming a film thickness of 1000 Å and a density similar to FeO<sub>2</sub> (about 5 g cm<sup>-3</sup> [22]) we have

$$\tilde{\rho}_{\text{magnetic}} = 2 \times 10^{17} \text{ bits kg}^{-1} \quad , \tag{6}$$

which is about seven orders of magnitude smaller than RNA.

Finally, we note that volume holographic storage techniques [30] are limited to a volumetric bit density of one bit per  $\lambda^3$  where  $\lambda$  is the wavelength. Thus, a hologram using  $\lambda = 500$  nm blue light could in principle hold  $8 \times 10^{12}$  bits cm<sup>-3</sup> and the mass information density, assuming a quartz-like storage medium would be about  $3 \times 10^{15}$  bits kg<sup>-1</sup>, about nine orders of magnitude smaller than biological. For holography using shorter wavelengths, say in the ultraviolet range of 50 nm, the density would scale by a factor of 1000 and in the far ultraviolet (5 nm) by a factor of

 $10^6$  which is about twice the information mass density of E-beam lithography and three orders of magnitude smaller than RNA.

We note that absolute bounds on the amount of information that can be stored using matter have been described in [31–34]. However, even though the capacities are larger by ten, twenty or more orders of magnitude than those so far described, it is unclear how information could be extracted from such dense storage media. Regardless, clear limits on the maximum possible density of *useful* storage for inscribed mass are unknown, although simple quantum mechanical bounds for less exotic media are provided in [29].

# 3 Minimizing Particle Transport Energy Under Delay Bounds

Here we derive lower bounds on the amount of energy necessary to drive a mass m from one point to another under some deadline  $\tau$ . We assume a *free* particle, untroubled by external forces from potential fields (i.e., gravity).<sup>2</sup> Though the results are well known, for continuity we derive them here. Also, in keeping with a communication theory flavor, we use only standard communications methods such as basic probability theory and Jensen's inequality.

#### **3.1** Jensen's Inequality

Let h() be a non-negative real-valued function of a single variable and let V be a bounded real random variable with mean  $E[V] = \overline{v}$ . We also assume that E[h(V)] exists. We first note that

$$\max_{v()} h(v) \ge E[h(V)] \tag{7}$$

and that when V is deterministic

$$\max_{v()} h(v) = E[h(V)] \quad .$$
(8)

Next we note that for h() strictly convex we have via Jensen's inequality [35, 36]

$$E[h(V)] \ge h(\bar{v}) \tag{9}$$

with equality iff V is a constant. We now use these relations to derive lower bounds on the amount of energy necessary to move particles under delay constraints.

<sup>&</sup>lt;sup>2</sup>Potential fields are treated explicitly in a longer version of this paper and summarized in section 3.3. The inclusion of gravity causes relatively minor modifications to the overall results. See http://www.winlab.rutgers.edu/ $\sim$ crose/papers/masschannel11.pdf for details.

### **3.2 Free Particles**

We wish to move a mass m over a distance D within time  $\tau$  where the only external force acting on the particle is what we apply. We will assume that the destination is at rest relative to the source, an initial mass velocity of zero and that we need not bring the mass to rest at the destination. That is, the mass is "caught" by the destination and the only problem is for the source to deliver it on time with minimum applied energy.

Let the particle position be x(t) and its velocity v(t) = dx(t)/dt. Let the intrinsic energy of the particle at velocity v be described by a nondecreasing strictly convex function h(v). In order for the particle to be delivered by time  $\tau$  when moved through distance D, the average velocity must be  $D/\tau$ . Specifically,

$$E[v(t)] = \frac{1}{\tau} \int_0^\tau v(t) dt = \bar{v} = D/\tau \quad .$$
 (10)

Equation (10) is equivalent to an expectation of v(t) over a random variable t, uniform on  $(0, \tau)$ .

We want to minimize the maximum total energy imparted to the particle under the arrival delay constraint. So we seek a trajectory v(t) such that

$$E^* = \min_{v(t)} \max_{t} h(v(t))$$
(11)

while requiring  $E[v(t)] = D/\tau$ . We note that

$$\min_{v()} \max_{t} h(v(t)) \ge \min_{v()} E[h(v(t))]$$
(12)

and that by Jensen's inequality

$$E[h(v(t))] \ge h(\bar{v}) \tag{13}$$

with equality iff v(t) is constant. Since h() and  $\bar{v}$  are given, E[h(v(t))] has a lower bound independent of the specific trajectory v(t). Therefore we can absolutely minimize E[h(v(t))] by requiring that the particle move at constant velocity. However, this choice of v(t) also causes equation (12) to be satisfied with equality. This leads to the well known result that minimum energy is expended when the particle is launched from its origin with constant velocity  $v(t) = D/\tau$ ,  $t \in (0, \tau]$ .

For particles approaching light speed we have

$$h_{\text{total}}(v) = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \quad . \tag{14}$$

However, this total energy includes the rest mass energy  $mc^2$ . The excess energy owing to velocity is

$$h(v) = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$$
(15)

and is strictly convex in v, so that the minimum applied energy is

$$E^* = mc^2 \left(\frac{1}{\sqrt{1 - \bar{v}^2/c^2}} - 1\right) \quad . \tag{16}$$

For particles traveling much slower than light speed ( $\bar{v} \ll c$ ) we have  $h(v) \approx \frac{1}{2}mv^2$  so that

$$E^* \approx \frac{1}{2}m\bar{v}^2 \quad . \tag{17}$$

## 3.3 Particles in Potential Fields: a summary

Although particle delivery might require overcoming a potential field (gravity), carefully computing its effect does not greatly affect the main points we will make. Detailed derivations can be found in [37]. A summary of the results follows.

#### 3.3.1 The Artillery Problem

In a terrestrial systems, one can imagine launching mass from source to destination much as one would launch an artillery shell. There is an energy penalty since both forward and vertical motion must be imparted to loft the projectile toward the target. Assuming a flat earth with gravitational acceleration g, for high speed transport ( $\bar{v} \gg \sqrt{gD/2}$ ) the energy necessary for on-time delivery is approximately the same as in free space. For low speed transport ( $\bar{v} \ll \sqrt{gD/2}$ ) the energy is dominated by the vertical velocity which must be imparted to keep the particle aloft a sufficiently long time. In between is a minimum energy trajectory which results in

$$E^* = \frac{1}{2}mgD \quad . \tag{18}$$

At this minimum the projectile reaches the destination after a time  $\tau$  with a delay (relative to a signal traveling at the speed of light)  $\delta = c\tau/D$ . The delay for the minimum energy trajectory is

$$\delta^* = c\sqrt{2/gD} \quad . \tag{19}$$

We pay a factor of two energy penalty over the free space case since exactly the same vertical velocity as horizontal velocity must be imparted to the particle. If we desire slower delivery ( $\delta > \delta^*$ ) the penalty increases as  $\delta^2$  and below the penalty increases as  $\delta^{-2}$ .

#### 3.3.2 Escape from a Potential Field

For great distances, the launched particle must exceed the escape velocity of at least the body from which it is launched, and perhaps even larger "bodies" formed by an agglomeration of matter whose physical dimension is small relative the distance to be traveled. Essentially, an escape

energy must be calculated and added to the overall energy budget. The net effect is that  $\delta$  is restricted to a feasible range. For terrestrial escape, we must have  $\delta < 3 \times 10^4$ . For solar system escape we need  $\delta < 2 \times 10^3$ . For galactic escape,  $\delta < 6 \times 10^2$  is required. For  $\delta$  corresponding to escape velocity, the energy required is approximately twice the energy to deliver the same mass with the same delay in the absence of gravity. This penalty decreases as  $\delta^{-2}$  for smaller  $\delta$ .

# 4 Energy Bounds on Information Delivery

To compare the energy needed to send a message as inscribed mass or radiation, we consider the situation shown in Figure 1. The information has to be sent over a distance D. The message receipt deadline,  $\tau$ , is the time allowed for the transmission of the message from the transmitter to the receiver. In the case of radiation, the transmission of the message takes T, so the entire message is available at the receiver after a delay of  $\tau = D/c + T$ . In the following, we will parameterize the acceptable delay by a dimensionless quantity  $\delta = c\tau/D$ .  $\delta \approx 1$  means that we require the message to be available at a time just greater than the light transit delay;  $\delta \gg 1$  means we can tolerate a long delay.

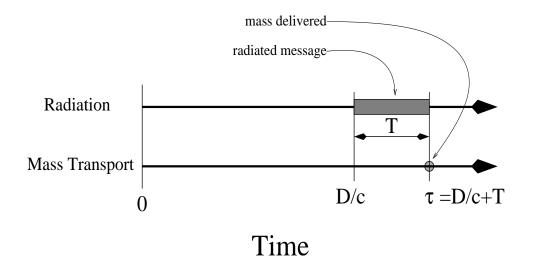


Figure 1: Temporal comparison of message delivery using radiation and mass transport. D: range to target, c: speed of light, T: radiated message duration,  $\tau$ : message delivery deadline.

### 4.1 Inscribed Mass

Because we have already shown that inscribed mass can support a very high information density, we will assume that the coding needed to assure reliable reception does not add significantly to the amount of mass that has to be sent. So, assuming some value for mass information density  $\tilde{\rho}$ , the

number of bits transported is  $B = m\tilde{\rho}$ . The energy necessary to transport mass m with deadline  $\tau = D/c + T$  in free space is then via equation (16)

$$E_w = \frac{B}{\tilde{\rho}} c^2 \left( \frac{1}{\sqrt{1 - \bar{v}^2/c^2}} - 1 \right) \quad . \tag{20}$$

In terms of the dimensionless delay  $\delta = c\tau/D = c/\bar{v}$ ,

$$E_w = \frac{B}{\tilde{\rho}}c^2 \left(\frac{\delta}{\sqrt{\delta^2 - 1}} - 1\right) \quad . \tag{21}$$

If  $\delta \gg 1$  (i.e., we can tolerate long a long delay), this reduces to

$$E_w \approx \frac{1}{2} \frac{B}{\tilde{\rho}} \left(\frac{c}{\delta}\right)^2 \quad . \tag{22}$$

If we require the message arrive as quickly as possible,  $\delta \approx 1$  and the energy required is

$$E_w \approx \frac{B}{\tilde{\rho}} \frac{c^2}{\sqrt{2(\delta - 1)}} \quad . \tag{23}$$

which is valid for relativistic motion. We can ask how short the delay must be before relativistic effect become important. Considering the last factor of equation (21), we see that it is equal to unity for  $\delta = \sqrt{3}/2$ , corresponding to a speed of 0.87 c. Significantly below this rather high speed, relativistic effects are small and can be ignored.

#### 4.2 Electromagnetic Transmission

If a transmitter radiates power P, a receiver at some distance D will capture some fraction of the radiated power  $P_r = \nu(D)P$  where  $\nu(D)$  is defined as the energy capture coefficient of the receiver. We assume square law isotropic propagation loss<sup>3</sup> and will neglect antenna gain for the moment since it can be taken into account by replacing the physical antenna area A with an effective area. We then have

$$\nu(D) = \frac{A}{4\pi D^2} \tag{24}$$

where A is the effective aperture of the receiver. Assuming additive Gaussian receiver noise, the Shannon capacity [36] in bits per second between the transmitter and receiver is

$$C = W \log_2 \left( \frac{PA}{4\pi D^2 N_0 W} + 1 \right) \tag{25}$$

<sup>&</sup>lt;sup>3</sup>For higher loss exponents such as those seen in terrestrial systems, we can multiply the result by the appropriate power of D.

where  $N_0$  is the background noise spectral intensity and W is the bandwidth of the transmission. If we assume a transmission interval long enough that the usual information theoretic results for long codes can be applied, the number of bits delivered for a transmission of duration T is

$$B = TC = TW \log_2 \left(\frac{PA}{4\pi D^2 N_0 W} + 1\right) \quad . \tag{26}$$

Notice that the time required for the arrival of the complete message is  $\tau = D/c + T$ , identical to the inscribed mass deadline shown in Figure 1.

Since  $E_r = PT$  we then have

$$E_r = TW N_0 \frac{4\pi D^2}{A} \left( 2^{\frac{B}{TW}} - 1 \right) \quad . \tag{27}$$

We note that although W is often interpreted simply as bandwidth, it is actually the sum of the bandwidths of all non-interfering channels connecting the transmitter to the receiver. For example, for line of sight paths in free space, two polarizations can be used, doubling W. In a scattering environment, multipath propagation can allow several channels to be used. For typical terrestrial propagation, this can multiply the effective W several times [38].

In terms of the required delay and  $\eta = Bc/DW$ , the ratio of message size to the timebandwidth product of the channel, the energy required is

$$E_r = B N_0 \frac{4\pi D^2}{A} \frac{\delta - 1}{\eta} \left[ 2^{\eta/(\delta - 1)} - 1 \right] \quad .$$
 (28)

For typical terrestrial radio channels, cellular telephony or digital television, we have  $\eta \approx 10^6$ . However, the acceptable delay is also large:  $\delta \approx 10^7$ . This is simply a consequence of most realtime messages taking tens of seconds (phone calls) to thousands of seconds (television programs), while terrestrial propagation delays are tens of microseconds to around a millisecond. Consequently the factor  $\eta/(\delta - 1) \approx 0.1$  or less. This reflects the fact that most radio channels are not bandwidth limited and therefore are not forced to use less energy efficient high-order modulation schemes. In the case where we are not bandwidth limited, we can assume  $\eta/(\delta - 1)$  is small and obtain the lower bound on the energy required to send a message by radio as

$$E_r \ge BN_0 \frac{4\pi D^2}{A} \ln 2 \quad . \tag{29}$$

While most terrestrial and near-earth (e.g., space probe) radios operate with  $\eta/(\delta - 1) \ll 1$ , we cannot exclude the possibility that large, urgent messages may have to be sent in a constrained bandwidth.<sup>4</sup> In that case,  $\eta/(\delta - 1) > 1$  and the energy requirement for radiative communication would grow exponentially.

<sup>&</sup>lt;sup>4</sup>For a fanciful example, see [39].

Finally, we note that in an essential reversal of Olber's paradox [40], it has been shown that radiative methods which posit a uniform distribution of cooperative repeaters scattered between source and destination can allow arbitrarily low energy requirements at the message source [41]. Likewise, Landauer [2, 25, 26, 42] has argued that an artfully arranged and synchronized sets of potential wells can allow arbitrarily low energy expenditure for communication between two points in space. However, as a general physical organizing principle such cooperation seems unlikely, especially over very large distances, and is therefore not considered here.

### 4.3 Directed Radiation

We have so far ignored the fact that electromagnetic radiation can be directed toward targets through means of a properly constructed antenna. Here we discuss simple physical limits on such directivity. The key results regarding aperture and field strength can be found in any elementary text on electromagnetic propagation such as [43].

As described in section 4.2, the energy capture coefficient of the receiving antenna is

$$\nu(D) = \frac{A}{4\pi D^2} \quad . \tag{30}$$

Energy falling outside the aperture of area A is wasted, so if the location of the receiver is known, the transmitter can use a directed beam of radiation. The energy capture coefficient is then

$$\nu(D) = \frac{AG}{4\pi D^2} \tag{31}$$

where G is the antenna gain. We see immediately that all of our previous results for isotropic radiators are applicable to directed beams, with the aperture A replaced by AG.

For diffraction limited beams from a circular aperture the maximum possible gain is

$$G_{\max} = \frac{8\pi^2 R^2}{\lambda^2} \tag{32}$$

where R is the radius of the transmitting antenna aperture and  $\lambda$  is the operating wavelength [44]. The equation (32) is only valid in the far-field, meaning distances at which the wavefronts are essentially spherical. We can find the distance  $D_{\text{far-fi} eld}$  by noting that energy conservation requires  $\nu(D) \leq 1$  which implies

$$G_{\max} \le \frac{4\pi D^2}{A} \tag{33}$$

and our gain equation is only valid for

$$D \ge D_{\text{far-fi} \text{ eld}} = \sqrt{2\pi} \frac{R^2}{\lambda}$$
 (34)

where we have assumed  $A = \pi R^2$ . (Sometimes  $D_{\text{far-fi} \text{ eld}}$  is defined with the numerical factor  $\sqrt{2\pi}$  replaced by 2. This difference has no effect on our argument.)

To obtain an estimate of the gains achievable with current technology, consider the radio telescope at Arecibo. It has R = 150 m and can operate at frequencies up to  $\approx 10$  GHz ( $\lambda = 3$  cm). Therefore,

$$G_{\text{Arecibo}} = 1.97 \times 10^9 \simeq 83 \text{ dB} \quad (\text{valid for } D \ge 3.33 \times 10^6 \text{ m}) \quad .$$
 (35)

At optical wavelengths ( $\lambda \approx 1 \ \mu m$ ), even higher gains are possible. For an aperture with R = 5 m, representative of the largest optical telescopes,

$$G_{\text{Optical}} = 1.97 \times 10^{15} \simeq 153 \text{ dB} \quad (\text{valid for } D \ge 1.11 \times 10^8 \text{ m}) \quad .$$
 (36)

Heroic engineering efforts may make even larger apertures possible. At microwave frequencies an aperture the size of the earth would require a relative surface accuracy similar to small (a few tens of centimeters across) mirrors used today for focussing soft X-rays. At 10 GHz, this antenna would have

$$G_{\text{Earth}} = 3.57 \times 10^{18} \simeq 175 \text{ dB} \quad (\text{valid for } D \ge 6.03 \times 10^{15} \text{ m}) \quad .$$
 (37)

In this last case, the far field begins two thirds of a light year from the antenna. Since this is quite a bit less than the distance to the nearest star, the gain formula is valid for interstellar distances.

# 5 The Radiation to Transport Energy Ratio

We define  $\Omega$ , the *radiation to transport energy ratio*, as

$$\Omega = \frac{E_r}{E_w} \tag{38}$$

and find that for free particle relativistic motion we have,

$$\Omega = \left[\frac{\tilde{\rho}N_0}{c^2}\right] \left[\frac{4\pi D^2}{AG}\right] \frac{\sqrt{\delta^2 - 1}}{\delta - \sqrt{\delta^2 - 1}} \frac{\delta - 1}{\eta} \left[2^{\eta/(\delta - 1)} - 1\right] \quad .$$
(39)

This result is easily understood as the product of three dimensionless factors:

$$\left[\frac{\tilde{\rho}N_0}{c^2}\right]$$
the relative density of information in  
radiation and inscribed mass, $\left[\frac{4\pi D^2}{AG}\right]$ the antenna efficiency factor (the frac-  
tion of radiated energy captured by  
the antenna),

$$\frac{\sqrt{\delta^2 - 1}}{\delta - \sqrt{\delta^2 - 1}} \frac{\delta - 1}{\eta} \left[ 2^{\eta/(\delta - 1)} - 1 \right]$$

the effects of the delay specification for radiation and inscribed mass and the channel bandwidth for radiation.

And if we assume that the radio channel is not bandwidth limited  $(\eta/(\delta-1) \ll 1)$ , we can simplify equation (39) to

$$\Omega \ge \left[\frac{\tilde{\rho}N_0}{c^2}\right] \left[\frac{4\pi D^2}{AG}\right] \frac{\sqrt{\delta^2 - 1}}{\delta - \sqrt{\delta^2 - 1}} \ln 2 \quad . \tag{40}$$

which for long delay ( $\delta \gg 1$ ) becomes

$$\Omega \ge \left[\frac{\tilde{\rho}N_0}{c^2}\right] \left[\frac{4\pi D^2}{AG}\right] (2\ln 2) \,\delta^2 \quad . \tag{41}$$

# 5.1 The Critical $\tilde{\rho}$

Suppose all the radiated energy is captured by the receiver. We can then ask what value of mass information density  $\tilde{\rho}$  makes mass transport more efficient: the *critical value* of  $\tilde{\rho}$  such that  $\Omega = 1$ . As before, for the terrestrial system we will assume a receiver temperature of  $300^{\circ}$ K and for the interstellar receiver, 3°K. In this case we can use equation (41) with  $AG = 4\pi D^2$  (all energy is captured). This is not physical, but establishes a lower limit on the relative efficiency of inscribed mass transport. For point to point links we have

 $\tilde{\rho}_{\text{terrestrial}} \stackrel{\text{write}}{\stackrel{>}{_{\sim}}} 1.57 \times 10^{37} \, \delta^{-2} \quad \text{bits kg}^{-1}$ (42) radiate

and

For the terrestrial receiver, if  $\delta = 10^7$  then mass will be more efficient than radiation if  $\tilde{\rho} >$ 

 $1.57 \times 10^{23}$  bits kg<sup>-1</sup>. This value falls within the range of our empirical "existence proofs" for mass information density. In contrast, for interstellar transmission at reasonable speeds ( $\delta \le 10^4$ ), we must have  $\tilde{\rho} \ge 1.57 \times 10^{31}$  bits kg<sup>-1</sup> which is about seven orders of magnitude larger than our largest empirical  $\tilde{\rho}$  of  $1.82 \times 10^{24}$ . So, if radiated energy could be perfectly focused on the intended destination, then interstellar radio channels would be much more efficient than inscribed mass channels, even using the most dense storage medium we know.

Of course, it is not always be possible to focus all radiated energy on the destination receiver. So let us refine these  $\tilde{\rho}$  bounds by considering what is possible given receiver and transmitter apertures. If the transmitter and receiver apertures are the same we have using equation (32) (and subject to equation (33)),

$$\tilde{\rho}_{\text{terrestrial}} \stackrel{\text{write}}{\stackrel{\geq}{\underset{\text{radiate}}{}}} 3.1 \times 10^{38} \left[ \frac{R^4}{D^2 \lambda^2} \right] \delta^{-2} \quad \text{bits kg}^{-1}$$
(44)

and

$$\tilde{\rho}_{\text{interstellar}} \stackrel{\text{write}}{\stackrel{\text{$\sim$}}{\underset{\text{$radiate$}}{\overset{\text{$\sim$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\overset{\text{$\sim$}}{\underset{\text{$radiate$}}{\overset{\text{$\sim$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\overset{\text{$\sim$}}{\underset{\text{$radiate$}}{\overset{\text{$\sim$}}{\underset{\text{$radiate$}}{\overset{\text{$\sim$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\overset{\text{$radiate$}}{\underset{\text{$radiate$}}{\overset{\text{$radiate$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\underset{\text{$radiate$}}{\underset{$radiate$}}{\overset{\text{$radiate$}}{\underset{\text{$radiate$}}{\underset{$radiate$}}{\overset{\text{$radiate$}}{\underset{$radiate$}}{\overset{\text{$radiate$}}{\underset{$radiate$}}{\overset{\text{$radiate$}}{\underset{$radiate$}}{\overset{\text{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{\underset{$radiate$}}{\overset{{$radiate$}}{\underset{$radiate$}}{$$

For the terrestrial system we will assume receive and transmit apertures of radius 0.05 m (handheld devices) and a wavelength of 3 cm corresponding to a transmission frequency of 10 GHz. For interstellar systems we will consider Arecibo-sized (150 m radius) and Earth-sized ( $6.38 \times 10^6$  m radius) apertures at the receiver and transmitter at the same wavelength. Corresponding critical  $\tilde{\rho}$ values are given in Table 1 along with *D* for which the expressions are valid.

For terrestrial transport and typical handheld apertures, a  $D\delta$  product greater than  $10^7$  meters makes inscribed mass more efficient than radiation. Given that  $\delta > 10^5$  for terrestrial transport, at distances greater than 100 meters, inscribed mass is always energetically favored.

For longer range transmissions and apertures typical of present day engineering practice, mass is favored for  $D\delta$  products greater than  $1.3 \times 10^{14}$  meters, or about  $10^{-2}$  light years. Since solar system escape requires roughly  $\delta < 10^3$ , mass is therefore favored for any distance less than  $10^{-5}$ light years or about  $10^{11}$  meters – roughly the distance from the earth to the sun.

If we assume a radio antenna the size of the earth, mass is favored for a  $D\delta$  product greater than  $2.4 \times 10^{23}$  meters or  $2.5 \times 10^7$  light years. So assuming that  $\delta$  is small enough to ensure escape from the solar system, mass is not more efficient than radiation until we have to go at least  $D > 10^4$  light years – a significant fraction of the Milky Way's diameter ( $10^5$  light years). For extragalactic messaging we require roughly  $\delta < 100$  for escape. Thus, if sending a message to the nearest galaxy (D roughly  $10^7$  light years) or beyond, inscribed mass is once again much more energy efficient than radiation.

Critical $\tilde{\rho}$ Values				
terrestrial (aperture radius 0.05 m)	$2.15 \times 10^{36} \left[\frac{D}{1 \text{ meter}}\right]^{-2} \delta^{-2} \text{ bits kg}^{-1}$	$D \ge 0.37 \text{ m}$		
interstellar (Arecibo sized aperture)	$\begin{cases} 1.74 \times 10^{52} \left[\frac{D}{1 \text{ meter}}\right]^{-2} \delta^{-2} & \text{bits kg}^{-1} \\ 1.95 \times 10^{20} \left[\frac{D}{1 \text{ light year}}\right]^{-2} \delta^{-2} & \text{bits kg}^{-1} \end{cases}$	$D \ge 3.3 \times 10^6 \mathrm{~m}$		
interstellar (Earth sized aperture)	$\begin{cases} 5.71 \times 10^{70} \left[\frac{D}{1 \text{ meter}}\right]^{-2} \delta^{-2} & \text{bits kg}^{-1} \\ 6.38 \times 10^{38} \left[\frac{D}{1 \text{ light year}}\right]^{-2} \delta^{-2} & \text{bits kg}^{-1} \end{cases}$	$D \ge 6.0 \times 10^{15} \mathrm{~m}$		

Table 1: Critical  $\tilde{\rho}$  for  $\lambda = 0.03$  m and the specified receiver and transmitter apertures. For convenience of comparison, interstellar distance are given in both meters and light years. The last column gives the domain of validity of the formula for  $\tilde{\rho}$ . For *D* less than the value shown, the critical  $\tilde{\rho}$  is less.

### **5.2** $\Omega$ vs. Distance

Though the critical  $\rho$  provides some sense of when mass inscription is favored over radiation, it does not provide a sense of just how large the gains can be in a variety of likely scenarios. Therefore, we plot the energy ratio  $\Omega$  for point to point links first assuming free space propagation over large distances (interstellar) and then terrestrial conditions.

The primary differences between these two scenarios are the receiver temperatures and apertures. For terrestrial systems we assume a receiver temperature of 300°K and an aperture of 10 cm diameter. For interstellar transmission we use a receiver temperature of 3°K and apertures with the same radius as the Arecibo radio telescope (R = 150 m) and with the radius of the earth ( $R = 6.38 \times 10^6$  m). In all cases, the radio links operate in the microwave range,  $\nu = 10$  GHz ( $\lambda = 0.03$  m). For both interstellar and terrestrial links, we consider two scenarios for the use of radio. The first is an isotropic transmitter, the second is a transmitter with gain equal to that of the receiving antenna.

In all cases, inscribed mass channels are many orders of magnitude more efficient than radiative channels. For example, for an isotropic transmitter and an earth-sized receiving aperture, Figure 2 shows that for a delay of  $\delta = 100$  (mean speed of  $\bar{v} = 10^{-2} c$ ), electromagnetic radiation requires  $10^9$  as much energy as inscribed mass at a range of one light year. At ten thousand light years, this gain is  $10^{17}$ . For an Arecibo-sized receiving aperture, the energy gain of mass over radiation is a factor of  $\approx 10^{18}$  at one light year and  $\approx 10^{26}$  at ten thousand light years. These gains are, for lack of a better word, astronomical.

Even for the more realistic case of transmitting antennas with significant gain, inscribed mass is energetically favored over a wide range of conditions. From Figure 3, we see that inscribed mass is more efficient than radiation between Arecibo-sized apertures for distance greater than  $10^{13}$  m, roughly the size of the solar system. Between truly enormous apertures, radiation remains more efficient than mass for links anywhere within the galaxy.

For terrestrial systems, the gains are not astronomical, but still impressive. Values of  $\Omega$  for distance up to 10 km are shown in Figures 4 and 5. The delays are for speeds corresponding to walking ( $\delta \approx 10^8$ ), driving ( $\delta \approx 10^7$ ) and flying ( $\delta \approx 10^6$ ). Compared to an isotropic radio transmitter, inscribed mass is more efficient for *any* distance. Even if a transmitting antenna with reasonably high gain is used, At distances greater than a few tens of meters, mass is more efficient.

Thus, for reasonable receiver aperture sizes and dense but empirically possible mass information density, inscribed mass transport is usually *much* more efficient than radiation when delay can be tolerated.

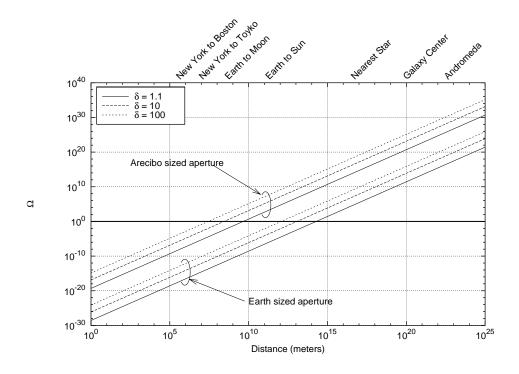


Figure 2: Energy ratio for free space particles versus distance using equation (40) for earth-sized and Arecibo-sized sized apertures. The bit per mass density is  $\tilde{\rho} = 1.8 \times 10^{24}$  bits kg<sup>-1</sup> and the receiver temperature 3°K. The slight difference in the spacing of the lines for different values of  $\delta$  is caused by relativistic effects.

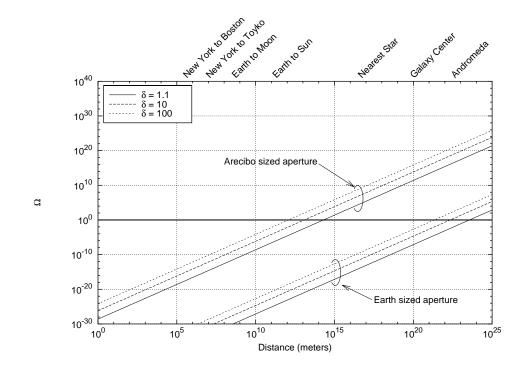


Figure 3: As with Figure 2, but for a transmitting antenna with gain equal to the receiving antenna.

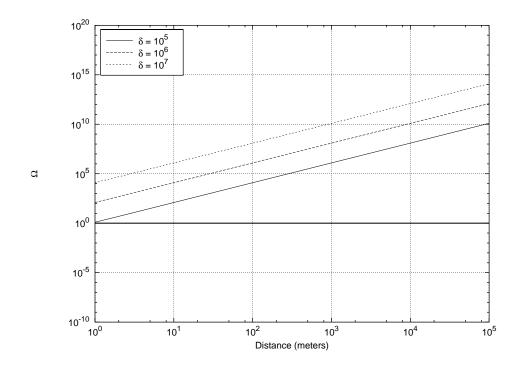


Figure 4: Energy ratio for free space particles versus distance using equation (40) an aperture with 0.05 m radius, characteristic of handheld radios. The bit per mass density is  $\tilde{\rho} = 1.8 \times 10^{24}$  bits kg<sup>-1</sup> and the receiver temperature 300°K.

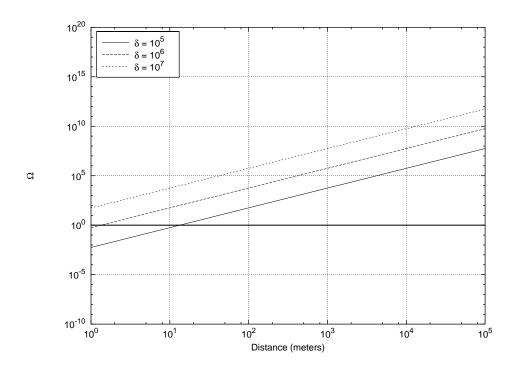


Figure 5: As with Figure 4, but for a transmitting antenna with gain equal to the receiving antenna.

# 6 Broadcast and Multicast

One benefit (or liability) of electromagnetic transmission is that it can disperse through space to multiple targets. In contrast, inscribed mass transport is conceptually a point to point method. We consider three cases:

- *blind broadcast* communication, where the locations of the receivers are unknown and each receiver gets the same message,
- *directed broadcast* where the same message is sent to all receivers but the receiver positions are known, and
- multicast where receiver positions are known and each receiver gets a different message

# 6.1 One Message, Many Receivers, Unknown Positions

If we want to send the same message to all receivers, then information theory for broadcast channels suggests we code the message to be received by the most distant receiver since those closer to the transmitter will always receive a stronger signal [36]. So, the amount of energy necessary to disperse the shared message to all receivers with aperture A within a radius U electromagnetically is

$$E_r \ge BN_0 \frac{4\pi U^2}{A} \ln 2 \quad . \tag{46}$$

Where again we have assumed that  $\eta/(\delta - 1) \ll 1$ .

To evaluate the energy required for mass transport, we could hold the deadline  $\tau$  constant – this would favor mass transport since destinations closer to the source could travel more slowly. However, the reduction in energy for variable mass transport velocity is only about 40%, and such velocity variation would complicate evaluation of scenarios where the initial velocity must be larger than some value (i.e., escape velocity). So we opt to use the slightly less optimal fixed particle speed assumption here and thus variable arrival times.

Therefore, the amount of energy necessary to deliver the shared message of B bits to all receivers in a volume of radius U using inscribed mass is the sum of the individual energies necessary for each receiver. For inscribed mass, each receiver is a piece of "flypaper" with area S. If the inscribed mass hits the receiver it is assumed to be read successfully. We assume that the receiver concentration is sufficiently dilute that shadowing effects are negligible. Since we do not know where the receivers are we must send enough inscribed mass particles so that one intercepts every patch of area S on a surface at the maximum range U. The energy required to deliver a message to all of the receivers is then

$$\bar{E}_w = \frac{Bc^2}{\tilde{\rho}} \frac{4\pi U^2}{S} \left( \frac{\sqrt{\delta^2 - 1}}{\delta - \sqrt{\delta^2 - 1}} \right) \tag{47}$$

and the relative efficiency of radiation as compared to inscribed mass is

$$\Omega = \left[\frac{\tilde{\rho}N_0}{c^2}\right] \left[\frac{S}{A}\right] \left(\frac{\sqrt{\delta^2 - 1}}{\delta - \sqrt{\delta^2 - 1}}\right) \quad . \tag{48}$$

For radio antennas and "flypaper" of equal area, and specializing to long delays we have,

$$\Omega = \left[\frac{\tilde{\rho}N_0}{c^2}\right] (2\ln 2)\,\delta^2 \quad . \tag{49}$$

### 6.2 One Message, Many Receivers, Known Positions

To calculate  $\Omega$  for known receiver locations, we need only modify the energy required for mass transport since the radiative broadcast energy budget is still given by equation (46). As before, we assume variable arrival deadlines with no paucity of bandwidth, and calculate the amount of energy necessary to deliver the shared message of *B* bits to all receivers in a volume of radius *U* using inscribed mass as the sum of the individual energies necessary for each receiver. The expected energy, assuming a Poisson density  $\sigma$  of receivers, long delay ( $\delta \gg 1$ ) and unlimited bandwidth is, following equation (22),

$$\bar{E}_w = E_w(U) \left(\sigma \frac{4}{3}\pi U^3\right) = \frac{1}{2} \frac{B}{\tilde{\rho}} \left(\frac{c}{\delta}\right)^2 \left(\sigma \frac{4}{3}\pi U^3\right) \quad .$$
(50)

If we then define  $\Omega(U)$  as the radiation to mass energy ratio for a spherical volume of radius U, for a receiver with aperture area A we have,

$$\Omega(U) \ge 6\ln 2 \left[\frac{\tilde{\rho}N_0}{c^2}\right] \left[\frac{1}{\sigma UA}\right] \delta^2 \quad .$$
(51)

For a terrestrial system it is more likely that receivers are arranged in a plane with areal density  $\tilde{\sigma}$ . So, if we define  $\Omega(U)$  as the radiation to mass energy ratio in a circular area of radius U, with a receiver having aperture A we have,

$$\Omega(U) \ge 8 \ln 2 \left[ \frac{\tilde{\rho} N_0}{c^2} \right] \left[ \frac{1}{\tilde{\sigma} A} \right] \delta^2 \quad .$$
(52)

### 6.3 Different Messages, Different Receivers

Now suppose that a different message must be sent to each receiver and that each of these messages is the same size. Let the distance to receiver *i* be  $D_i$  and assume that  $D_i \leq D_{i+1}$  i = 1, 2, ...M. The transmitter has power budget *P* which must be split *M* ways. Let  $\alpha_i$  be the fraction of power allocated for receiver *i* so that  $\sum_i \alpha_i = 1$ . Applying information theory for the Gaussian broadcast channel [36] provides that the rate  $\mathcal{R}_i$  seen by user *i* satisfies

$$\mathcal{R}_{i} \leq W \log_{2} \left( \frac{\alpha_{i} P \nu(D_{i})}{\sum_{j=i+1}^{M} \alpha_{j} P \nu(D_{i}) + W N_{0}} + 1 \right) = W \log_{2} \left( \frac{\alpha_{i} P}{\sum_{j=i+1}^{M} \alpha_{j} P + N_{i}} + 1 \right)$$
(53)

where  $N_i = W N_0 / \nu(D_i)$  and we note that  $N_i$  increases in *i*.

To understand rate bounds we can set  $R_i = B/T$  and solve for appropriate  $\alpha_i$ 

$$B = WT \log_2 \left( \frac{\alpha_i P}{\sum_{j=i+1}^M \alpha_j P + N_i} + 1 \right)$$
(54)

which we rewrite as

$$\frac{N_i}{P} = \frac{\alpha_i}{\beta} - \sum_{j=i+1}^M \alpha_j \tag{55}$$

where  $\beta \equiv 2^{B/WT} - 1 = 2^{\eta/(\delta-1)} - 1$ . We can then rewrite equation (55) in matrix form as

$$\frac{1}{P}\mathbf{N} = \begin{bmatrix} 1/\beta & -1 & \cdots & -1 \\ 0 & 1/\beta & -1 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & 1/\beta & -1 \\ 0 & \cdots & \cdots & 0 & 1/\beta \end{bmatrix} \boldsymbol{\alpha}$$
(56)

where  $\mathbf{N}^{\top} = [N_1, \cdots, N_M]$  and  $\boldsymbol{\alpha}^{\top} = [\alpha_1, \cdots, \alpha_M]$ . We then have through matrix inversion,

$$\boldsymbol{\alpha} = \frac{\beta^2}{P} \begin{bmatrix} 1/\beta & 1 & (\beta+1) & \cdots & (\beta+1)^{M-3} & (\beta+1)^{M-2} \\ 0 & \ddots & \ddots & \ddots & & (\beta+1)^{M-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & (\beta+1) \\ \vdots & & & \ddots & \ddots & & 1 \\ 0 & \cdots & \cdots & 0 & & 1/\beta \end{bmatrix} \mathbf{N} \quad .$$
(57)

Since  $\beta$  and all the  $N_i$  are non-negative, all the  $\alpha_i$  will be non-negative. Since we seek the energy

 $E_r$  necessary to broadcast B bits independently to each location, we must have

$$E_{r} = PT = (\mathbf{1}^{\top} P \boldsymbol{\alpha})T = T\beta^{2} \mathbf{1}_{M}^{\top} \begin{bmatrix} 1/\beta & 1 & (\beta+1) & \cdots & (\beta+1)^{M-3} & (\beta+1)^{M-2} \\ 0 & \ddots & \ddots & \ddots & & (\beta+1)^{M-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & (\beta+1) \\ \vdots & & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & 0 & 1/\beta \end{bmatrix} \mathbf{N}$$
(58)

where  $\mathbf{1}_M^{\top}$  is an *M*-dimensional vector all of whose entries are 1.

We can rewrite equation (58) as

$$E_{r} = BN_{0} \frac{\delta - 1}{\eta} \beta^{2} \mathbf{1}_{M}^{\top} \begin{bmatrix} 1/\beta & 1 & (\beta + 1) & \cdots & (\beta + 1)^{M-3} & (\beta + 1)^{M-2} \\ 0 & \ddots & \ddots & \ddots & & (\beta + 1)^{M-3} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & & 1 \\ 0 & \cdots & \cdots & 0 & & 1/\beta \end{bmatrix} \begin{bmatrix} \frac{1}{\nu(D_{1})} \\ \vdots \\ \frac{1}{\nu(D_{M})} \end{bmatrix} .$$
(59)

Using

$$\frac{\delta - 1}{\eta} \beta = \frac{\delta - 1}{\eta} \left( 2^{\eta/(\delta - 1)} - 1 \right) \ge \ln 2 \quad , \tag{60}$$

we obtain a lower bound on  $E_r$  by taking  $\eta/(\delta - 1) \ll 1$ , giving

$$E_{r} \geq \ln 2 B N_{0} \mathbf{1}_{M}^{\top} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\nu(D_{1})} \\ \vdots \\ \frac{1}{\nu(D_{M})} \end{bmatrix} = \ln 2 B N_{0} \sum_{i=1}^{M} \frac{4\pi D_{i}^{2}}{A_{i}} \quad .$$
(61)

Equation (61) implies that  $E_r$  is minimum when each receiver has its own channel, independent of (orthogonal to) the others – essentially a collection of M independent point to point channels. If we assume identical receivers (all  $A_i$  the same), this allows us to calculate the expected minimum energy required for multipoint distinct message radiation. If receivers have a spatial density  $\sigma$ ,

$$\bar{E}_r \ge 2\sigma \ln 2 B N_0 \int_0^{\pi} \int_0^{2\pi} \int_0^U \frac{4\pi r^2}{A} r^2 dr \, d\phi \, \sin \theta d\theta = \frac{32}{5} \pi^2 \ln 2 B N_0 \left[\frac{U^5 \sigma}{A}\right] \quad . \tag{62}$$

We can compare this directly to the inherently multipoint long delay mass transport energy of

equation (51) to obtain

$$\Omega \ge \frac{48\pi}{5} \ln 2 \left[ \frac{\tilde{\rho} N_0}{c^2} \right] \left[ \frac{U^2}{A} \right] \delta^2 \quad . \tag{63}$$

#### 6.4 $\Omega$ vs. Distance for Broadcast and Multicast

Here we consider the energy ratio  $\Omega$  when the same message is to be delivered everywhere (shared) and when different messages are intended for different destinations (distinct). For shared messages with blind broadcast we take the results of equation (58) and assume equal receiving areas for radiation and inscribed mass. We have first assumed that the locations of the receivers are unknown, so using directive antennas confers no benefit. The formula for  $\Omega$  then depends only on the information density of inscribed mass, the background noise density for radiation and the tolerable delay  $\delta$ . In Figure 6,  $\Omega$  is plotted against the acceptable delay for the two cases of receiver temperature considered earlier, 3 K and 300 K.

For delays of a few times  $10^6$ , inscribed mass is more efficient than radiation. These speeds are about 100 m s<sup>-1</sup> or less, typical of terrestrial transportation. Over interstellar distances, blind broadcast communication by inscribed mass is significantly less efficient than radiation for any reasonable delay since  $\delta$  must be greater than about  $10^4$  simply to escape the solar system. This is not surprising, since true broadcast communication using inscribed mass requires an enormous number of message particles, all of which must be accelerated.

In contrast, when the position of the receiver is known, inscribed mass can have significant advantages over radiation in a variety of regimes assuming reasonable densities of receivers in the interstellar and terrestrial cases. Results are shown in Figure 7, Figure 8 and Figure 9.

For multiple distinct messages, the results are similar to those seen for point to point links. Figure 10 shows the efficiency of mass for large distances, with an isotropic transmitter – each message being transmitted on a different frequency, for instance, so that all messages can be transmitted simultaneously. As before, inscribed mass is more efficient for any distance outside of the solar system, even for enormous receiving apertures.

If the transmitting antennas have gain equal to the receiving antennas, mass is still more efficient for multicast communication until the apertures are very large. In Figure 11 we see that at a distance of a light year,  $\Omega \approx 10^7$  for Arecibo sized apertures at the transmitter and receiver. For apertures the size of the earth, radiation wins for links within the galaxy, mass wins for distances beyond.

For terrestrial systems we see similar behavior, shown in Figures 12 and 13. At low speeds  $(\delta > 10^5)$  and with an isotropic transmitter, mass is always more efficient than radiation. Even with moderate transmit antenna gain mass almost always wins. Also note that mass would confer even more advantage had we assumed a more typical  $D^{-3.5}$  to  $D^{-4}$  propagation loss for the terrestrial radio channel.

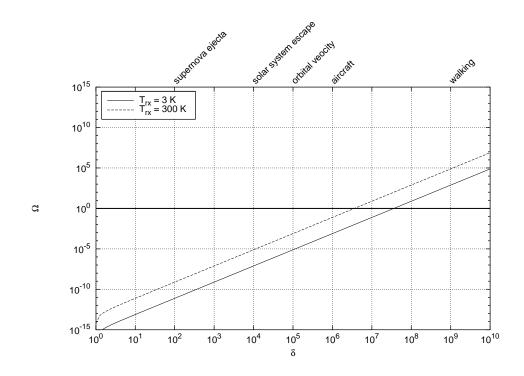


Figure 6: Energy ratio for a broadcast message with unknown receiver locations, as a function of the acceptable delay  $\delta$  according to equation (49). Curves for receiver temperatures of 3 K and 300 K are shown. The downturn in the curves near  $\delta = 1$  is caused by relativistic effects.

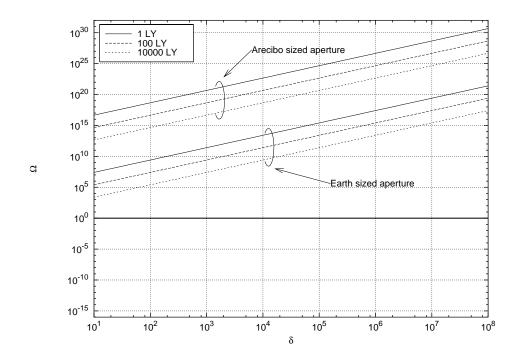


Figure 7: Energy ratio for a broadcast message with known receiver locations, as a function of the acceptable delay  $\delta$  according to equation (51). Receiver temperature of 3 K and receiver density  $\sigma = 6.4 \times 10^{-3}$  light year<sup>-3</sup> (stellar density of Milky Way).

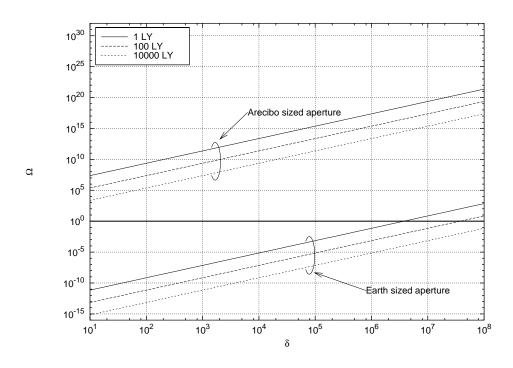


Figure 8: Same as Figure 7 but with transmitter aperture equal to receiver aperture.

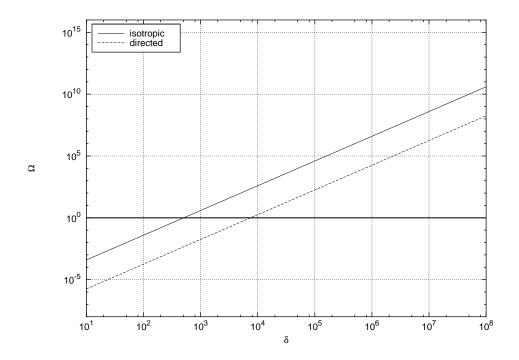


Figure 9: Energy ratio for a broadcast message with known receiver locations, as a function of the acceptable delay  $\delta$  according to equation (52). Receiver temperature 300 K, receiver aperture 0.05 m and receiver density  $\sigma = 0.01 \text{ m}^{-2}$ . Isotropic transmitter and equal transmitter/receiver aperture cases shown.

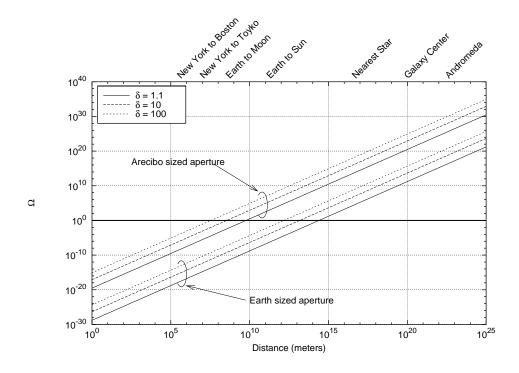


Figure 10: Energy ratio for distinct message problem in free space for earth-sized and Arecibo-sized apertures according to equation (63). The bits per mass density is  $\tilde{\rho} = 1.8 \times 10^{24}$  bits kg<sup>-1</sup>, the receiver temperature is 3°K.

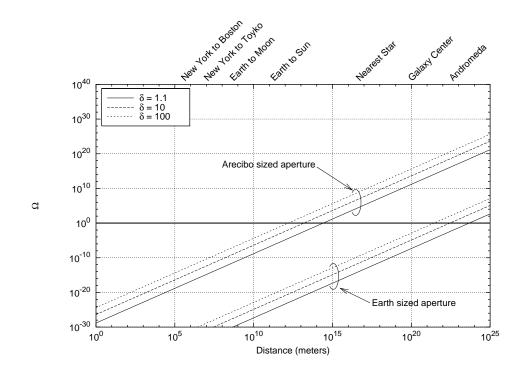


Figure 11: As in Figure 10, but with transmitting antennas with gain equal to that of the receiving antennas.

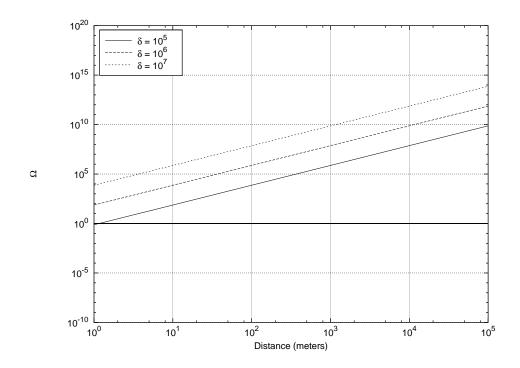


Figure 12: Energy ratio for distinct message problem in free space for a receiver with aperture R = 0.05 m, characteristic of handheld devices according to equation (63). The bits per mass density is  $\tilde{\rho} = 1.8 \times 10^{24}$  bits kg<sup>-1</sup>, the receiver temperature is 300°K.

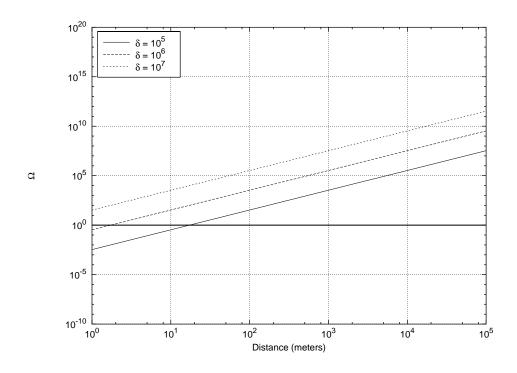


Figure 13: As in Figure 12, but with transmitting antennas with gain equal to that of the receiving antennas.

# 7 Discussion and Conclusion

In the previous sections we have seen that inscribed mass channels can be many many orders of magnitude more efficient than channels which use electromagnetic radiation – even when assumptions are made which favor the radiative channel such as large bandwidth ( $WT/B \gg 1$ ) as well as best case  $D^2$  propagation loss in terrestrial systems. The only situation where it might be difficult to make inscribed mass transport more efficient are for what seem heroically large (earth-sized) receive and transmit apertures. We now discuss selected implications of these results.

#### 7.1 Message Transcription Energy: empirical costs

We have mentioned previously that from a theoretical perspective, the energy cost of transcription and readout can be made as small as necessary so that no energy (or rate) penalty need be paid for the inscription process [2, 25, 26]. Nonetheless, it is still useful to consider an empirical example of inscription energetics.

The genome of E.Coli has 4, 639, 221 bases [45]. According to a popular biochemistry text (Lehninger [46]), it takes a cell of E. Coli approximately 20 minutes to divide. The rate of energy consumption devoted to replicating the genetic material is 60, 000 ATP per second. With  $8 \times 10^{-20}$  Joules per ATP we infer that replication consumes  $1.24 \times 10^{-18}$ J bit<sup>-1</sup>. For simplicity we will assume that all the energy is consumed by inscription as opposed to readout.

In comparison, the minimum energy necessary for delivery of a message inscribed on DNA at c/100 ( $\delta = 100$ ) is  $2.5 \times 10^{-12}$ J bit<sup>-1</sup> – quite a bit larger than the energy used by E. Coli for genome replication. Of course, the delivery energy varies as  $\delta^{-2}$ . Thus, for much lower speed transport ( $\delta = 10^4, 10^5$ ) the inscription energy costs would be comparable to or much larger than the energy required for delivery. However, depending upon the radiation to mass energy ratio margin, inscribed mass could still be much more energetically efficient than radiation even for this simple empirically derived estimate of necessary transcription energy.

#### 7.2 Delays for Terrestrial, Interstellar and Intergalactic Mass Transport

For terrestrial systems, the optimum speed for sending an inscribed mass message is given by equation (19) so that

$$\tau \approx 0.45 \left[\frac{D}{1 \text{ meter}}\right]^{1/2}$$
 seconds (64)

on earth. For D ranging from one meter to ten kilometers we have a range of 450 msec to 45 sec. The trip to Boston from New York City (320 km) would require 255 seconds ballistically (with the projectile traveling at roughly Mach 4). These delivery times may be long compared to the light travel time, but they are surprisingly short compared to the time scales of many human activities that utilize large amounts of information. If delays of seconds to minutes or more are tolerable,

inscribed mass can be attractive over terrestrial distances.

For interstellar mass delivery even at high speed, transport delays can easily be geological in scale. Thus, in Figure 14 we provide a plot of message transport delay versus distance in lightyears for different fractions of light speed and place it explicitly in a geological context. We note that

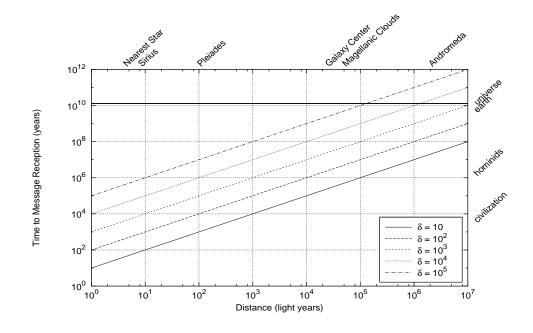


Figure 14: Message transport delay versus distance for different transport velocities. Approximate ages of human civilization, the hominids, the earth and the universe shown for reference. The bold horizontal line is at  $1.3 \times 10^{10}$  years, the age of the visible universe.

the solar system is on the order of four billion years old and the visible universe is on the order of thirteen billion years old. So, limiting delivery delays to some fraction of the earth's age (say 1 billion years) seems reasonable. For distances within the Milky Way ( $10^5$  light years diameter) a transport speed of  $10^{-3}c$  allows delivery within  $10^8$  years. Local extra-galactic messaging ( $\approx 10$  million light years) at  $\delta = 10^3$  would result in delays comparable to the age of the visible universe, so perhaps we would have to assume  $\delta = 100$ . We note that although increased velocity reduces the efficiency of inscribed mass transport, the greatly increased distance more than makes up for this loss. Specifically, by equation (45), increasing D by a factor of 1000 while decreasing  $\delta$  by a factor of 10 results in an overall  $10^4$  gain for inscribed mass efficiency.

### 7.3 Open Issues for Interstellar Channels

We have completely ignored the channel characteristics for inscribed mass by essentially assuming that what is sent arrives intact. For terrestrial systems, this is probably not a bad assumption. However, for interstellar transport, a mass packet would be subject to a variety of high energy insults for a long period of time. This issue is important and the subject of ongoing work [29]. However, we note that the relative efficiency of inscribed mass can be at times so enormous, that incredibly high error rates could be tolerated using simple redundancy codes, by sending large numbers of separate messages, or even by encasing the message in a hardened transport carrier. Nonetheless, the effect of insults to the information integrity of mass packets requires investigation.

We have also skirted the issue of what sort of messages one might want to send, how they might be detected or where they might be sent [47–50]. The large delays associated with interstellar travel and the seeming fragility of species to cosmic insults suggests that messages should be constructed "for posterity" as opposed to for initiating a chat. Alternately, "colonization" via some process akin to panspermia [51] might be a goal. Regardless, one ostensible virtue of inscribed mass channels is that once the message arrives, it is persistent as compared to electromagnetic radiation which is transient and thus must be sent repeatedly in order to assure the message is received. Of course, one must also consider the technical problems of constructing mass packets to be hearty and easily detected. Nonetheless, the seeming simplicity of mass packet delivery does raise interesting questions about terrestrial biological history and perhaps SETI/xenobiological studies as well.

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