Molecular Communication Using Timing & Payload

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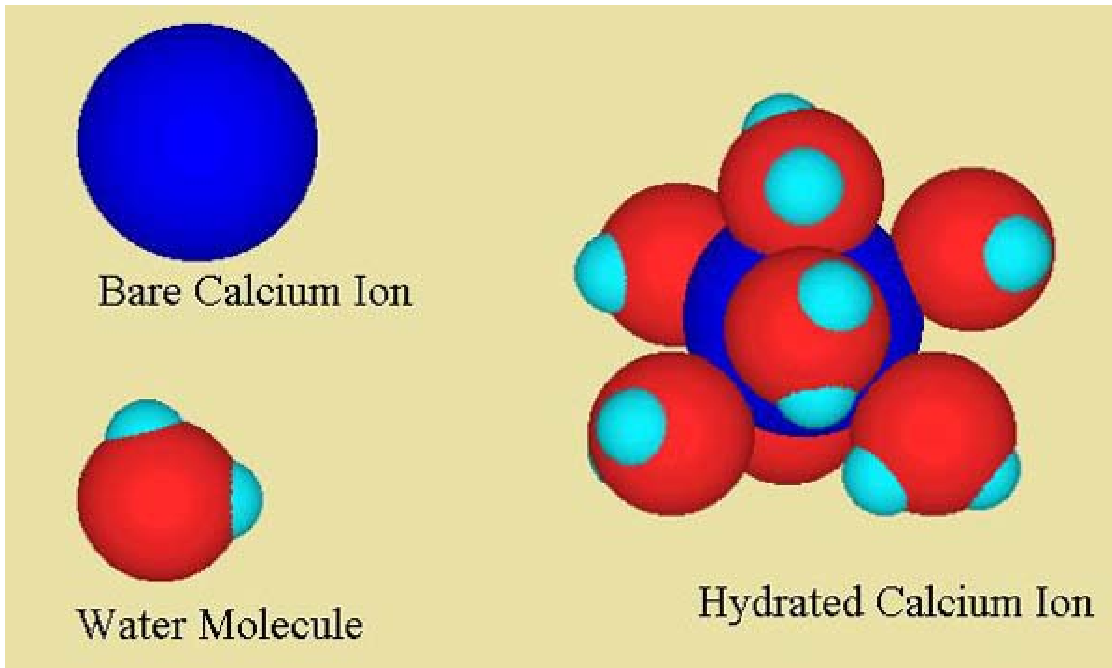
ITA'15
San Diego
February 3, 2015

What Is A ...

What Is A ...

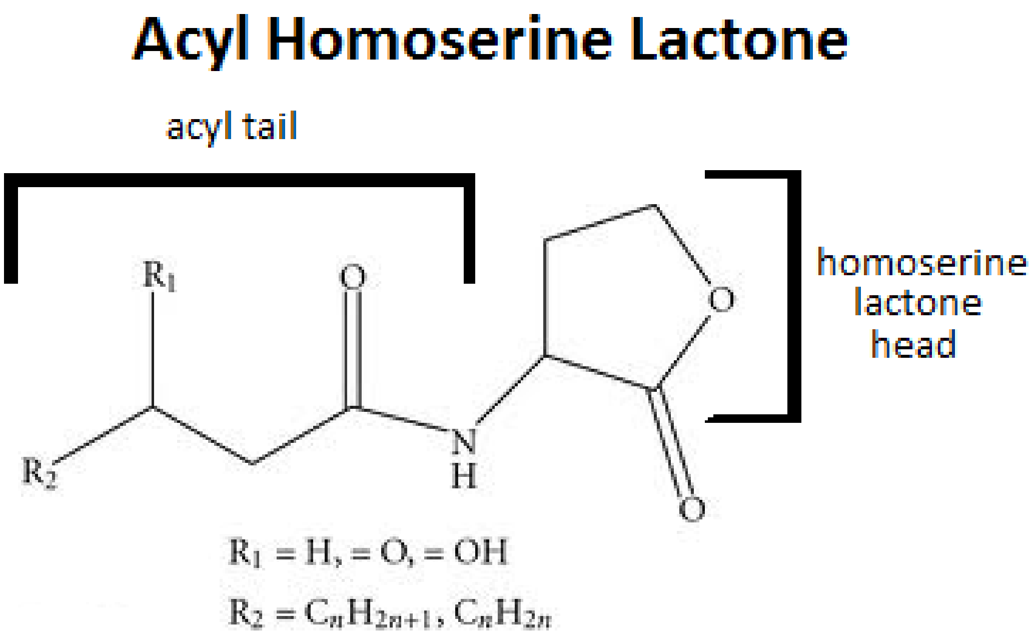
Signaling Molecule

A REALLY Simple Signaling Molecule (Token)



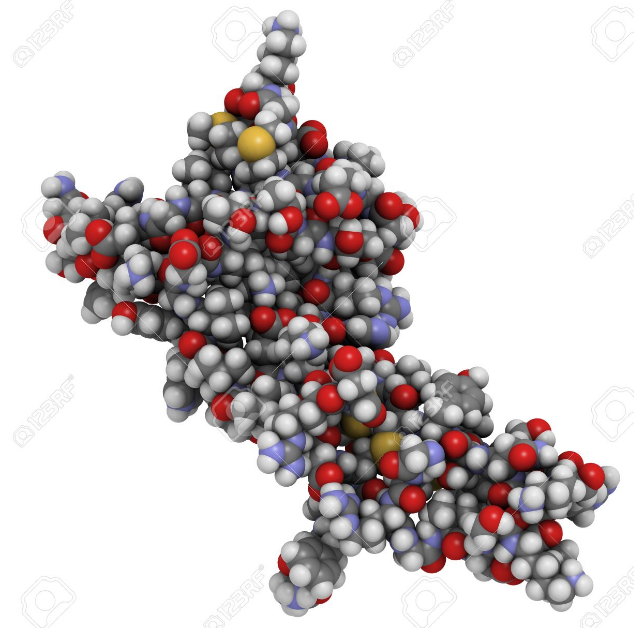
Naked (and clothed) Ca^{++}

A Simple Signaling Molecule (Token)



Quorum sensing signal

A More Complex Signaling Molecule (Token)



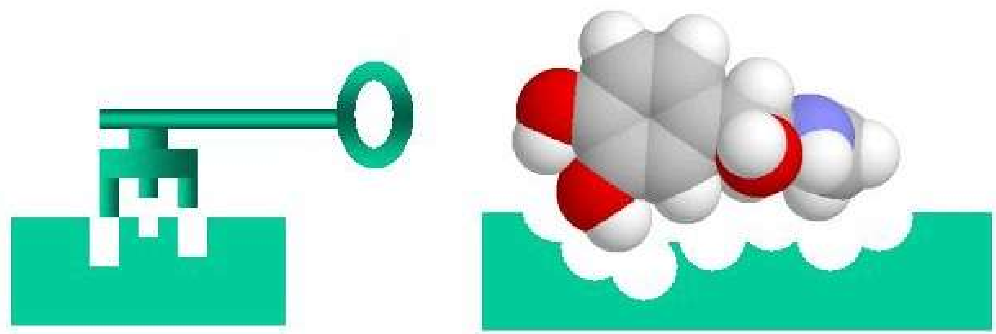
Nerve Growth Factor (protein)

What Is A ...

What Is A ...

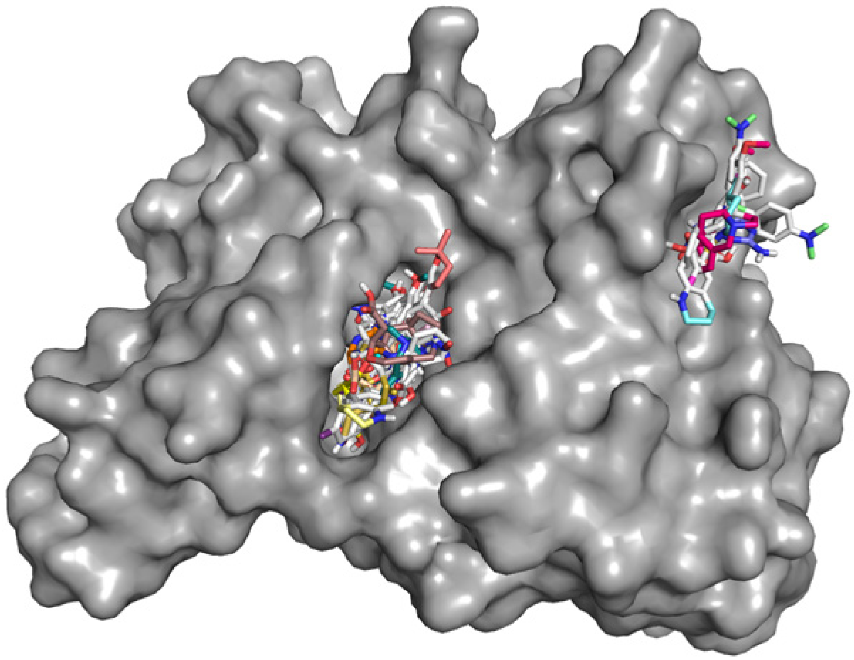
Signal Receptor

Receptor Specificity Cartoon



Ligand (token) docks with receptor (protein)

A More Detailed Receptor Specificity Cartoon



Ligands (tokens) dock with receptor (protein)

What Are Some ...

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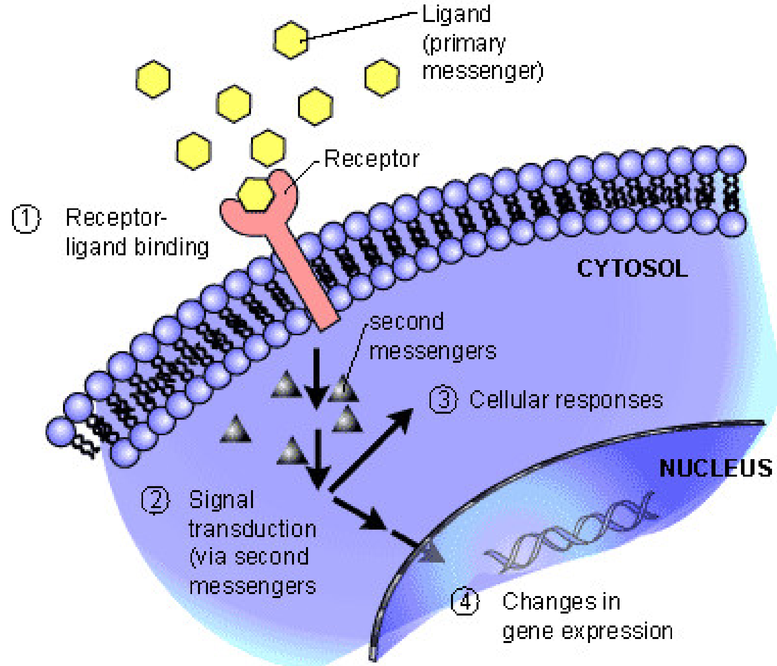
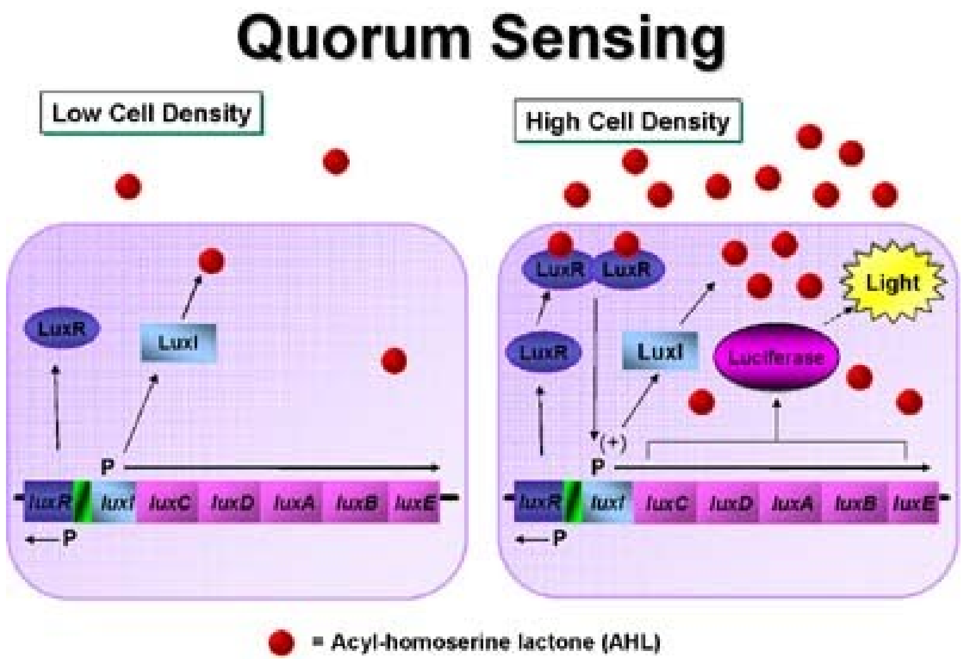
Communication Examples

Reception and Transduction Cartoon

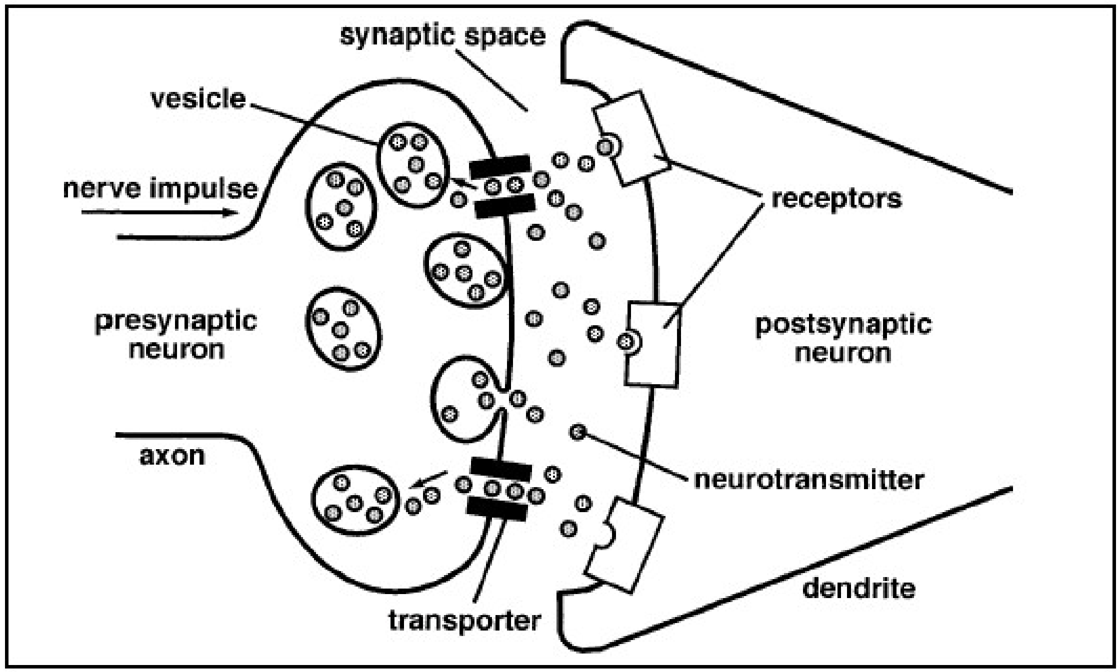


Ligand → Receptor → Gene Tickling

Identical Tokens: bacteria

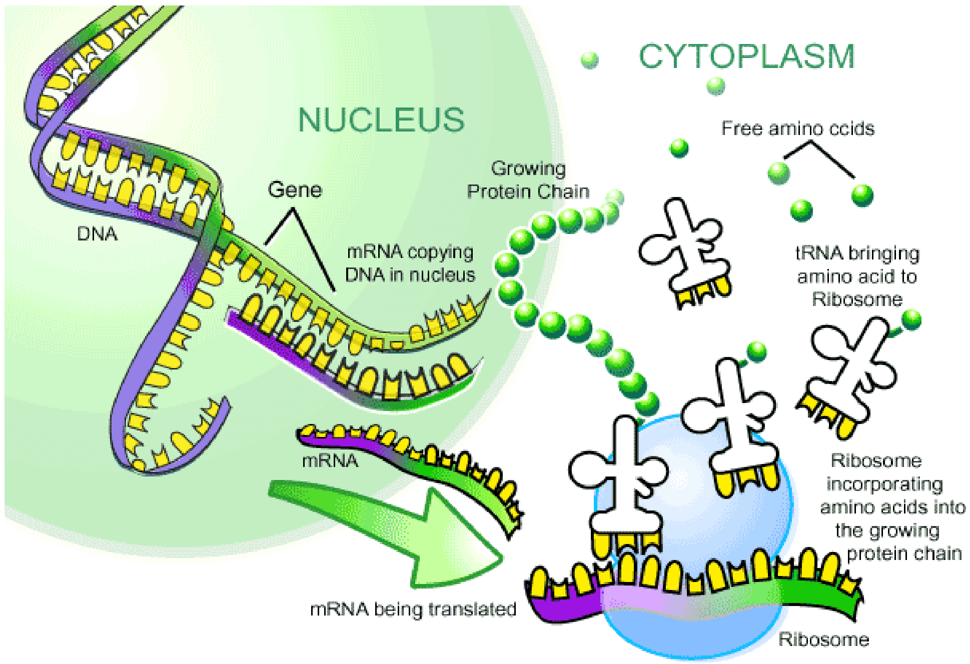


Identical Tokens: neurons



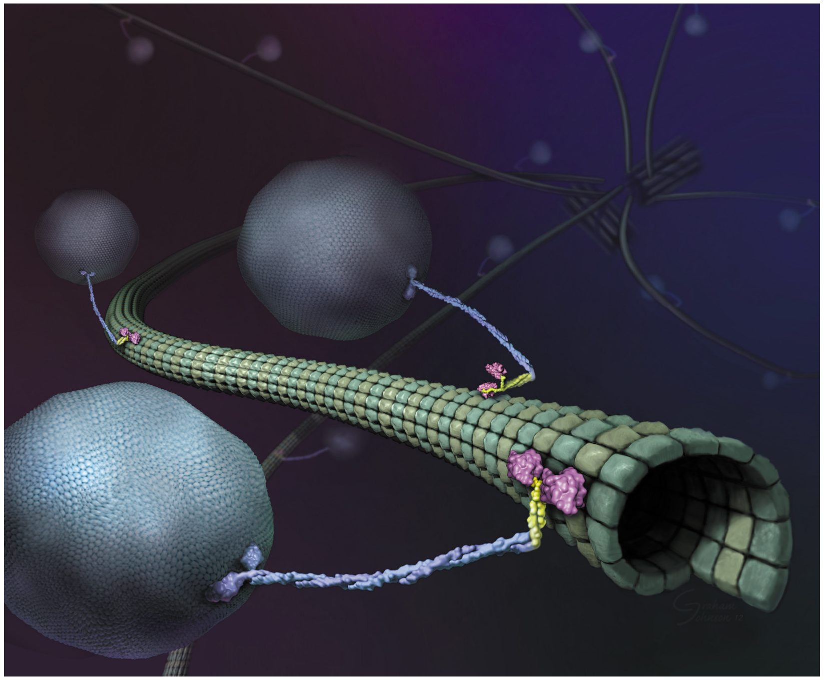
ACh release $\rightarrow$ postsynaptic uptake

Tokens with Payloads: transcription



Nuclear DNA $\rightarrow$ mRNA $\rightarrow$ Ribosome $\rightarrow$ Protein

Active Transport



Bacterial Microtubules

Inner Life of a Cell

TRES cool movie

`http://naturedocumentaries.org/3964/inner-life-cell/`

Today's Talk

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PURPOSEFUL MYOPIA

Transport (passive) Receptor Kinetics (ignore)

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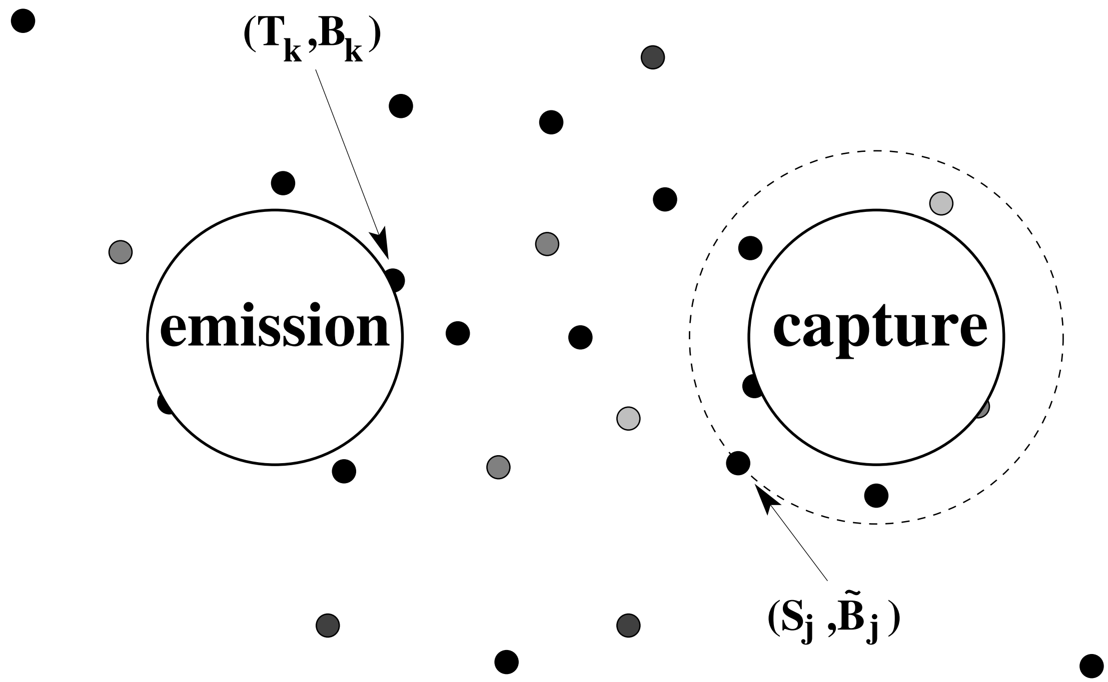
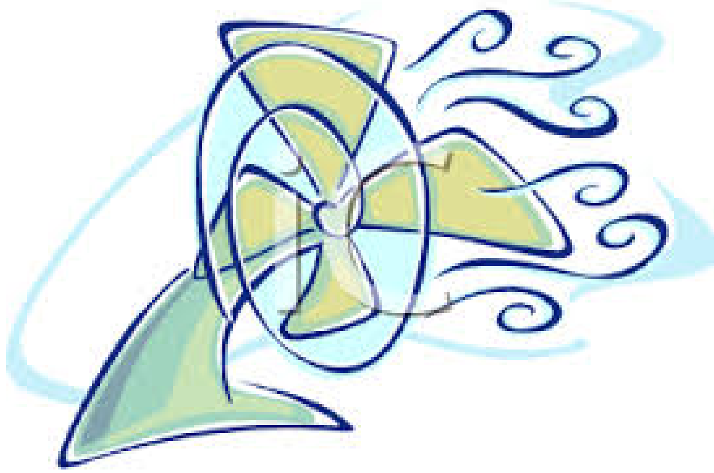
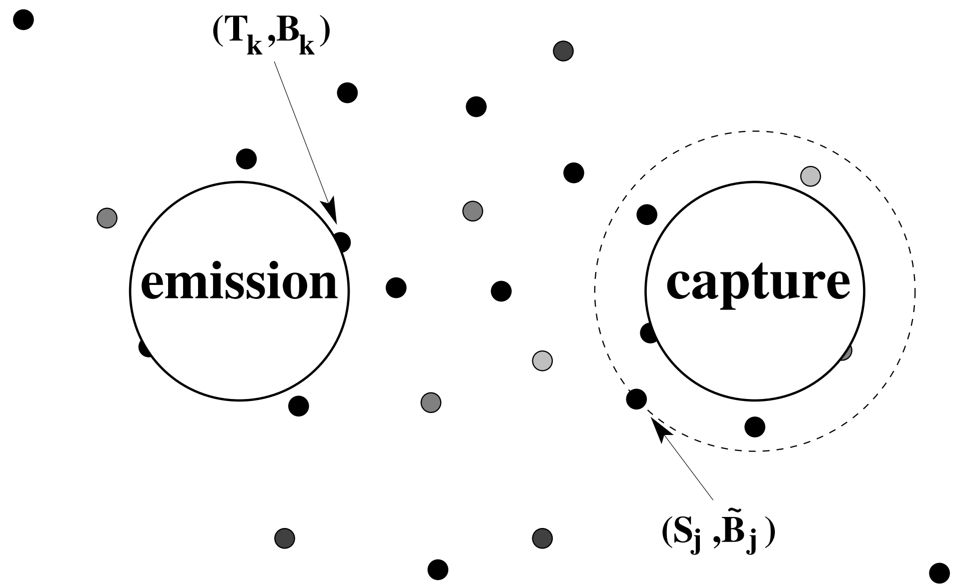


Coding → Emission → Transport → Capture → Decoding

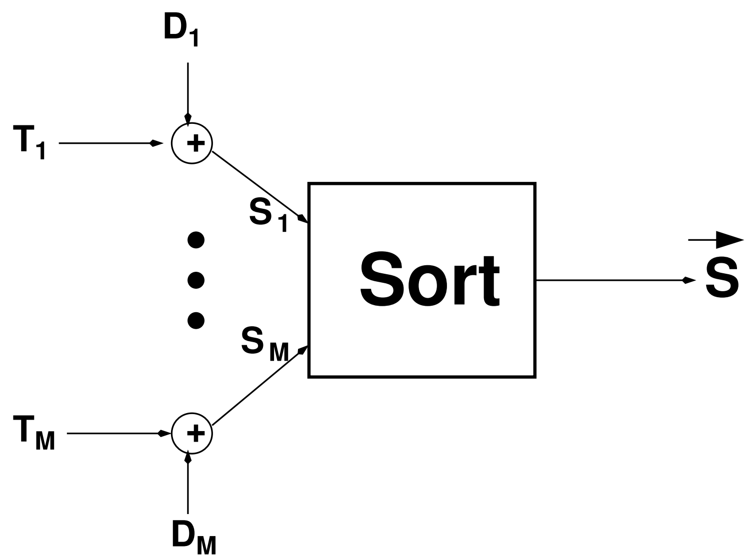
Could Even Add Some Drift





Coding $\rightarrow$ **Emission** $\rightarrow$ **Transport** $\rightarrow$ **Capture** $\rightarrow$ **Decoding**

Mathematical Abstraction



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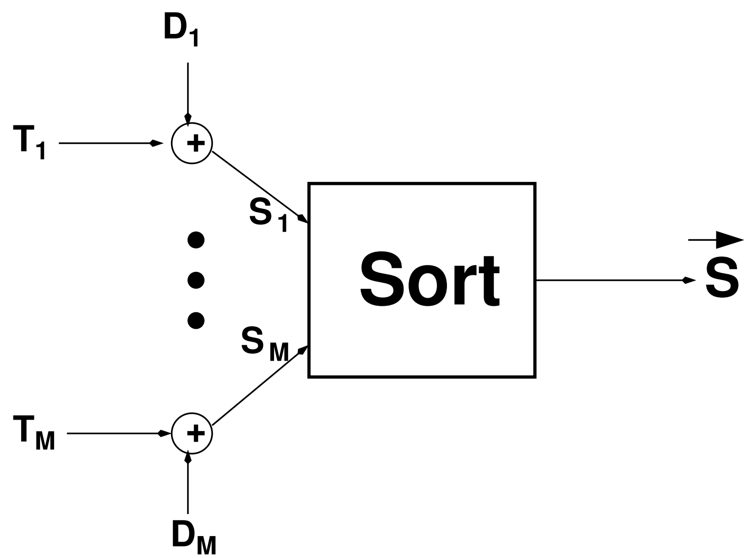

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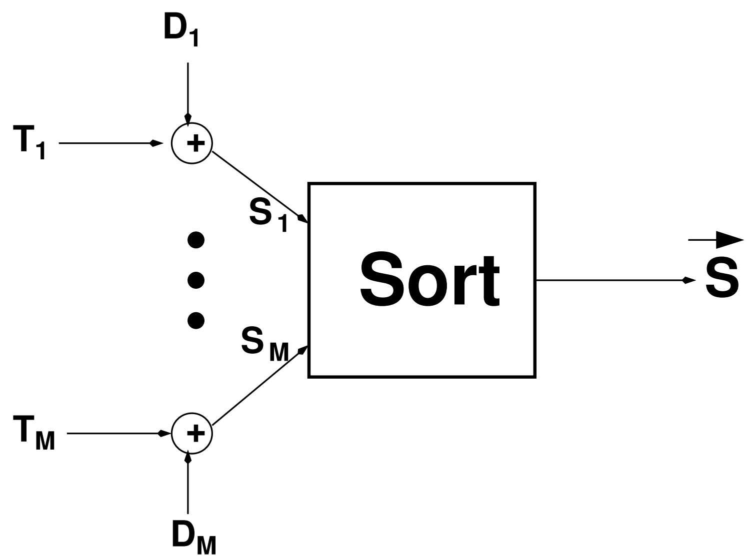

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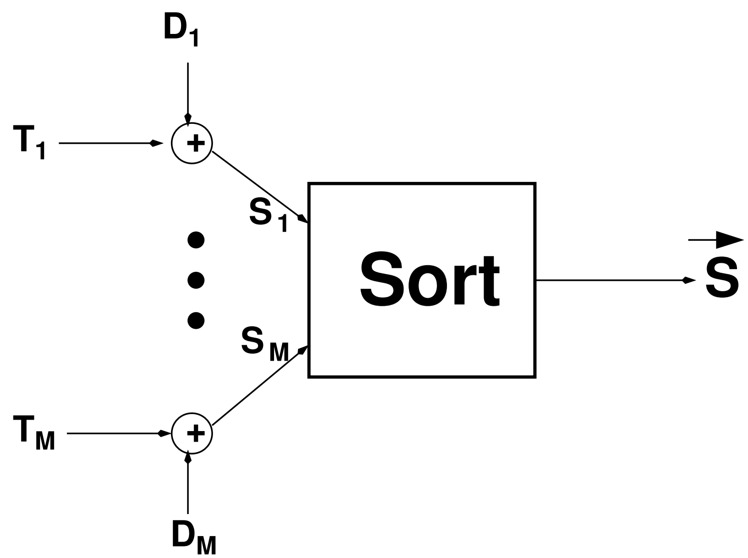

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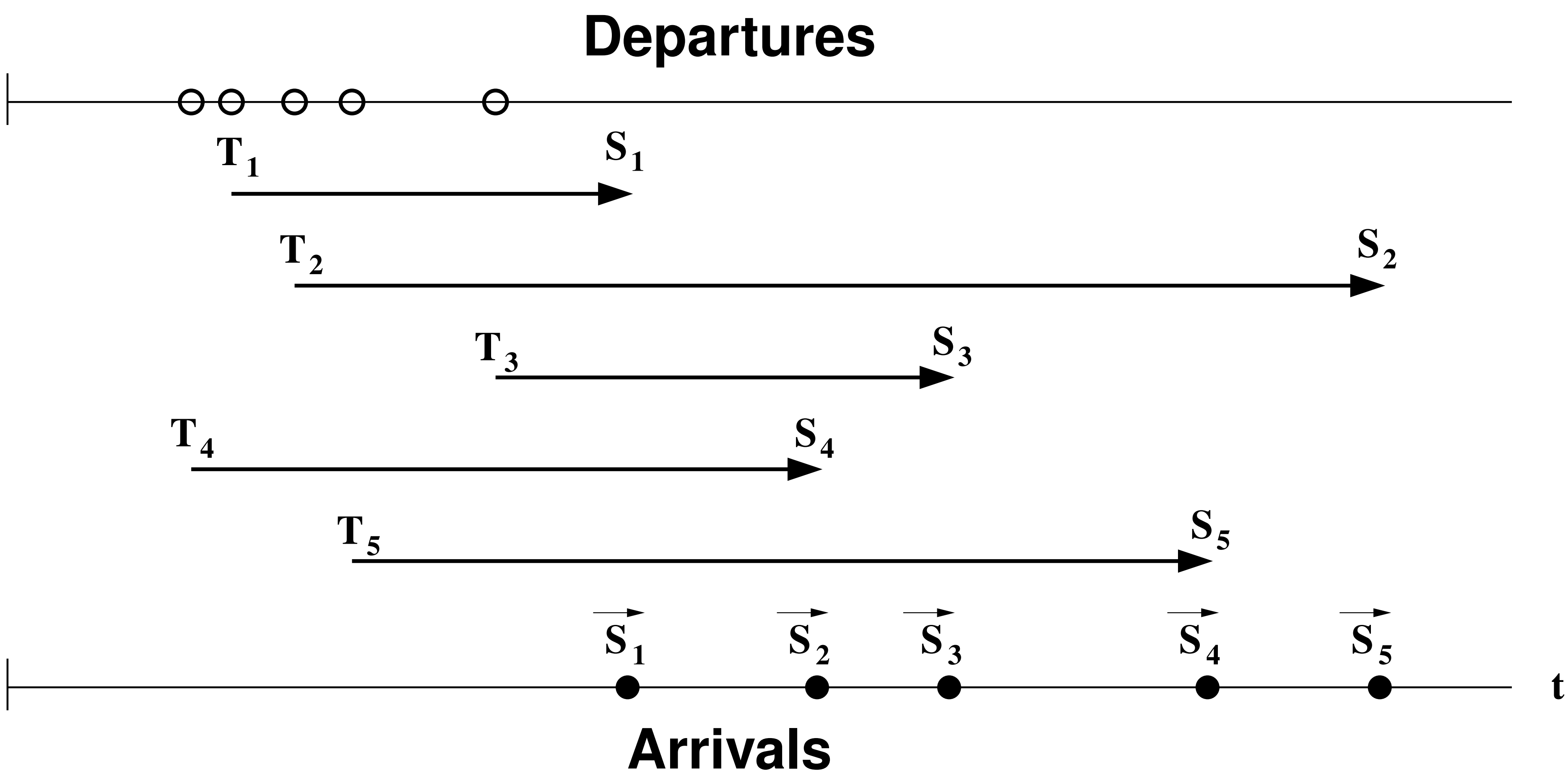
First passage time: $E[D] = 1/\mu$

Token Timing



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M tokens on an interval $\tau(M)$

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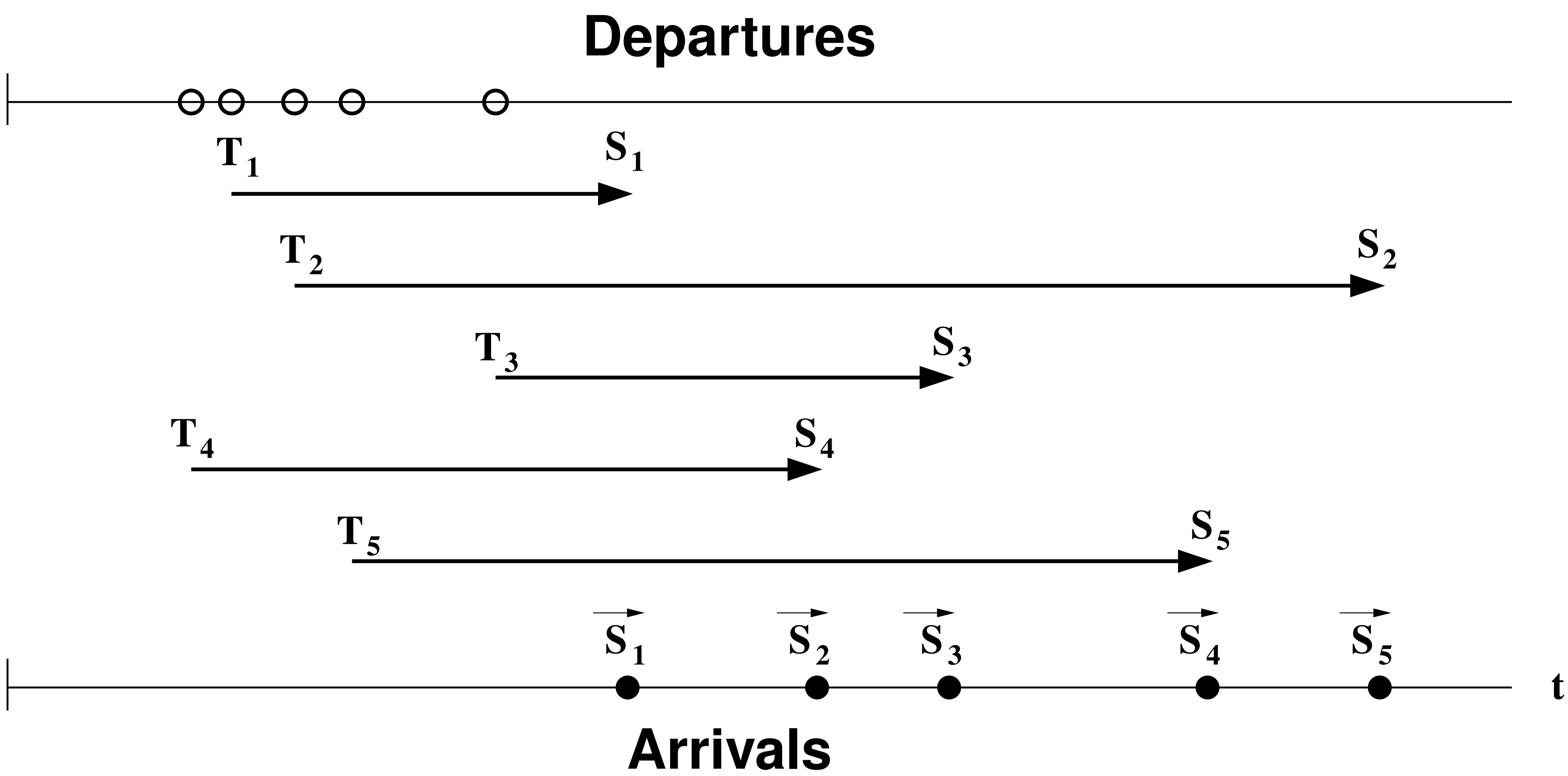
$$I(\vec{\mathbf{S}};\mathbf{T}) = h(\vec{\mathbf{S}}) - h(\vec{\mathbf{S}}|\mathbf{T}) = ?$$

Hypersymmetries



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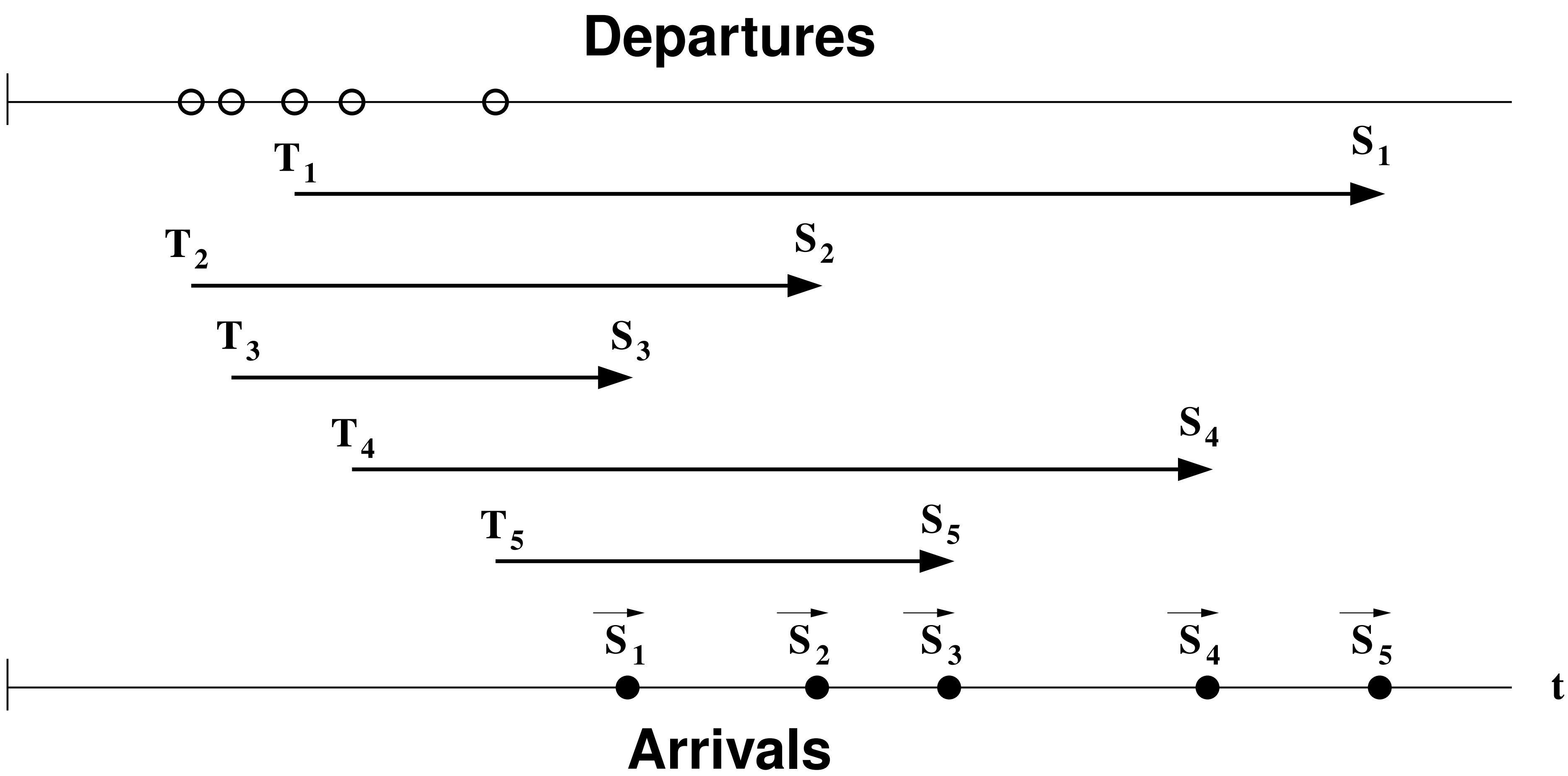

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$$\Downarrow$$

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$$I(\vec{\mathbf{S}}; \mathbf{T}) = \underbrace{h(\mathbf{S}) + H(\Omega|\vec{\mathbf{S}}, \mathbf{T})}_{\text{The Money!}} - \underbrace{(\log M! + h(\mathbf{D}))}_{\text{constant}}$$

Omitting the Details


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Set: $\rho \equiv \frac{M}{\tau(M)}$

Require: $E[D] < \infty$

Define: $\chi \equiv \frac{\mu \text{ (first passage rate)}}{\rho \text{ (token launch rate)}}$

$$C_m(M) = \max_{\text{hypersymm } f_{\mathbf{T}}()} \left(I(\vec{\mathbf{S}};\mathbf{T})M \right)$$

$$C_m = \lim_{M \to \infty} C_m(M)$$

$$C_t = \rho C_m$$

A Preparatory Exponential-Passage Motif

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$$\lim_{M \to \infty} \frac{1}{M} H(\Omega|\vec{\mathbf{S}}, \mathbf{T}) = e^{-\frac{1}{\chi}} \sum_{k=2}^{\infty} \left(\frac{1}{\chi} \right)^k (k\chi - 1) \frac{\log k!}{k!}$$

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Timing Channel Workup $\rightarrow$ Sequence# Size

Some Sleight of Hand Required

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Hypersymmetric input $\mathbb{T}$

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Remaining disorder: $H(\Omega|\vec{\mathbf{S}},\mathbf{t})$

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Hypersymmetric input $\mathbf{T}$

But CAN use *ordered* $\mathbf{T}$ at input

Decoding identifies sent $\mathbf{t}$

Remaining disorder: $H(\Omega|\vec{\mathbf{S}}, \mathbf{t})$

Average disorder: $H(\Omega|\vec{\mathbf{S}}, \mathbf{T})$

Energy

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Identical Tokens: c_0 joules per token

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Inscribed Tokens:

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substrate:	c_1 joules per token
payload B **bits:**	$B\Delta c_1$ joules per token
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Inscribed Tokens:

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$$H(\Omega|\vec{\mathbf{S}}, \mathbf{T}) \leq MK \leq \log M!$$

Timing-Only Bits/Joule

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Theorem 1.

$$\mathcal{C}_T \geq \frac{1}{c_0}\left(\log\chi + \underbrace{e^{-\frac{1}{\chi}}\sum_{k=2}^{\infty}\left(\frac{1}{\chi}\right)^k (k\chi - 1)\frac{\log k!}{k!}}_{H(\Omega|\vec{\mathrm{S}},\mathrm{T})/M\text{: average per-token order-uncertainty}}\right)$$

Payload-Only Bits/Joule

Theorem 2.

$$\mathcal{C}_P = \frac{B}{c_1 + \Delta c_1 \left(B + \min_{\mathbf{t}} \frac{1}{M} H(\Omega | \vec{\mathbf{S}}, \mathbf{t}) \right)}$$

Lemma 3.

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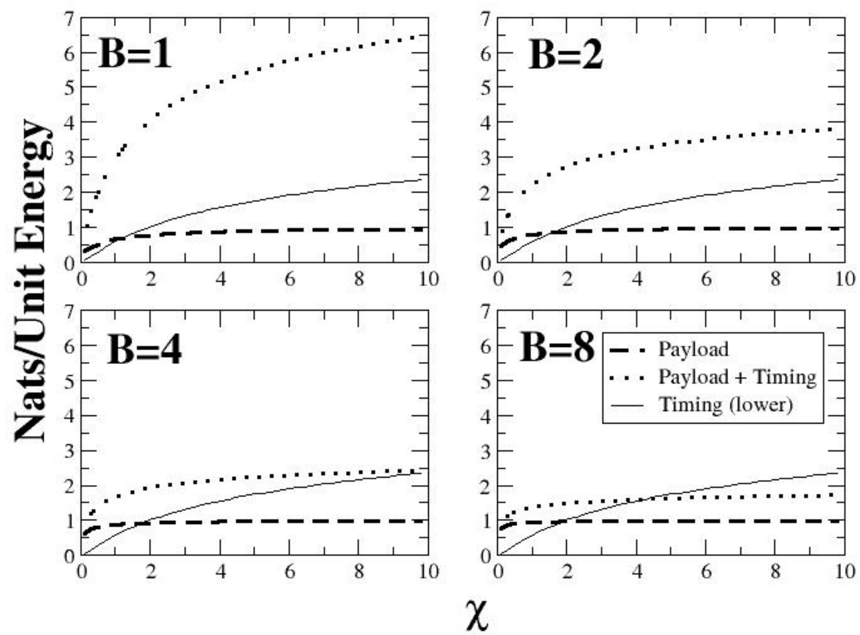
Payload + Timing Bits/Joule Lower Bound

Theorem 4.

$$\mathcal{R}_{P+T} \approx \frac{\log\left(1+\frac{\chi M}{e}\right)+B}{c_1+\Delta c_1\left(B+\underbrace{e^{-\frac{1}{\chi}}\sum_{k=2}^{\infty}\left(\frac{1}{\chi}\right)^k(k\chi-1)\frac{\log k!}{k!}}_{H(\Omega|\vec{\mathrm{S}},\mathrm{T})/M:\ \text{average per-token order-uncertainty}}\right)}$$

where $\mathcal{R}_{P+T} \leq \mathcal{C}_{P+T}$.

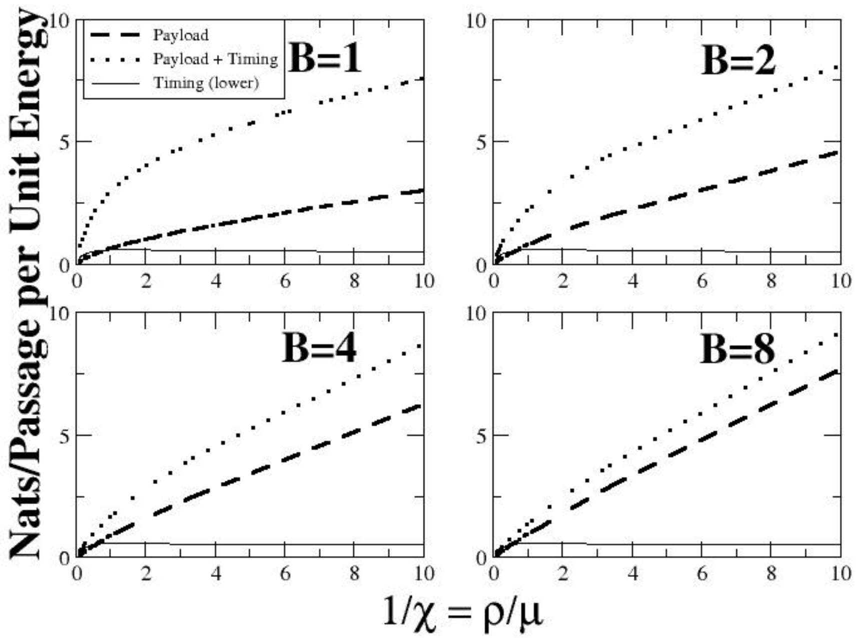
Info per Unit Energy



$\chi \leftrightarrow$ **passage rate per launch rate**

$c_0 = 1$, $c_1 = 0$, $\Delta c_1 = 1$

Info per Passage per Unit Energy



$\frac{1}{\chi} \leftrightarrow$ **launch rate per passage rate**

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And Now ....

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Numerical Play Time

Play Time Setup

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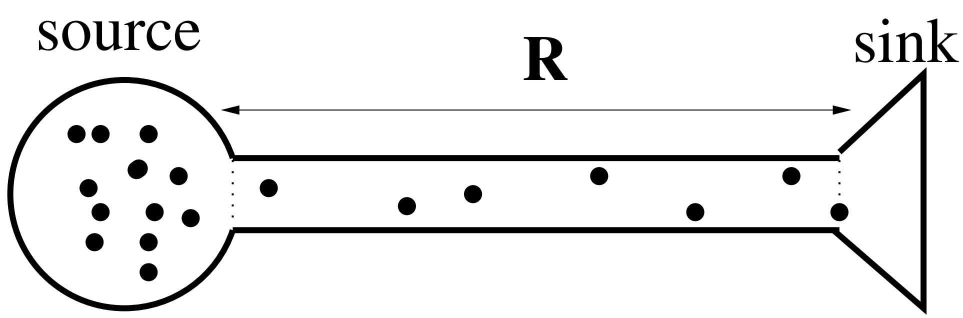

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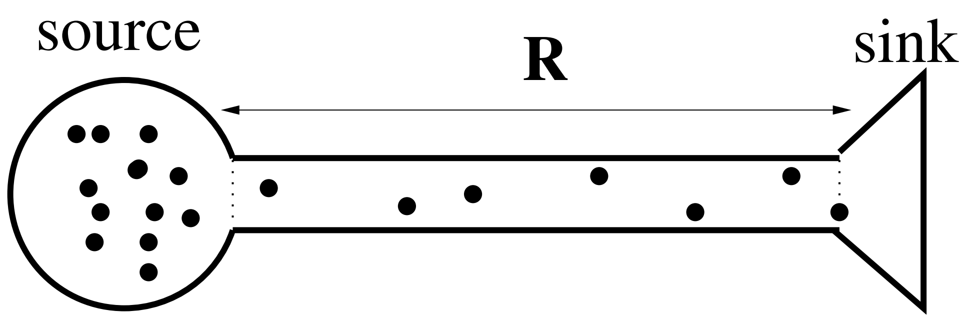

Protein Token Construction $4B$**ATP** $= 3.2B \times 10^{-19} J$

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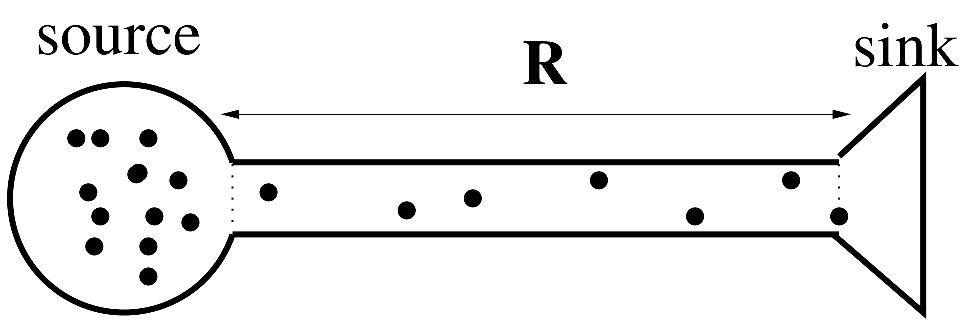

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Diffusion Coefficient, $\mathcal{D}$ **in air:** $\approx 10^{-5}m^2/s$

Mean First Passage Time, $E[D] \approx \frac{R^2}{2\mathcal{D}}$

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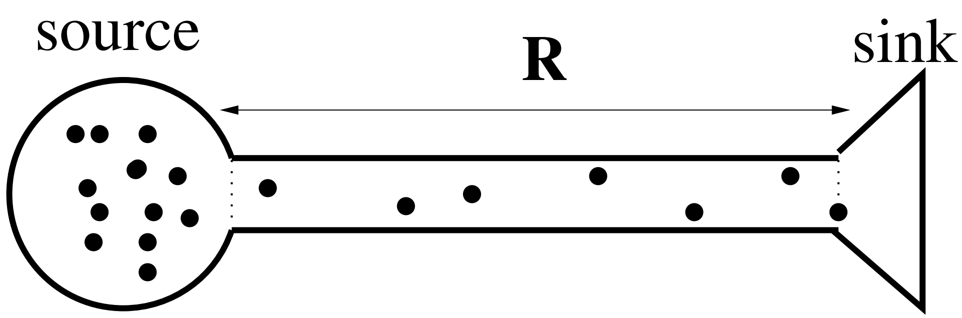

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Across a table ($1m$**):** $E[D] \approx 14hrs$ (need fan ☺)

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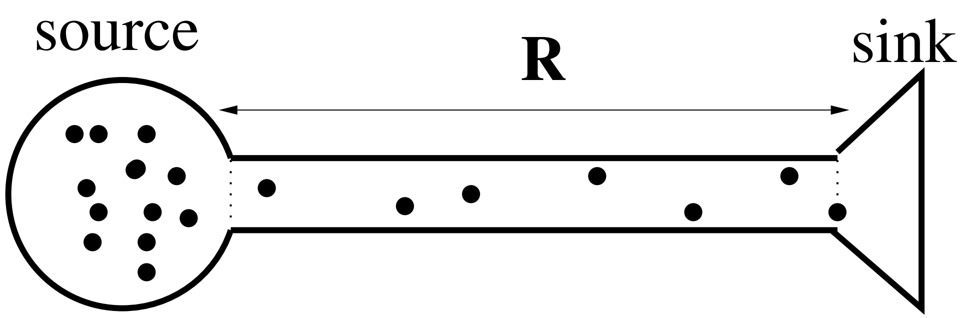

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Traipsing Over To The Payload Wild Side

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- **20 lb paper @** 1000dpi: 2×10^{10} bits/kg
- **DVD**: 3×10^{12} bits/kg
- **Magnetic Storage** with FeO_2: 2×10^{17} bits/kg
- **Optical Lithography** with SiO_2: 3.85×10^{18} bits/kg
- **E-beam Lithography** with SiO_2: 1.54×10^{21} bits/kg
- **STM** with Xe on Ni: 1.74×10^{22} bits/kg
- **RNA**: 3.6×10^{24} bits/kg
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BLACK HOLE: $\boldsymbol{r = 1\mu m \rightarrow 1.5 \times 10^{39}}$ **bits/kg**

Netflix *House of Cards* Delivery

Disk Farm Fantasy

Suppose token construction energy cost $\ll$ fan energy cost

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$1\mu g$ RNA per second $\Rightarrow 3.6 \times 10^{15}$ bits/sec

Appropriately Awed Response

Molecular Communication

Molecular Communication

Timing + Payload Framework

Molecular Communication

Timing + Payload Framework

Lower Bounds

Molecular Communication

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Molecular Communication

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Lower Bounds

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Molecular Communication

Timing + Payload Framework

Lower Bounds

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Scary Efficiencies and Rates

Molecular Communication

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Slow release with timing &/or small payload

Need Rate Efficiency?

Fast release with payload + timing or large payload

Scary Efficiencies and Rates

(beware transport latency)

If You Only Remember One Slide

A truck filled with storage media, driven across town, is a very reliable high bit rate channel.

–Comm. Theory Collective Subconscious

BUT ...

If You Only Remember One Slide

A truck filled with storage media, driven across town, is a very reliable high bit rate channel.

–Comm. Theory Collective Subconscious

BUT ...

A swarm of timed gnats

If You Only Remember One Slide

A truck filled with storage media, driven across town, is a very reliable high bit rate channel.

–Comm. Theory Collective Subconscious

BUT ...

A swarm of timed gnats with backpacks

If You Only Remember One Slide

A truck filled with storage media, driven across town, is a very reliable high bit rate channel.

–Comm. Theory Collective Subconscious

BUT ...

A swarm of timed gnats
with backpacks
in a breeze

If You Only Remember One Slide

A truck filled with storage media, driven across town, is a very reliable high bit rate channel.

–Comm. Theory Collective Subconscious

BUT ...

A swarm of timed gnats
with backpacks
in a breeze
could be better.