Molecular Communication Using Timing & Payload

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Bio-Inspired Wireless

Overview

Biology-Inspired Molecular Communication

• Eu/Prokaryotic systems intercommunicate

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 - Emission/reception of special molecules (tokens)

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Intriguing Science & Engineering

It's a Snake!

It's a Snake! It's a Tree!

It's a Snake! It's a Tree! It's a Wall!

It's a Snake! It's a Tree! It's a Wall! It's a Spear!

It's a Snake! It's a Tree! It's a Wall! It's a Spear! It's a Rope!

Chris Is Getting Old (and cranky?)

Overview

Chris Is Getting Old (and cranky?)

Is there a

Overview

Chris Is Getting Old (and cranky?)

Is there a

Unifying Elephant?

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Framework + Fundamental Limits Applications

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A (stab at a) Unified Framework

Energy Use Is Fundamental

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Token construction + Transport

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Token construction + Transport

Inscribed Matter Is Fundamental

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 $\text{m-RNA} \rightarrow 3.6 \times 10^{24} \frac{\text{bits}}{\text{kg}}$

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Timing Is Fundamental

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Token construction + Transport

Inscribed Matter Is Fundamental

 $m-RNA \rightarrow 3.6 \times 10^{24} \frac{bits}{kg}$

Timing Is Fundamental

Mean first passage time is key







Information-Theoretic Modeling

(for Roy, Narayan, Predrag, Waheed and Anand 🙂)



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Energy



Information-Theoretic Modeling

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Energy Bounds

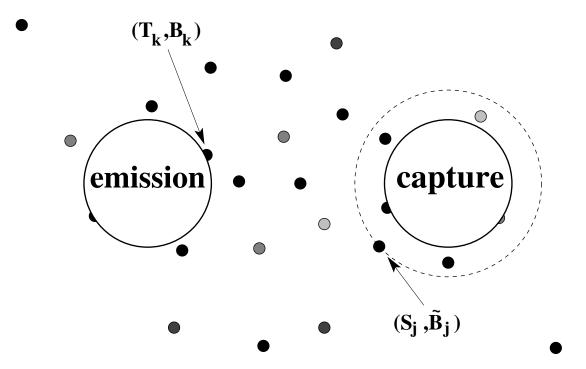


Information-Theoretic Modeling

(for Roy, Narayan, Predrag, Waheed and Anand 🙂)

Energy Bounds Ball Park Calculations

Diffusion Cartoon



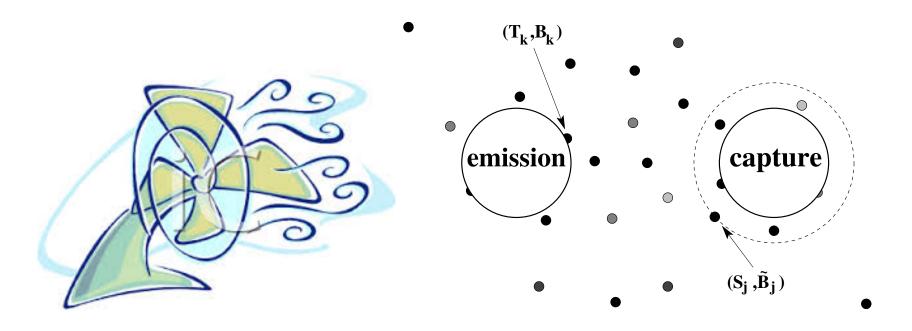
 $\textbf{Coding} \rightarrow \textbf{Emission} \rightarrow \textbf{Transport} \rightarrow \textbf{Capture} \rightarrow \textbf{Decoding}$

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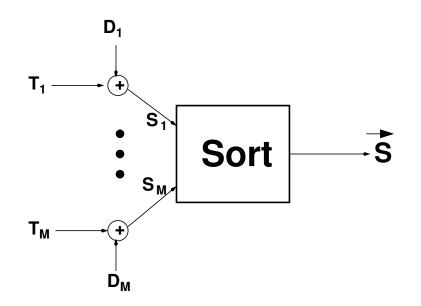
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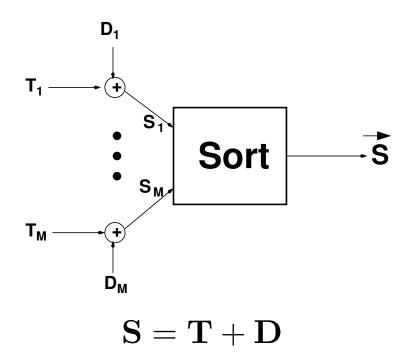
C. Rose

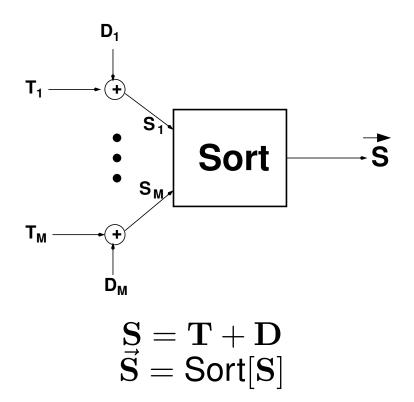
Diffusion with Drift Cartoon



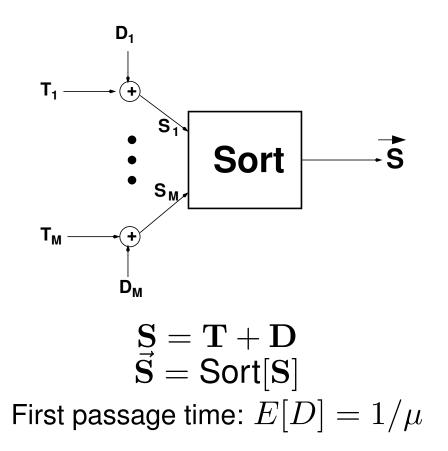
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Mutual Information M tokens on an interval $\tau(M)$

Mutual Information

$M \text{ tokens on an interval } \tau(M)$ $I(\mathbf{S}; \mathbf{T}) = h(\mathbf{S}) - h(\mathbf{S}|\mathbf{T})$ $= h(\mathbf{S}) - h(\mathbf{D})$ $\leq M (h(S) - h(D)), \quad \text{(i.i.d. D)}$

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Easy, Right?

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Easy, Right? $I(\vec{\mathbf{S}}; \mathbf{T}) = h(\vec{\mathbf{S}}) - h(\vec{\mathbf{S}}|\mathbf{T}) = ?$

 $\exists M! \mathbf{T} \stackrel{\Omega}{\to} \vec{\mathbf{T}}$

(permutation operator $P_{\Omega}()$, index Ω)

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We can balance any given I()-maximizing $f_{\mathbf{T}}()$ so that:

$$f_{\mathbf{T}}(\mathbf{T}) = f_{\mathbf{T}}(P_{\Omega}(\mathbf{T})) \quad \forall \Omega$$

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We can balance any given I()-maximizing $f_{\mathbf{T}}()$ so that:

$f_{\mathbf{T}}(\mathbf{T}) = f_{\mathbf{T}}(P_{\Omega}(\mathbf{T})) \quad \forall \Omega$ Consider Only Hypersymmetric \mathbf{T} $\max_{f_{\mathbf{T}}} I(\vec{\mathbf{S}}, \mathbf{T})$

 $f_{\mathbf{T}}()$ hypersymmetry $\rightarrow f_{\mathbf{S}}()$ hypersymmetry

 $f_{\mathbf{T}}()$ hypersymmetry $\rightarrow f_{\mathbf{S}}()$ hypersymmetry $f_D()$ non-singular $\rightarrow f_{\mathbf{S}}()$ continuous

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:: "Edges and Corners" of $f_{\mathbf{S}}()$ have **zero measure**

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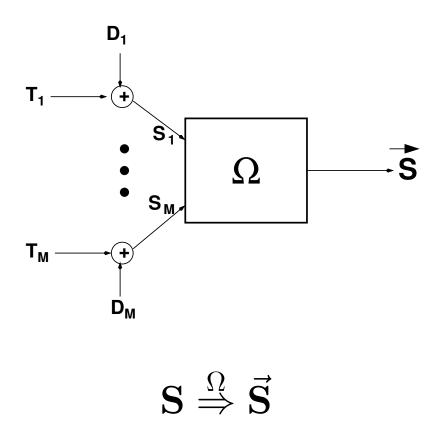
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 \therefore "Edges and Corners" of $f_{\mathbf{S}}()$ have **zero measure**

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$$h(ec{\mathrm{S}}) = h(\mathrm{S}) - \log M!$$

Channel Redux



 $\{\vec{\mathbf{S}},\Omega\}\leftrightarrow \mathbf{S}$

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 $\{\vec{\mathbf{S}}, \Omega\} \leftrightarrow \mathbf{S}$ $h(\mathbf{S}|\mathbf{T}) = h(\vec{\mathbf{S}}, \Omega|\mathbf{T}))$ $= h(\vec{\mathbf{S}}|\mathbf{T}) + H(\Omega|\vec{\mathbf{S}}, \mathbf{T})$

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$$I(\vec{\mathbf{S}};\mathbf{T}) = I(\mathbf{S};\mathbf{T}) - \left(\log M! - H(\Omega|\vec{\mathbf{S}},\mathbf{T})\right)$$

$$\{ \vec{\mathbf{S}}, \Omega \} \leftrightarrow \mathbf{S}$$

$$h(\mathbf{S}|\mathbf{T}) = h(\vec{\mathbf{S}}, \Omega | \mathbf{T}))$$

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$$I(\vec{\mathbf{S}};\mathbf{T}) = \underbrace{h(\mathbf{S}) + H(\Omega|\vec{\mathbf{S}},\mathbf{T})}_{\text{The Money!}} - \underbrace{(\log M! + h(\mathbf{D}))}_{\text{constant}}$$



TENSION!

Entropy maximized by independent ${\bf T}$

 $h(\mathbf{S}) \le \sum h(S_m)$

m

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$$h(\mathbf{S}) \le \sum_m h(S_m)$$

 $H(\Omega|\vec{\mathbf{S}},\mathbf{T})$ maximized by correlated \mathbf{T}

$$H(\Omega | \vec{\mathbf{S}}, \mathbf{T}) = \log M!$$

identical launch times $T_1 = T_2 = \cdots = T_M$

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My Past Personal Struggles

 \bigcirc \exists closed form results/bounds for $H(\Omega | \vec{\mathbf{S}}, \mathbf{T})$

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$$igcup_{f_{\mathbf{T}}()} \max h(\mathbf{S}) + H(\Omega|ec{\mathbf{S}},\mathbf{T}) \geq \mathbf{?}$$
 (ISIT'13)

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$$\stackrel{\scriptstyle \bullet \bullet}{=} \max_{f_{\mathbf{T}}(\mathbf{0})} h(\mathbf{S}) + H(\Omega | \vec{\mathbf{S}}, \mathbf{T}) \leq \mathbf{?} \text{ (ISIT'14)}$$

Timing Channel Details

Channel Use Formalities Handwaving



Timing Channel Details

Channel Use Formalities Handwaving



PUNCHLINE

$ho \equiv rac{M}{ ext{launch epoch}}$ all ok if mean first passage time $E[D] < \infty$

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Capacity Per Token

Define:

$$C_m(M) = \frac{1}{M} \max_{f_{\mathbf{T}}(I)} I(\vec{\mathbf{S}}; \mathbf{T})$$

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And:

$$C_t = \rho C_m$$

Construction Energy

Identical Tokens: *c*₀ joules per token

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Inscribed Tokens:

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Inscribed Tokens:

substrate: c_1 joules per tokenpayload B bits: $B\Delta c_1$ joules per tokensequence# K bits: $K\Delta c_1$ joules per token, where

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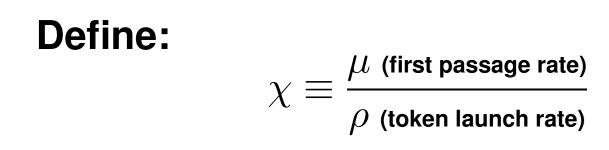
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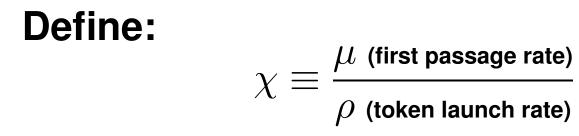
 $\frac{1}{M}H(\Omega|\vec{\mathbf{S}},\mathbf{T}) \le K \le \frac{1}{M}\log M!$





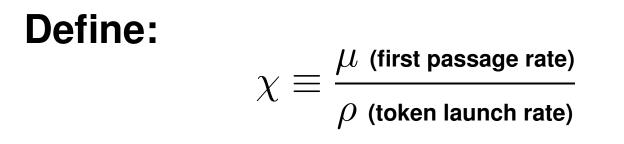






Min Max Lower Bound Parade

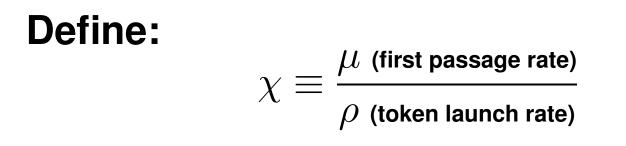
And Now ...



Min Max Lower Bound Parade

exponential first passage

And Now ...



Min Max Lower Bound Parade

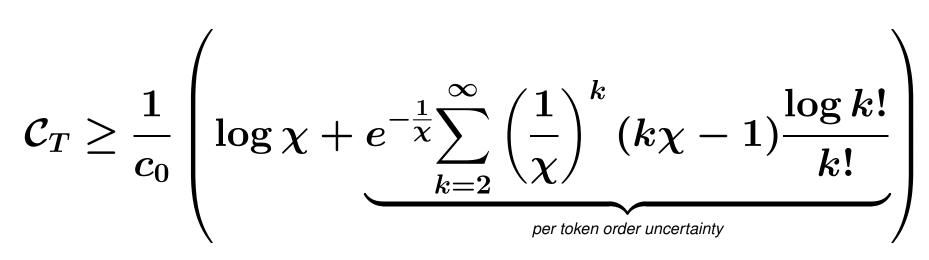
exponential first passage

(it's kinda the timing channel's "Gaussian")

Timing-Only Bits/Joule

Timing-Only Bits/Joule





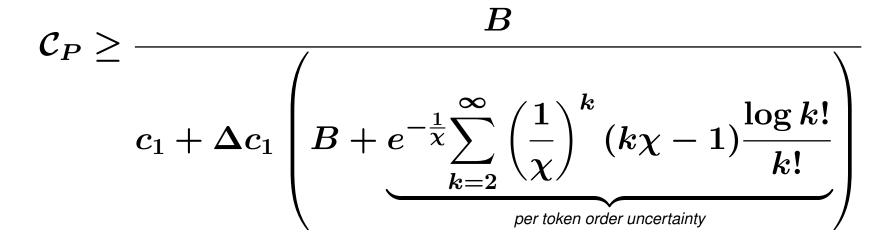
Payload-Only Bits/Joule

Theorem 2.

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$$C_P = \frac{B}{c_1 + \Delta c_1 \left(B + \min_{\mathbf{t}} \frac{1}{M} H(\Omega | \vec{\mathbf{S}}, \mathbf{t}) \right)}$$

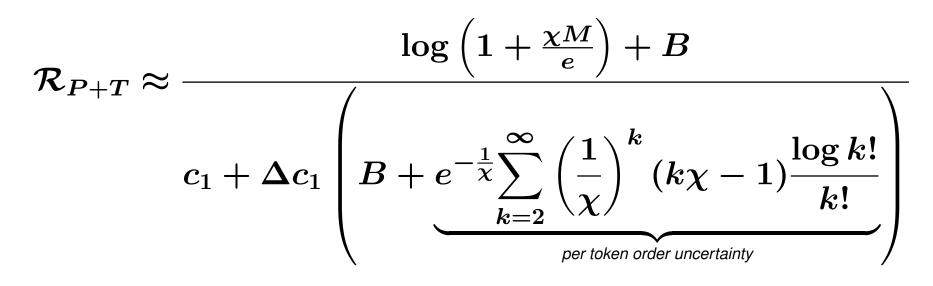
Lemma 3.



Payload + Timing Bits/Joule Lower Bound



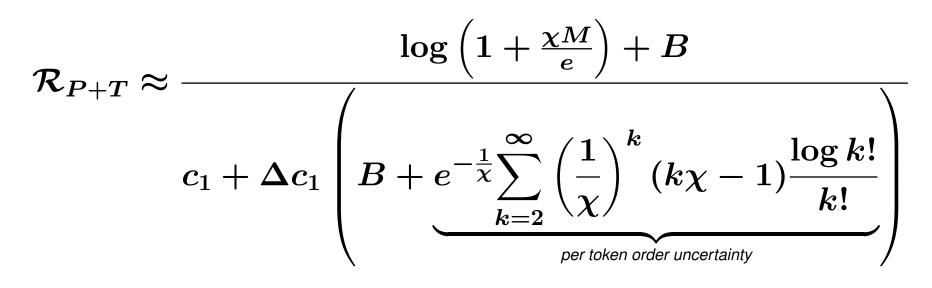
22



where $\mathcal{R}_{P+T} \leq \mathcal{C}_{P+T}$.

Payload + Timing Bits/Joule Lower Bound

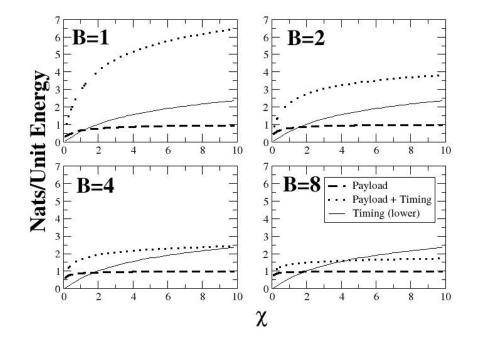




where $\mathcal{R}_{P+T} \leq \mathcal{C}_{P+T}$.

ASIDE: dumb header $(\frac{1}{M} \log M!)$: $\mathcal{C}_{P+T} \to 0$ in M

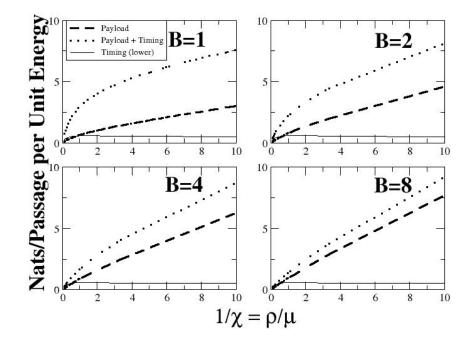
Info per Unit Energy



 $\chi \leftrightarrow$ passage rate per launch rate $c_0 = 1, c_1 = 0, \Delta c_1 = 1$

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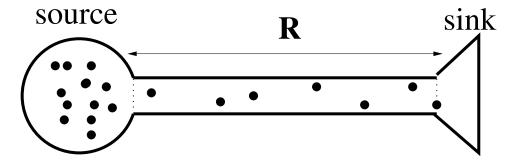
Info per Passage per Unit Energy

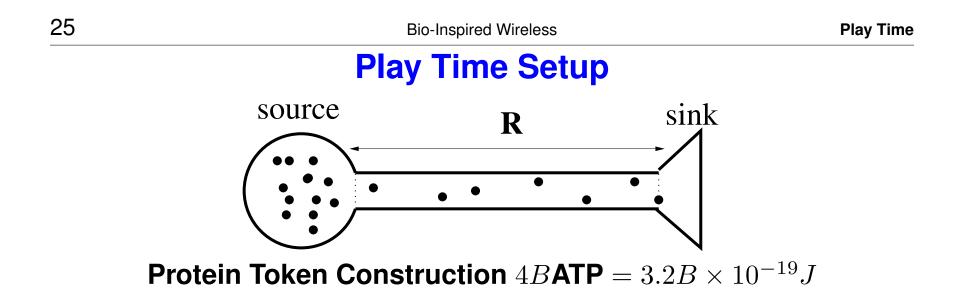


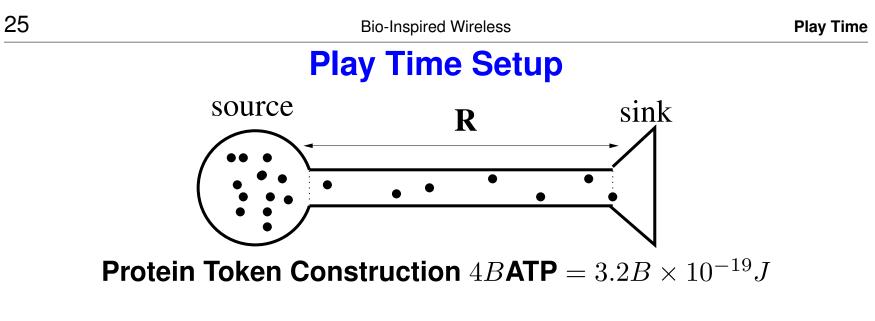
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Play Time Setup

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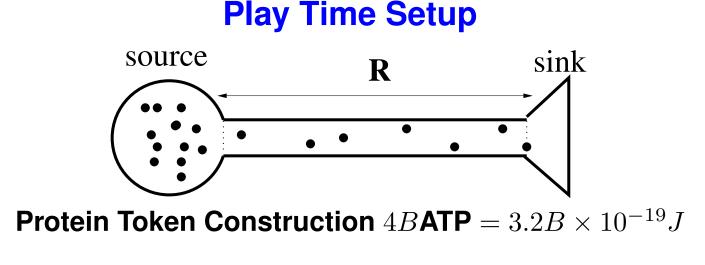




Diffusion Coefficient, \mathcal{D} :

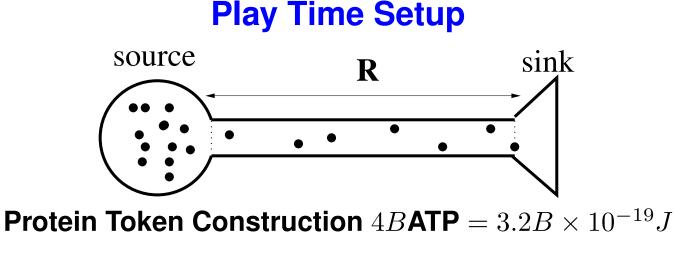
 $pprox 10^{-5} m^2/s$ in air $pprox 10^{-5} cm^2/s$ in water

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Diffusion Coefficient, \mathcal{D} : $\approx 10^{-5}m^2/s$ in air $\approx 10^{-5}cm^2/s$ in water

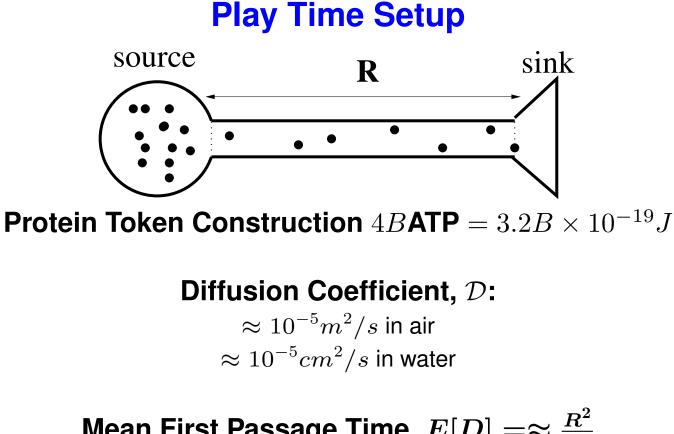
Mean First Passage Time, $E[D] = \approx \frac{R^2}{2D}$



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Mean First Passage Time, $E[D] = \approx \frac{R^2}{2D}$

Across a table (1*m*): $E[D] \approx 14hrs$ (need fan \bigcirc)



Mean First Passage Time, $E[D] = \approx \frac{R^2}{2D}$ Across a table (1m): $E[D] \approx 14hrs$ (need fan \bigcirc) Across a synapse (20nm): $E[D] = 0.2\mu s$

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$$rac{1}{\chi}=rac{
ho}{\mu}=1=B$$

$$\frac{1}{\chi} = \frac{\rho}{\mu} = 1 = B$$

Across a table: \approx bits/day/attojoule

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Across a table: \approx bits/day/attojoule Across a synapse: \approx Mb/s/attojoule

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Across a table: \approx bits/day/attojoule Across a synapse: \approx Mb/s/attojoule

$$\frac{1}{\chi} = \frac{\rho}{\mu} = 1000 = B$$
:

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$rac{1}{\chi}=rac{ ho}{\mu}=1000=B$: Across a table: pprox Kb/day/femtojoule

$$\frac{1}{\chi} = \frac{\rho}{\mu} = 1 = B$$

Across a table: \approx bits/day/attojoule Across a synapse: \approx Mb/s/attojoule

$\frac{1}{\chi} = \frac{\rho}{\mu} = 1000 = B$:

Across a table: \approx Kb/day/femtojoule Across a synapse: \approx Gb/s/femtojoule

Tantalizing



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Tantalizing



Suppose token construction energy cost \ll fan energy cost

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Tantalizing



Suppose token construction energy cost \ll fan energy cost

1mg RNA per second $\Rightarrow 3.6 \times 10^{18}$ bits/sec

Appropriately Awed Response



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Timing + Payload Framework

Summary

Timing + Payload Framework Lower Bounds

Timing + Payload Framework Lower Bounds Need Bit Efficiency?

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Slow release with timing &/or small payload

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Slow release with timing &/or small payload

Need Rate Efficiency?

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Slow release with timing &/or small payload

Need Rate Efficiency?

Fast release with payload + timing or large payload

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Slow release with timing &/or small payload

Need Rate Efficiency?

Fast release with payload + timing or large payload

Scary Efficiencies and Rates

Timing + Payload Framework

Lower Bounds

Need Bit Efficiency?

Slow release with timing &/or small payload

Need Rate Efficiency?

Fast release with payload + timing or large payload

Scary Efficiencies and Rates

(beware transport latency)