Supplementary Discussion for ET Might Write Not Radiate

Christopher Rose[1,] and Gregory Wright[2]

[1]Department of Electrical and Computer Engineering
Rutgers University
Piscataway, New Jersey 08854 USA

[2]Antiope Associates
18 Clay Street
Fair Haven, New Jersey 07704 USA

Energy Requirements for Radiative Communication

Consider the scenario of FIGURE A. The energy used to send an electromagnetic message is PT where P is the radiated power. A receiver at some distance D will capture some fraction of this power $\nu(D)P$ where $\nu(D)$ is defined as the energy capture coefficient of the receiver. We assume square law isotropic propagation loss with transmitting antenna gain

$$G_{\max} = \frac{8\pi^2 R^2}{\lambda^2} = 2\pi^2 \mathcal{A}^2 \quad , \tag{1}$$

for diffraction limited beams from a circular filled aperture of radius R operating at a wavelength λ [1]. The normalised aperture, $\mathcal{A} = 2R/\lambda$ is the aperture in units of the wavelength. (Our usage corresponds to that common in astronomy, where 'aperture' is the diameter of the antenna.) Equation (1) is only valid in the far-field, meaning distances at which the wavefronts are essentially spherical. We then have $\nu(D) = AG/4\pi D^2$ where A is the effective collecting area of the receiver. We note that $\frac{AG}{4\pi D^2} \leq 1$ by energy conservation.

Assuming additive Gaussian receiver noise, the Shannon capacity [2] in bits per second between the transmitter and receiver is

$$C = W \log_2 \left(\frac{PAG}{4\pi D^2 N_0 W} + 1 \right) \quad , \tag{2}$$

where N_0 is the background noise spectral intensity and W is the channel bandwidth. If we assume a transmission interval long enough that the usual information theoretic results for long codes can

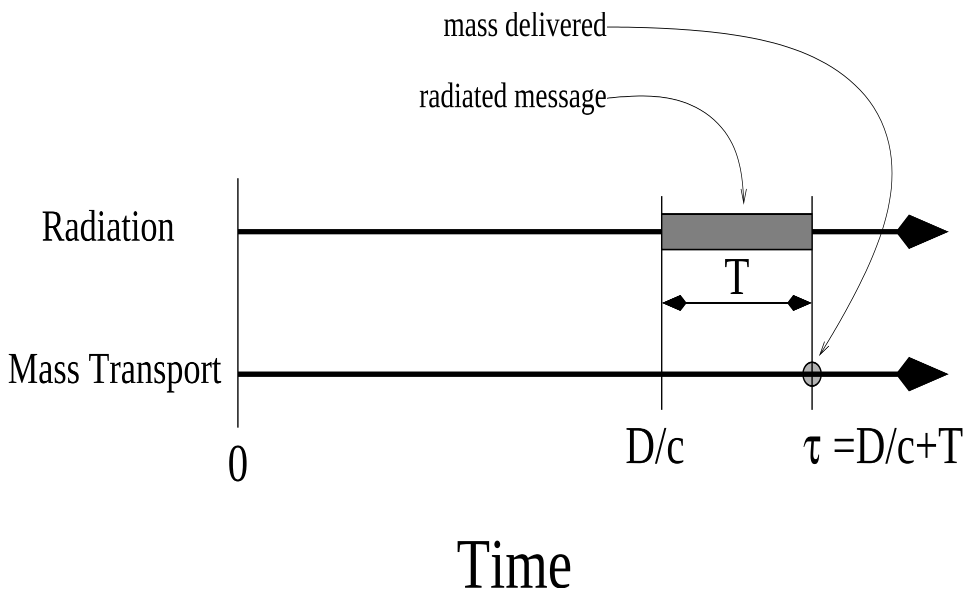

FIGURE A: Delivery timing diagram for written and radiated messages. *A message of B bits is sent over a distance D and received by a time* τ*. In the case of radiation, the transmission is of duration* T*, so the entire message is available at the receiver after a delay of* $\tau = D/c + T$*. In this way, the standard measure of communication efficiency, bits per joule, applies to both the electromagnetic radiation and inscribed matter cases thereby allowing direct comparison.*

be applied, the number of bits delivered for a transmission of duration T is $B = TC$. Notice that we have set the time required for the arrival of the complete message to $\tau = D/c + T$, identical to the inscribed matter delivery time shown in FIGURE A.

Since $E_r = PT$, we bound E_r from below by assuming a large "time-bandwidth" product TW to obtain

$$E_r \geq BN_0 \frac{4\pi D^2}{AG} \ln 2 \quad . \tag{3}$$

Equation (3) is a best case scenario with unlimited degrees of communication freedom and sidesteps the issues of preferred frequencies and bandwidths considered by Cocconi and Morrison [3], Townes [4, 5] and others. Thus, our energy estimate is conservative since no method of electromagnetic communication can use *less* energy than given in equation (3).

Assuming transmit and receive apertures both of size R, using equation (1), setting $A = \pi R^2$, and defining $\mathcal{D} = D/2R$ as normalised target distance results in

$$E_r \geq \frac{8 \ln 2}{\pi^2} BN_0 \left[\frac{\mathcal{D}}{\mathcal{A}} \right]^2 \quad . \tag{4}$$

Inscribed Matter Messages Can Be Dense

Calculating the energy required to deliver an inscribed matter message requires knowing the mass information density $\tilde{\rho}$ in bits kg^{-1}. Absolute bounds on mass information density have been described in [6]. These limits assume matter in its densest possible state and are far larger than what can be obtained practically with ordinary matter. In contrast, clear limits on the maximum possible information storage density for ordinary inscribed matter are unknown.

It is therefore useful to consider current empirical limits on mass information density. For example, information coded as single stranded RNA (such as the polio virus) has an information density of two bits per base (four possible bases) which at 330 kDaltons (5.5×10^{-19} grams) per kilobase gives an information density of $\tilde{\rho}_{\mathrm{RNA}} = 3.63 \times 10^{24}$ bits kg^{-1}. This figure makes singled stranded RNA the medium with the largest mass information density for which we have an "existence proof."

If we could build stable alloys of the lightest solid elements (Li, Be) with arbitrary placement of the atoms (average molecular weight 8), we could achieve $\tilde{\rho}_{\mathrm{LiBe}} = 6.022 \times 10^{23}/8g = 7.5 \times 10^{25}$ bits kg^{-1}. A scanning tunneling microscope (STM) can place an equivalent of about 10^{15} bits per square inch using individual xenon atoms on a nickel substrate [7]. The per bit dimension is then 0.8 nm on a side. By somewhat arbitrarily assuming a 10 nm nickel buffer between layers and that the density of nickel (8.9 g cm^{-3}) will predominate owing to the relatively thick layering, we have $\tilde{\rho}_{\mathrm{STM}} = 1.8 \times 10^{22}$ bits kg^{-1} or about two orders of magnitude smaller than RNA storage. We can also define a more conservative nickel-based material with exactly 1000 nickel atoms per bit. The associated mass information density is $\tilde{\rho}_{\mathrm{Ni}} = 6.022 \times 10^{23}/(1000 \times 58.7g) \approx 10^{22}$ bits kg^{-1}.

At the lower end of the $\tilde{\rho}$ spectrum, consider that the Voyager spacecraft (comparison suggested by L. Sage) carry plaques bearing inscribed messages — pictograms, images and audio recordings — encoded as on a phonograph record and including a stylus with which to play it. If we assume that the total information content of these messages is under 10^9 bits, then at a total weight of approximately one ton (about 909 kg) [8], the mass information density of the Voyager craft is approximately $\tilde{\rho}_{\mathrm{voyager}} = 1.1 \times 10^6$bits kg^{-1}.

Inscribed Matter Message Assembly Energy Is Small

Message assembly is part of the inscribed matter energy budget, and even if in theory this assembly energy is zero [9], it is useful to have empirical bounds. At the densest encodings we can envision of one atom per bit, we can imagine that assembling a message might require its construction from constituent atoms. The energy required is on the order of 2 eV per bond. Thus, for RNA storage with an average of about 32 atoms and 36 covalent bonds per base, we have about 36 eV per bit as an upper bound. For our $\tilde{\rho}_{\mathrm{Ni}} = 10^{22}$ bits kg^{-1} nickel-based material with 1000 atoms per bit, the maximum construction energy is 2000 eV.

For RNA inscribed matter, metabolic measurements provide another empirical bound on assembly energy. The genome of E.Coli has $4,639,221$ bases [10] and a cell takes approximately 20 minutes to divide. Replicating the genetic material consumes $60,000$ ATP molecules per second [11]. With 8×10^{-20} Joules per ATP molecule we infer that message assembly consumes

6.2×10^{-19}J bit$^{-1} \approx 3.9$eV bit^{-1}. The implication is that actual construction energy can be much lower than the per-atom upper bound.

Now, assume a launch velocity of $\bar{v} = 42$ km sec^{-1} (solar escape from earth orbit). For RNA inscribed matter we have launch energy $(4.2 \times 10^4 \text{m sec}^{-1})^2/2\tilde{\rho}_{\text{RNA}} = 2.45 \times 10^{-16}$ or approximately 1500 eV bit^{-1}. For $\tilde{\rho}_{\text{Ni}} = 10^{22}$ bits kg^{-1} we have 5.5×10^5 eV bit^{-1}. Thus, we expect inscription energy costs to be negligible relative the launch cost at the speeds required for interstellar delivery.

Escape from Gravitational Wells

Suppose v_e is the escape velocity from the launch platform. We want the terminal velocity far from the platform to be αv_e with $\alpha > 1$. We then require $\frac{1}{2}m\alpha^2 v_e^2$ to accelerate the mass appropriately in the absence of the potential well. In the presence of the potential well we spend energy $\frac{1}{2}m\alpha^2 v_e^2 + \frac{1}{2}mv_e^2$. We then have the ratio

$$\frac{E_{\text{potential}}}{E_{\text{no potential}}} = 1 + 1/\alpha^2 \quad , \tag{5}$$

which represents a gravitational well escape penalty factor. As α gets large, the final velocity is much greater than the escape velocity, so the energy needed to escape is relatively less significant. For $\alpha = 2$, the penalty is 1.25.

Radiated Messages Must Be Repeated

A radiated message is lost if the target is not listening when the message passes by, and over interstellar distance, it seems unlikely that the sender will know with high probability whether the target is listening. Therefore, we ask a question: how many electromagnetic message repetitions are necessary to meet a successful detection probability criterion Φ_d?

We will assume as a best case that the listener is sure to decode any incoming radiated message. That is, sure detection seems a not unreasonable assumption if the signal purposely contains correlation peaks at a variety of time scales, and we simply ignore the issue of whether the listener's antennas are pointed in the right direction. Under these assumptions, the problem for the sender is to compose a transmission schedule which meets the detection probability criterion Φ_d and minimises energy. Although precisely stated, the problem is imprecise mainly because the underlying random variables associated with listening civilization emergence and decline cannot be characterised given our empirical sample size of 1.

Nonetheless, we will formally define optimal scheduling problems under two different assumptions on the time course for birth and death of listening civilizations. We first assume a civilization emerges, declines, and does not re-emerge. We then assume a renewal process where civilizations repeatedly emerge and decline.

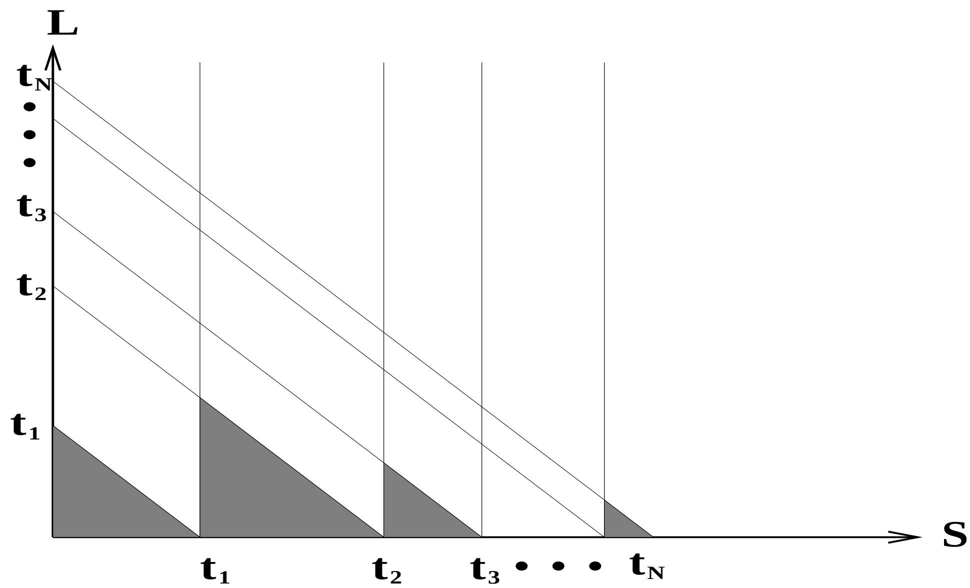

FIGURE B: No-detection regions for single civilization emergence and decline. *The arrival schedule is* $\{t_i\}$, $i = 1,2,\cdots,N$ *and the shaded regions comprise events* (S,L) *such that no message in the schedule arrives while the civilization is listening. We seek to minimise the probability that no message is received, and therefore to maximise the detection probability* Φ_d.

Single Transient Civilizations

We define the non-negative independent random variables S and L as the times when a civilization starts listening for messages and the civilization lifetime, respectively. $f_S(s)$ and $f_L(\ell)$ are the probability densities for S and L. Then, given message arrival times of $\{t_i\}$, $i = 1,2,\cdots,N$, the probability of failure (decoding no messages) is

$$1-\Phi_d = \sum_{k=1}^{N}\int_{t_{k-1}}^{t_k} f_S(s)\int_0^{t_k-s} f_L(\ell)d\ell ds = \sum_{k=1}^{N}\int_{t_{k-1}}^{t_k} f_S(s)F_L(t_k-s)ds \quad , \tag{6}$$

where $F_L()$ is the cumulative distribution function of L and the time reference t_0 is zero. The right side of equation (6) is simply integral of the joint PDF $f_L(\ell)f_S(s)$ over the shaded area shown in FIGURE B. That is, the message is not detected for a schedule $\{t_i\}$, $i = 1,2,\cdots,N$ when random variable pairs (S,L) lie in the shaded regions.

As a worst case analysis, we compute the optimal schedule by assuming exponential distributions on S and L with means $1/\lambda$ and $1/\mu$ respectively. We choose exponential distributions since they maximise distribution entropy subject to a constraint on the mean [2] and thus imply maximum uncertainty about S and L.

With $f_S(s) = \lambda e^{-\lambda s}$ and $F_L(\ell) = 1-e^{-\mu\ell}$ equation (6) becomes

$$\Phi_d = \frac{\lambda}{\mu-\lambda}\sum_{k=1}^{N} e^{-\lambda t_k}\left(1-e^{-(\mu-\lambda)(t_k-t_{k-1})}\right) \quad . \tag{7}$$

Taking partials with respect to t_k yields

$$\frac{\partial \Phi_d}{\partial t_k} = \frac{\lambda}{\mu-\lambda}\left[-\lambda e^{-\lambda t_k} + \mu e^{-\lambda t_k} e^{-(\mu-\lambda)(t_k-t_{k-1})} - (\mu-\lambda) e^{-\lambda t_{k+1}} e^{-(\mu-\lambda)(t_{k+1}-t_k)}\right] \quad . \tag{8}$$

Simplifying, defining $\Delta_k = t_k - t_{k-1}$, and setting to zero to find stationary points yields

$$\mu e^{-(\mu-\lambda)\Delta_k} - (\mu-\lambda)e^{-\mu\Delta_{k+1}} = \lambda \quad . \tag{9}$$

Then, defining $x_k = e^{-\mu\Delta_k}$ we have the second order difference equation

$$x_{k+1} = \frac{\mu}{\mu-\lambda} x_k^{1-\frac{\lambda}{\mu}} - \frac{\lambda}{\mu-\lambda} \quad , \tag{10}$$

for stationary points $\{x_k\}$. Φ_d can therefore be numerically optimised by choice of Δ_1.

Examining equation (10), we note that if $x_{k+1} > x_k > 0$ then x_k will increase monotonically without bound. Likewise, if $x_k > x_{k+1} > 0$ then x_k will decrease toward zero monotonically. To determine which is the case, we solve

$$x = \frac{\mu}{\mu-\lambda} x^{1-\frac{\lambda}{\mu}} - \frac{\lambda}{\mu-\lambda} \quad , \tag{11}$$

rewrite it as

$$x^{1-\frac{\lambda}{\mu}} - (1-\frac{\lambda}{\mu})x = \frac{\lambda}{\mu} \quad , \tag{12}$$

and note that $x^{1-\frac{\lambda}{\mu}}$ is concave in x. Thus, $x^{1-\frac{\lambda}{\mu}} - (1-\frac{\lambda}{\mu})x$ is also concave in x. By the definition of x_k we must have $0 \leq x_k \leq 1$ and the extremum of $x^{1-\frac{\lambda}{\mu}} - (1-\frac{\lambda}{\mu})x$ occurs at $x = 1$. So we must have

$$x_{k+1} \leq x_k \quad , \tag{13}$$

with equality if and only if $x_k = 1$. Then, since $\nexists\, x_k$ other than $x_k = 1$ for which $x_{k+1} = 1$, the sequence x_k decreases monotonically toward zero for all initial conditions except $x_1 = 1$. Thus, the sequence $\{\Delta_k\}$ must increase without bound unless $\Delta_1 = 0$, and a unique stationary sequence $\{x_k\}$ exists for each Δ_1. We can then numerically seek the Δ_1 which maximises Φ_d for a given N.

With mean times of $1/\lambda = 10^9$ years for civilization start and a mean lifetime of $1/\mu = 10^8$ years, iterative numerical calculations show that $N = 2\times10^3$ and $N = 2\times10^5$ messages are necessary to achieve $\Phi_d = 0.99$ and $\Phi_d = 0.9999$, respectively.

Since iterative numerical calculation can be time-consuming, it is also useful to analytically approximate the required N by assuming uniform interarrival intervals Δ which are small compared to the variation in $f_S(s)$ and $F_L(\ell)$. We then have

$$1-\Phi_d \approx \sum_{k=1}^{N} \frac{\Delta}{2}\left(f_S(t_{k-1})F_L(\Delta) + f_S(t_k)F_L(0)\right) \approx \frac{1}{2}F_L(\Delta) \quad , \tag{14}$$

assuming sufficiently large N.

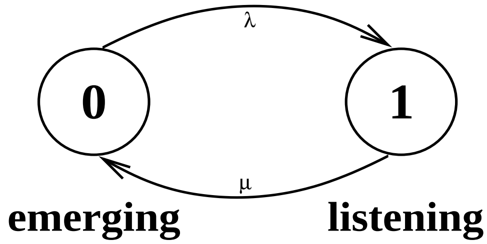

FIGURE C: Markov model for sequential transient civilizations. *Time to civilization emergence is exponential with mean* $1/\lambda$ *and lifetime is also exponential with mean* $1/\mu$*.*

We can compare the exact result to the small fixed Δ approximation by first finding Δ such that the criterion is satisfied. Using equation (14) and exponential distributions we have

$$\Delta = -\frac{1}{\mu}\log(1-2\Phi_f) \quad , \tag{15}$$

where $\Phi_f = 1-\Phi_d$. We then must choose $N\Delta$ large enough so that we are reasonably sure (to within our criterion level Φ_d) to have an arrival after the target is listening. To this end we set $F_S(N\Delta) = 1-\Phi_f/10$. Solving for N and rearranging yields

$$N = \frac{\mu}{\lambda}\frac{\log(\Phi_f/10)}{\log(1-2\Phi_f)} \quad . \tag{16}$$

We then have $N = 3149$ and $N = 5.7\times10^5$ for criteria of $\Phi_d = 0.99$ and $\Phi_d = 0.9999$ respectively, assuming $1/\lambda = 10^9$ years and $1/\mu = 10^8$ years. These N are comparable to the exact results derived from iterative calculations. The most important feature of equation (16) is that N scales as the ratio of μ to λ. So smaller mean civilization lifetime in comparison to mean civilization start time implies larger N for the same criterion level Φ_d.

Sequential Transient Civilizations

When listening civilizations are sequential, the target civilization is either listening when a message arrives, or has not yet emerged since the last decline. We simplistically represent this sequence as a Markov (renewal) process with i.i.d. times to civilization emergence S and lifetime L with means $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively.

We can write the probability that a given schedule $\{t_i\}$, $i = 1,2,\ldots N$ does not result in detection as

$$\Phi_f = 1-\Phi_d = \prod_{k=1}^{N}\text{Prob}\left(X(t_k)=0|X(t_{k-1}=0)\right) \quad , \tag{17}$$

where $X(t)\in\{0,1\}$ is the civilization state (emerging or listening) at time t according to FIGURE C.

Again assuming exponential distributions for S and L results in

$$1-\Phi_d = \prod_{k=1}^{N}\left(\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)\Delta_k}\right) \quad . \tag{18}$$

We see that Φ_d is monotone increasing in Δ_k, so $\Delta_k \to \infty$ maximises Φ_d for fixed N. So we have as a bound

$$\Phi_d \leq 1-\left(\frac{\mu}{\lambda+\mu}\right)^N \quad . \tag{19}$$

For $\Phi_d \leq 0.99$ and 0.9999 we have $N \gtrsim 50$ and 100 respectively when $\lambda = 10^{-9}$ and $\mu = 10^{-8}$

Of course, large interarrival times are impractical and we would more practically imagine a deadline, Γ for radiated messages attempts. It is easy to show using Lagrange multipliers that the optimal Δ_k should be constant which implies $\Delta_k = \Delta = \frac{\Gamma}{N}$. Thus, we would have

$$(1-\Phi_d) = \left(\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)\frac{\Gamma}{N}}\right)^N \tag{20}$$

With $1/\lambda = 10^9$, $1/\mu = 10^{-8}$ and $\Gamma = 10^{10}$ (roughly the average time required for ten emergence/decline cycles) we find that $\Phi_d = 0.99$ requires $N \approx 56$ and $\Phi_d = 0.9999$ requires $N \approx 610$.

We conclude that the necessity of repeating radiated messages increases the total radiated energy necessary to assure reception by an order of magnitude or more. For the same criterion level Φ_d, "one shot" civilizations which never re-emerge after decline require significantly more repetitions than sequential civilizations since reception by any one of the sequential civilizations constitutes a success.

Speculation on Long Messages

We have shown that the marginal energy per bit is almost always much greater for radiated messages than for inscribed matter messages. However, owing to practical issues of shielding and message discovery at the destination, inscribed matter favours long messages. Thus, the thorny question remains as to exactly why long messages might be preferred for interstellar communications, and we offer our speculations.

Interstellar distances with c as the speed limit for any communication method implies that round trip transit times might be very long. The stochastic nature of evolution, the potential fragility of a competitive technologically sophisticated species, a potential dearth of habitable locations and the inherent dangers associated with solar systems [12, 13] combined suggest that the probability of simultaneously communicating civilizations could be low. Such low probability makes longer messages reasonable if only to provide a synchronization strategy for civilizations at different levels of technical sophistication – by including detailed instructions about how to reply [14].

Of course, there is still the issue of motive. Why would a civilization devote significant effort to any kind of interstellar communication? Again, this question is thorny. However, if civilizations at any given stellar location are short-lived owing to various sorts of celestial or self-inflicted calamity, there could be a survival benefit to "seeding" the cosmos with compelling records of a civilization in the hopes that it would be eventually adopted by other fledgeling civilizations (or subsumed by older ones). Perhaps most plausible in this context is literal biological seeding [15] of likely habitats.

Under such assumptions, communicating civilizations may more likely have been spawned (or influenced) by older communicating civilizations at other locations while non-communicating civilizations may more likely disappear. This scenario, played out over galactic time scales, could support a bias toward long interstellar messages.

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