

Interference Avoidance and Power Control for Uplink CDMA Systems

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Abstract— The paper presents an algorithm for joint codeword adaptation and power control in which users in a CDMA system so as to achieve a specified set of target signal-to-interference plus-noise ratios (SINR). Codeword adaptation is based on greedy interference avoidance which decreases the effective interference seen by users and is followed eventually by power adjustment if the resulting SINR after codeword adaptation is below the specified target. Provided that the targets are admissible the algorithm yields a codeword ensemble and power allocation that satisfy a water filling distribution. Numerical examples which illustrate the algorithms are also included.

I. INTRODUCTION

Efficient use of radio resources becomes important in wireless communication systems. Main components of radio resource management at the physical layer are efficient use of the allocated spectrum and transmitter power control, both of which contribute to minimizing interference and increasing system capacity. In addition, transmitter power control extends also the battery life in mobile stations. Until recently, power control and codeword adaptation were treated as distinct problems, with researchers concentrating on either transmitter power control [2], [10], [11], [19], or spectrum utilization through signal design for efficient multiple access [4], [7]–[9], [13], [15], [16].

In the traditional approach transmitted power is regulated to provide each user with an acceptable connection by limiting the interference caused by other users, and the power control problem requires that a vector of users' transmitter powers be computed such that a specified set of constraints is met [19]. An alternative approach models power control as a noncooperative game, in which the quality of service is expressed in terms of user utility and pricing functions [10], [11]. Results from game theory are then used to derive a distributed power control procedure based on maximizing the net utility (utility minus the price).

Signal design for efficient multiple access considers the design of signature sequences (codewords) or waveforms to be used by users in a CDMA system such that a given criterion is optimized. This can be an individual criterion that defines performance or quality of service achieved by a given user like the signal-to-interference ratio [7], [8], [13] or the required signal bandwidth [4], [15], or a global criterion like sum capacity or total squared correlation [9], [17], [18]. Algorithms for

signal design can be either centralized, in which case optimal signatures are computed at the base station receiver and then assigned to users [4], [9], [15], [17], [18], or distributed, in which case users independently update signatures based on some common information broadcast by the base station [7], [8], [13].

Joint signal design and power allocation has generated interest in the research community lately. We note that, for the downlink of a CDMA system an analysis can be found in [1], and for the uplink case algorithms for optimal allocation of powers and signatures can be found in [17], [18]. More recent work [3], [6] presents algorithms for signal design and power control subject to quality of service expressed in terms of signal-to-interference ratios or RMS bandwidth constraints. We note that these are centralized schemes in which calculations are performed at the receiver and results must be transmitted to users.

With more computational power becoming available in mobile stations, distributed algorithms are desirable for joint power control and codeword optimization. In this paper we present one such algorithm that combines a power control mechanism with interference avoidance [7], [8], [13] in order to provide better performance and minimize transmitted power.

II. GREEDY INTERFERENCE AVOIDANCE: A BRIEF REVIEW

Consider the uplink of a CDMA system with processing gain N and K users in which users are received with different powers at the base station. The received signal at the base station is given by

$$\mathbf{r} = \sum_{\ell=1}^K b_{\ell} \sqrt{p_{\ell}} \mathbf{s}_{\ell} + \mathbf{n} \quad (1)$$

We define the $K \times K$ diagonal matrix \mathbf{P} containing received powers of all users

$$\mathbf{P} = \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_K \end{bmatrix} \quad (2)$$

and rewrite the received signal in matrix-vector form as

$$\mathbf{r} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n} \quad (3)$$

where $\mathbf{b} = [b_1 \dots b_K]^\top$ the vector containing the information symbols sent by users.

Assuming that simple matched filters are used at the receiver for all users, the SINR for a given user k is

$$\begin{aligned} \gamma_k &= \frac{(\sqrt{p_k}\mathbf{s}_k^\top \mathbf{s}_k)^2}{\sum_{\ell=1, \ell \neq k}^K (\mathbf{s}_k^\top \mathbf{s}_\ell \sqrt{p_\ell})^2 + E[(\mathbf{s}_k^\top \mathbf{n})^2]} \\ &= \frac{p_k}{\mathbf{s}_k^\top \left(\sum_{\ell=1, \ell \neq k}^K p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} \right) \mathbf{s}_k} \end{aligned} \quad (4)$$

We define the correlation matrix of the interference-plus-noise seen by user k

$$\begin{aligned} \mathbf{R}_k &= \sum_{\ell=1, \ell \neq k}^L p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} \\ &= \mathbf{S}\mathbf{P}\mathbf{S}^\top - p_k \mathbf{s}_k \mathbf{s}_k^\top + \mathbf{W} \\ &= \mathbf{R} - p_k \mathbf{s}_k \mathbf{s}_k^\top \end{aligned} \quad (5)$$

where $\mathbf{R} = \mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W}$ is the correlation matrix of the received signal in equation (3) and $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top]$ is the noise covariance matrix. Thus, the SINR for user k becomes

$$\gamma_k = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} \quad (6)$$

The denominator in equation (6) represents the Rayleigh quotient of matrix \mathbf{R}_k and is absolutely minimized when \mathbf{s}_k is equal to the minimum eigenvector of \mathbf{R}_k [12, p. 348]. Therefore, in order to maximize user k 's SINR through codeword adaptation one should replace the current codeword of user k with the minimum eigenvector of \mathbf{R}_k .

In this framework, greedy interference avoidance is defined by replacement of user k codeword \mathbf{s}_k with the minimum eigenvector of \mathbf{R}_k . This procedure is referred to as greedy interference avoidance since by replacing its current codeword with the minimum eigenvector of the interference-plus-noise correlation matrix, user k avoids interference by placing its transmitted energy in that region of the signal space with minimum interference-plus-noise energy and greedily maximizes SINR without considering potentially negative effects on other users. We note that user power is not changed with greedy interference avoidance.

Sequential application by all users of this greedy SINR maximization procedure defines the eigen-algorithm for interference avoidance [8], formally stated below:

- 1) Start with an initial codeword ensemble and power allocation for users specified by the codeword matrix \mathbf{S} and power matrix \mathbf{P}
- 2) For each user $k = 1 \dots K$

- replace user k codeword \mathbf{s}_k with the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process \mathbf{R}_k

- 3) Repeat step 2 until a fixed point is reached.

It has been shown that for fixed user powers application of greedy interference avoidance monotonically decreases the total weighted squared correlation (TWSC) defined as

$$\text{TWSC} = \text{Trace} [\mathbf{R}^2] = \text{Trace} [(\mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W})^2] \quad (7)$$

This form of the TWSC is an extension of the TWSC in [6], [14] for colored noise with covariance matrix \mathbf{W} and has been used before in connection with interference avoidance algorithms [7], [8]. The monotonic decrease in TWSC along with the fact that TWSC is lower bounded ensure convergence of the eigen-algorithm to a fixed point [8]. Properties of such fixed points are investigated in more detail in [7] where it is shown that a variant of this algorithm in which Step 3 is augmented with a procedure to escape suboptimal fixed points, converges to the optimal fixed point where the resulting codeword ensemble absolutely minimizes the TWSC. However, we note that in practice, such escape procedures have never been necessary when starting from random initial codewords.

III. INTERFERENCE AVOIDANCE AND POWER CONTROL

In the case of standard power control [19] user power is the only adjustable parameter, and power control algorithms are employed to ensure that each user's SINR is equal to or above a specified target SINR, provided these are feasible. However, codeword optimization methods provide additional degrees of freedom, by allowing users to change their codewords in addition to their power. In this section we present an algorithm which combines codeword adaptation through greedy interference avoidance with a power control mechanism. The algorithm decreases effective interference seen by a given user by means of codeword adaptation through greedy interference avoidance followed by power adjustment if the resulting SINR after codeword adaptation is below the specified target SINR.

We note that when there is no constraint on allotted user power, K users with SINR requirements $\{\gamma_1^*, \dots, \gamma_k^*, \dots, \gamma_K^*\}$ are admissible in the uplink of a CDMA system with processing gain N if and only if the sum of their effective bandwidths is less than the processing gain [17], [18]

$$\sum_{k=1}^K \frac{\gamma_k^*}{1 + \gamma_k^*} < N \quad (8)$$

The proposed combined interference avoidance and power control algorithm is formally stated below:

- 1) For a given background noise with covariance matrix \mathbf{W} , start with a random set of user codewords and powers specified by matrices \mathbf{S} and \mathbf{P} respectively.
- 2) Specify a set of desired target SINRs $\gamma_1^*, \dots, \gamma_K^*$ satisfying the condition in equation (8)
- 3) For each user $k = 1, \dots, K$ do
 - a) Compute \mathbf{R}_k using equation (5) and determine the minimum eigenvalue λ_k and eigenvector \mathbf{x}_k

- b) Minimize the effective interference for user k by replacing its current codeword \mathbf{s}_k with the minimum eigenvector \mathbf{x}_k of \mathbf{R}_k
- c) If user k 's SINR after codeword replacement is below the specified target γ_k^* , adjust user k power to meet the target SINR:

$$p_k = \gamma_k^* \lambda_k$$

Otherwise, leave user k 's power unchanged.

- 4) Repeat step 3 until a fixed point is reached.

It can be shown that such combined interference avoidance and power control monotonically decreases a quantity we will call the *normalized total weighted squared correlation* (NTWSC) by analogy to the TWSC in [6], [14]. NTWSC is defined as

$$\text{NTWSC} = \text{Trace} \left[\left(\frac{1}{E} (\mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W}) \right)^2 \right] \quad (9)$$

where $E = \text{Trace}[\mathbf{P}] + \text{Trace}[\mathbf{W}]$. The NTWSC represents the TWSC of the received signal correlation matrix $\mathbf{R} = \mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W}$ normalized by the total signal plus noise energy E at a given instance. In order to prove that the NTWSC is monotonically decreased by the combined interference avoidance and power control algorithm we follow the same line of reasoning as in [8] where it is shown that for fixed user powers TSC is monotonically decreased by application of interference avoidance. We omit this proof for brevity.

State the result as a formal theorem.

We note that, while $\text{Trace}[\mathbf{R}]$ is in general not constant during the combined interference avoidance and power control algorithm due to variations in user power, $\text{Trace}[(1/E)\mathbf{R}]$ is constant and equal to 1. We also note that, similar to the TWSC, the NTWSC is a Schur-convex function [5] in the eigenvalues of matrix $(1/E)\mathbf{R}$ and is lower bounded by when \mathbf{R} has N equal eigenvalues. The fact that the NTWSC is lower bounded and monotonically decreased by the combined interference avoidance and power control algorithm ensures that the algorithm will always reach a fixed point where user codewords are minimum eigenvectors of corresponding interference plus noise correlation matrices \mathbf{R}_k , and user SINRs are equal to or above the specified target SINRs. That is, at a fixed point user codewords and powers no longer change and satisfy

$$\mathbf{R}_k \mathbf{s}_k = \lambda_k \mathbf{s}_k \quad \text{and} \quad \gamma_k \geq \gamma_k^* \quad \forall k \quad (10)$$

with λ_k being the minimum eigenvalue of \mathbf{R}_k and implies the actual user k SINR value $\gamma_k = p_k / \lambda_k$. Using the expression of \mathbf{R}_k in equation (5) we note that at a fixed point all codewords are also eigenvectors of the received signal correlation matrix \mathbf{R} since we can rewrite equation (10) as

$$\mathbf{R} \mathbf{s}_k = (p_k + \lambda_k) \mathbf{s}_k = p_k \left(1 + \frac{1}{\gamma_k} \right) \mathbf{s}_k \quad (11)$$

Among all fixed points of the combined interference avoidance and power control algorithm, an optimal fixed point

corresponds to a codeword ensemble and power allocation that performs an aggregate water filling of the signal space with eventual ‘‘oversized’’ users commandeering dimensions with minimum noise energy [17], [18]. With no oversized users, all eigenvalues of the received signal correlation matrix \mathbf{R} in equation (11) will be equal¹ to the ‘‘water mark’’ c^* in the resulting water filling distribution and we can write $c^* = p_k(1 + 1/\gamma_k)$, $\forall k$. In the case of an oversized user ℓ the corresponding eigenvalue of \mathbf{R} is equal to $p_\ell + \sigma_\ell$ with σ_ℓ being the noise energy (variance) along the signal space dimension for which user ℓ has sole use.

Extensive simulations of the combined interference avoidance and power control algorithm have shown that it always reaches an optimal fixed point when initialized with random user codewords, provided that the specified target SINRs are admissible as defined by equation (8). Although we have not yet proven this result theoretically, we note that it is consistent with empirical observations made on interference avoidance algorithms which show that with random codeword initialization the algorithms always converge to the optimal fixed point [7], [8].

IV. NUMERICAL EXAMPLES AND DISCUSSIONS

We consider a system with $K = 5$ users and processing gain $N = 4$ in a colored noise environment with covariance matrix

$$\mathbf{W} = \text{diag}\{0.9501, 0.2311, 0.6068, 0.4860\}$$

and initial, randomly chosen, codeword matrix

$$\mathbf{S}_i = \begin{bmatrix} -0.2473 & -0.5629 & 0.3914 & -0.2594 & 0.7857 \\ -0.9522 & 0.5847 & 0.2088 & 0.9626 & 0.0437 \\ 0.0717 & 0.5839 & -0.2233 & -0.0601 & -0.0704 \\ 0.1645 & -0.0185 & 0.8680 & 0.0502 & -0.6130 \end{bmatrix}$$

We initialize all user powers $p_k = 1$ and set uniform SINR targets $\gamma_k^* = 1.8$, $\forall k = 1, \dots, K$. The targets are admissible since by adding up the effective bandwidths as in equation (8) we get $3.2143 < N$. The combined interference avoidance and power control algorithm yields the optimal codeword matrix

$$\mathbf{S} = \begin{bmatrix} 0.3772 & 0.0943 & 0.5753 & 0.3929 & 0.6955 \\ -0.0553 & 0.8336 & 0.1513 & -0.7931 & 0.2288 \\ 0.8967 & 0.3686 & -0.2635 & 0.1533 & -0.4263 \\ 0.2250 & -0.4005 & 0.7594 & -0.4394 & -0.5312 \end{bmatrix}$$

and power allocation matrix

$$\mathbf{P} = \text{diag}\{2.9591, 2.8184, 3.0434, 2.7457, 2.7074\}$$

which correspond to a water filling distribution with received signal correlation matrix

$$\mathbf{R} = \text{diag}\{4.1370, 4.1370, 4.1370, 4.1370\}$$

We note that the resulting SINRs, which are equal to

$$\{2.5123, 2.1374, 2.7829, 1.9734, 1.8937\}$$

¹We assume that users have enough power to span all available signal space dimensions. If this is not the case, then we can discard those dimensions with large noise energy unoccupied by users [7], [8].

are not uniform although all the specified targets are identical. We also note that initializing user powers with different, yet still uniform values, results in a different solution. For example for initial power $p_k = 0.5, \forall k$, we get

$$\mathbf{S} = \begin{bmatrix} 0.1248 & 0.3317 & 0.3472 & 0.7018 & 0.6497 \\ -0.3269 & -0.3034 & 0.6620 & -0.6596 & 0.5714 \\ 0.8991 & -0.5357 & 0.1644 & -0.0717 & 0.0535 \\ 0.2631 & 0.7148 & 0.6436 & -0.2592 & -0.4985 \end{bmatrix}$$

and power allocation matrix

$$\mathbf{P} = \text{diag}\{3.2607, 2.7262, 2.7917, 2.6996, 2.7359\}$$

which correspond also to a water filling distribution with received signal correlation matrix

$$\mathbf{R} = \text{diag}\{4.1220, 4.1220, 4.1220, 4.1220\}$$

and for which the resulting SINRs are

$$\{3.7853, 1.9531, 2.0984, 1.8980, 1.9737\}$$

If initial power is changed to $p_k = 2, \forall k$, we get

$$\mathbf{S} = \begin{bmatrix} 0.3451 & 0.0094 & 0.6587 & 0.3737 & 0.6362 \\ -0.0129 & 0.7394 & 0.3244 & -0.8530 & 0.1036 \\ 0.8250 & 0.5604 & -0.3728 & 0.3082 & -0.1852 \\ 0.4474 & -0.3730 & 0.5674 & -0.1943 & -0.7418 \end{bmatrix}$$

and power allocation matrix

$$\mathbf{P} = \text{diag}\{3.0519, 3.1565, 3.4632, 3.0207, 3.2530\}$$

The received signal correlation matrix for this case is

$$\mathbf{R} = \text{diag}\{4.5548, 4.5548, 4.5548, 4.5548\}$$

for which the resulting SINRs are

$$\{2.0306, 2.2573, 3.1725, 1.9689, 2.4987\}$$

It is worth noting that, when uniform SINRs are desired one should use regular interference avoidance and initialize user powers with the same value which will not change during the codeword adaptation process. For the same example presented above, if one applies the regular eigen-algorithm without power control starting with the same codeword matrix \mathbf{S}_i and noise covariance matrix \mathbf{W} with all user powers $p_k = 2$ we obtain the codeword matrix

$$\mathbf{S} = \begin{bmatrix} -0.3028 & 0.0470 & 0.5856 & 0.2536 & 0.7471 \\ 0.1159 & 0.6639 & 0.3187 & -0.9263 & 0.0698 \\ -0.8483 & 0.5842 & -0.2738 & 0.2008 & -0.2341 \\ -0.4188 & -0.4645 & 0.6932 & -0.1933 & -0.6183 \end{bmatrix}$$

which corresponds also to a water filling distribution with received signal covariance matrix

$$\mathbf{R} = \text{diag}\{3.0685, 3.0685, 3.0685, 3.0685\}$$

We note that in this case resulting SINRs are uniform and equal to 1.8717, which is also above the desired target of 1.8.

We now illustrate now the oversized user case for the same $K = 5$ users and $N = 4$ signal dimensions. We start with the same background noise covariance matrix \mathbf{W} and initial

codeword matrix \mathbf{S}_i as in the previous examples, initialize user powers with $p_k = 1, \forall k$, but require user 1 to have a target SINR equal to 9 while for the remaining users we keep the target SINR equal to 1.8. After running the combined interference avoidance and power control algorithm in this case we get the codeword matrix

$$\mathbf{S} = \begin{bmatrix} 0 & 0.1406 & 0.1235 & 0.5329 & 0.9601 \\ 1 & 0.0050 & 0.0027 & -0.0065 & 0.0016 \\ 0 & 0.9877 & -0.4401 & -0.4539 & 0.1527 \\ 0 & 0.0686 & 0.8894 & -0.7141 & 0.2342 \end{bmatrix}$$

and power allocation matrix

$$\mathbf{P} = \text{diag}\{6.9618, 4.5118, 4.9748, 4.5118, 5.0137\}$$

The received signal covariance matrix for this case is

$$\mathbf{R} = \text{diag}\{7.0184, 7.1929, 7.0184, 7.0184\}$$

and the resulting SINRs in this case are

$$\{30.1173, 1.8, 2.4344, 1.8, 2.5011\}$$

One can notice that in this case user 1, whose target SINR is large relative to the other users' target SINRs, resides in signal space dimension 2 which has the lowest noise energy, while the other 4 users share the remaining 3 dimensions.

V. CONCLUSION

An algorithm for joint codeword adaptation and power control in the uplink of a CDMA system has been presented in the paper. The algorithm uses greedy interference avoidance to adapt user codewords followed by an eventual power update in case the SINR after codeword adaptation is below the specified target SINR. Empirical evidence shows that if the specified target SINRs are admissible [17], [18] then the algorithm yields codeword ensembles and power allocations which satisfy an aggregate water filling of the signal space, with eventual oversized users that have codewords which are orthogonal to the other users' codewords. The algorithm lends itself to distributed implementation and users can update codewords/powers sequentially based only on knowledge of the correlation matrix of the received signal, or alternately, but making estimates of the received correlation from sequential broadcasts of the received signal \mathbf{r} .

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