Shannon-Theoretic Prescriptions for Outdoor Wireless Comm.

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Acknowledgements

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Shannon Theory and Practice

- Shannon theory provides fundamental limits
- What do practical system designers think?
  - Before the 90's: info theory = ivory tower research
  - After 1993: info theory gives performance benchmarks and design guidelines for practical comm systems
- Post-turbo Design Axiom: Shannon limit on any channel can be approached with "reasonable" complexity

(assuming sufficient ingenuity)

Great achievements (by others)

- Capacity over AWGN channel, binary errors channel, binary erasures channel
  - Random-looking codes on graphs
  - Iterative decoding
- Many code constructions: Turbo codes, LDPCs, repeat-accumulate codes
- What about wireless?

Today's focus: Outdoor Wireless

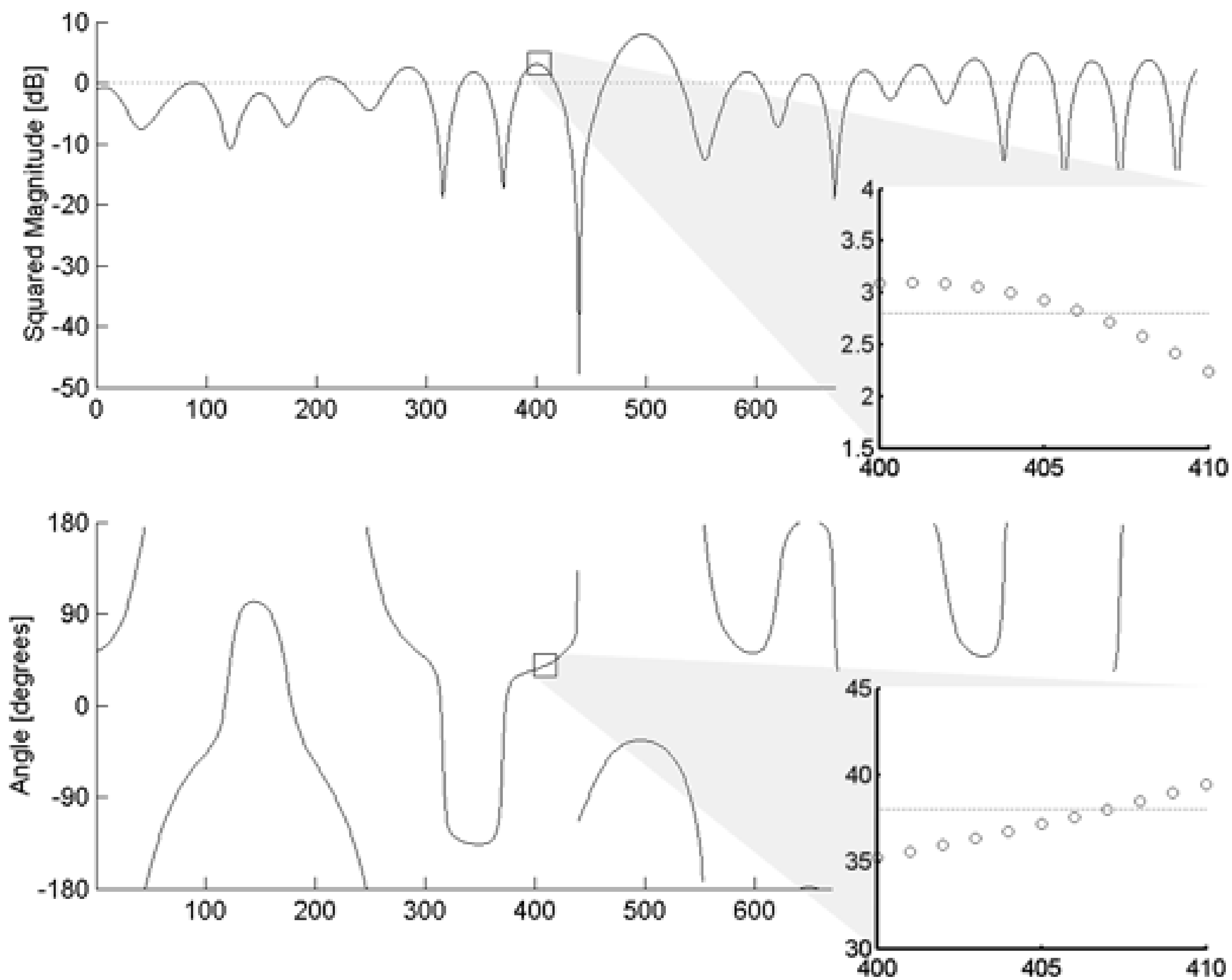
- Channel varies in time
  - Idea of "known" channel no longer applicable
  - Must account for channel estimation and tracking in Shannon theory and design
- Channel varies in frequency
  - Multipath propagation can cause nulls in transfer fn.
  - Wideband systems provide diversity
- Channel varies in space
  - Multiple antennas can be used to enhance performance

Summary of Results

- Handling channel time variations
  - I. Compute and approach capacity for moderate mobility and moderate SNR
  - II. Speeding at high SNR is a bad idea
- Frequency and space diversity
  - Bandwidth, Power-delay profile, Power-angle profile
  - III. Compact characterization of the effect of physical characteristics on performance

Narrowband fading

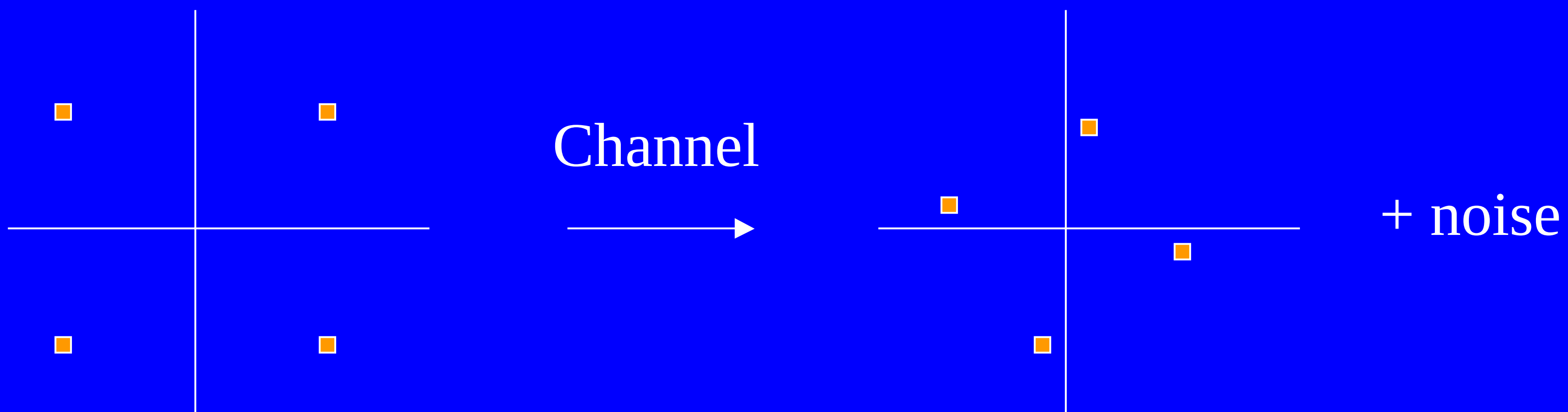
$$Y(n) = h[n] X[n] + W[n]$$



The effect of fading

Sent X

Received $Y = h X + N$



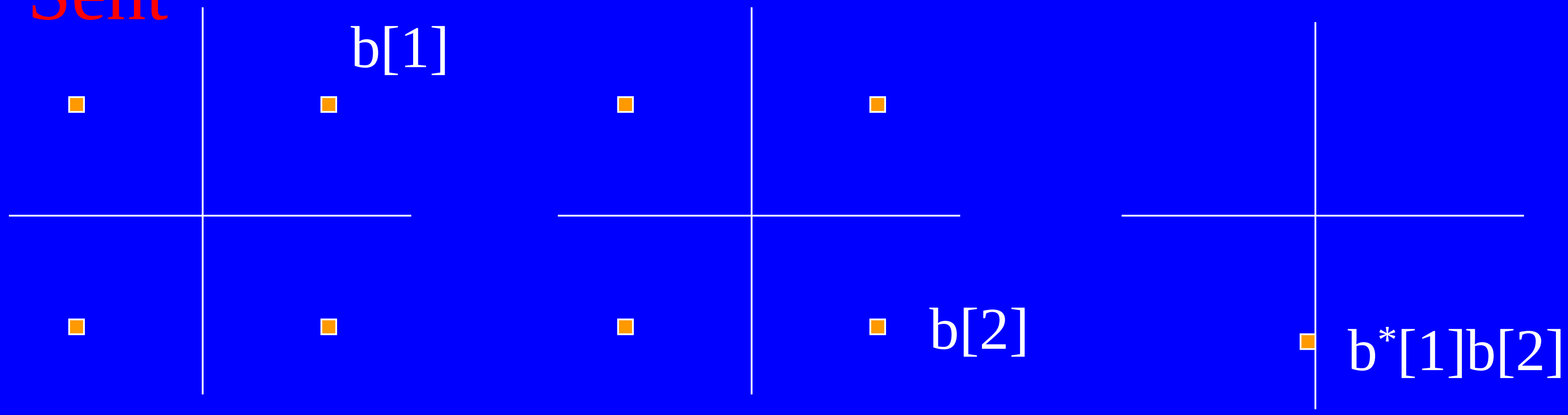
Wireline: estimate and undo channel amplitude and phase

Wireless: Tracking time-varying channel expensive

“Perfect” tracking impossible

Differential Modulation

Sent



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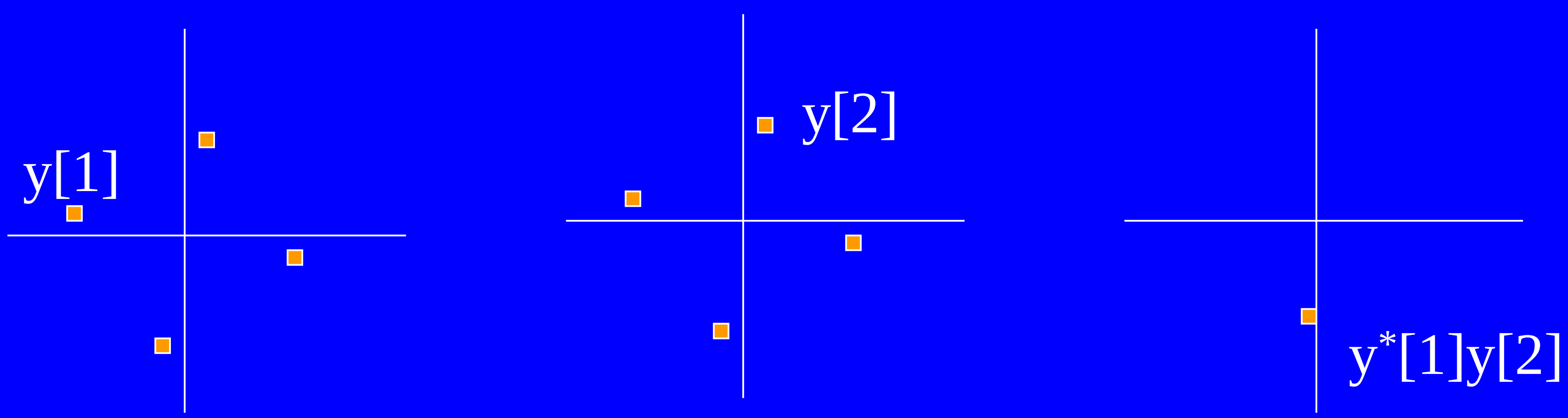
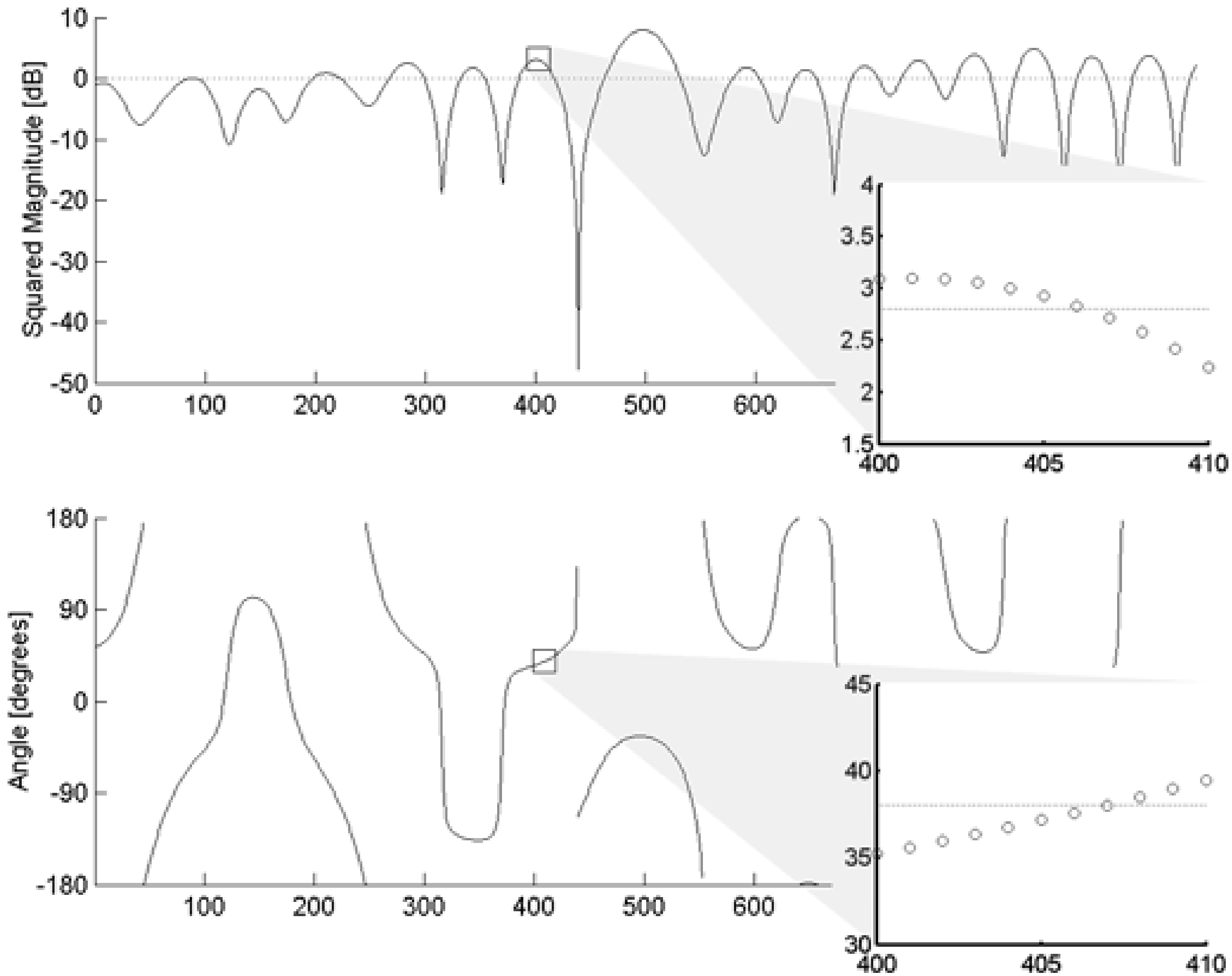
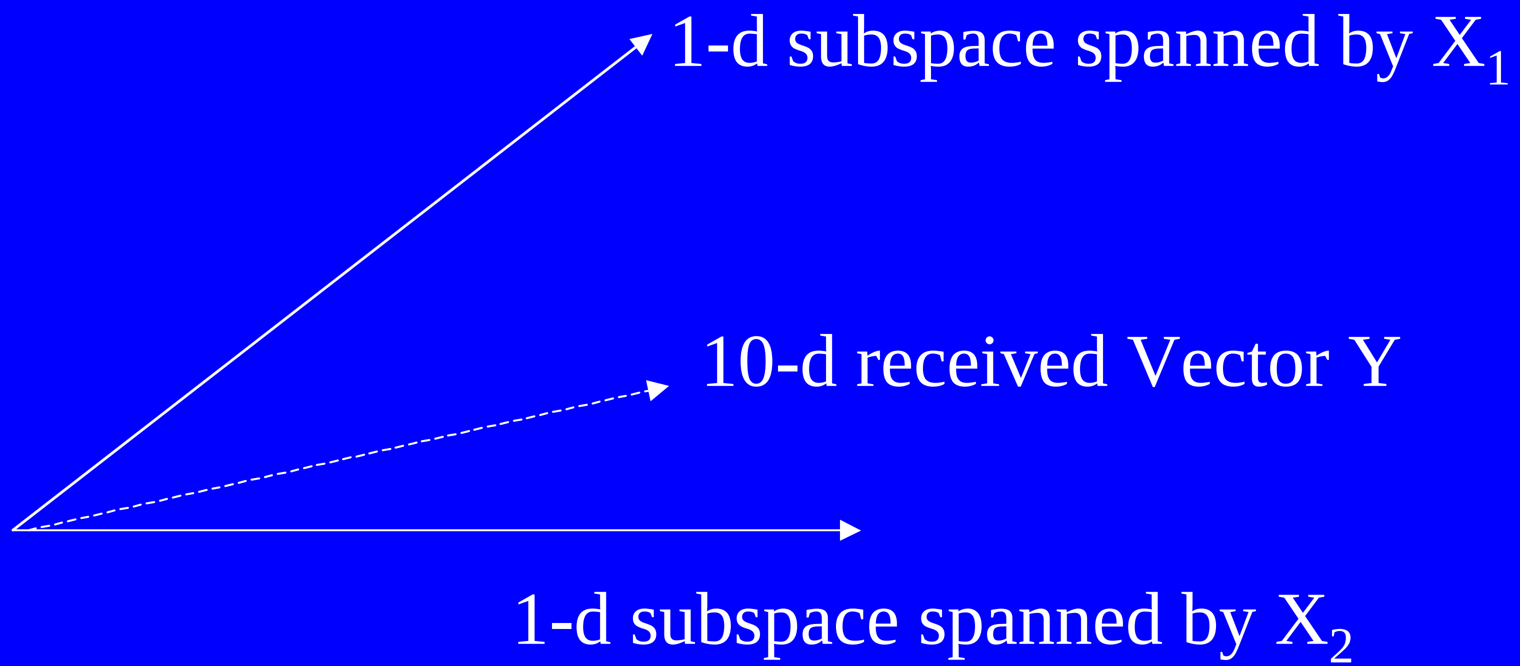


Problem: 3 dB penalty!

The block fading approximation
(channel roughly constant over several symbols)



Block noncoherent demodulation



4 bits/symbol, block length 10 symbols

➔Pick the closest subspace among 1 million possible ones

Eliminates 3 dB penalty, but with exponential complexity!

Low-complexity block demod

- Y = h X + N 10-dimensional

$$Y_1 = hX_1 + N_1, \ldots, Y_{10} = hX_{10} + N_{10}$$

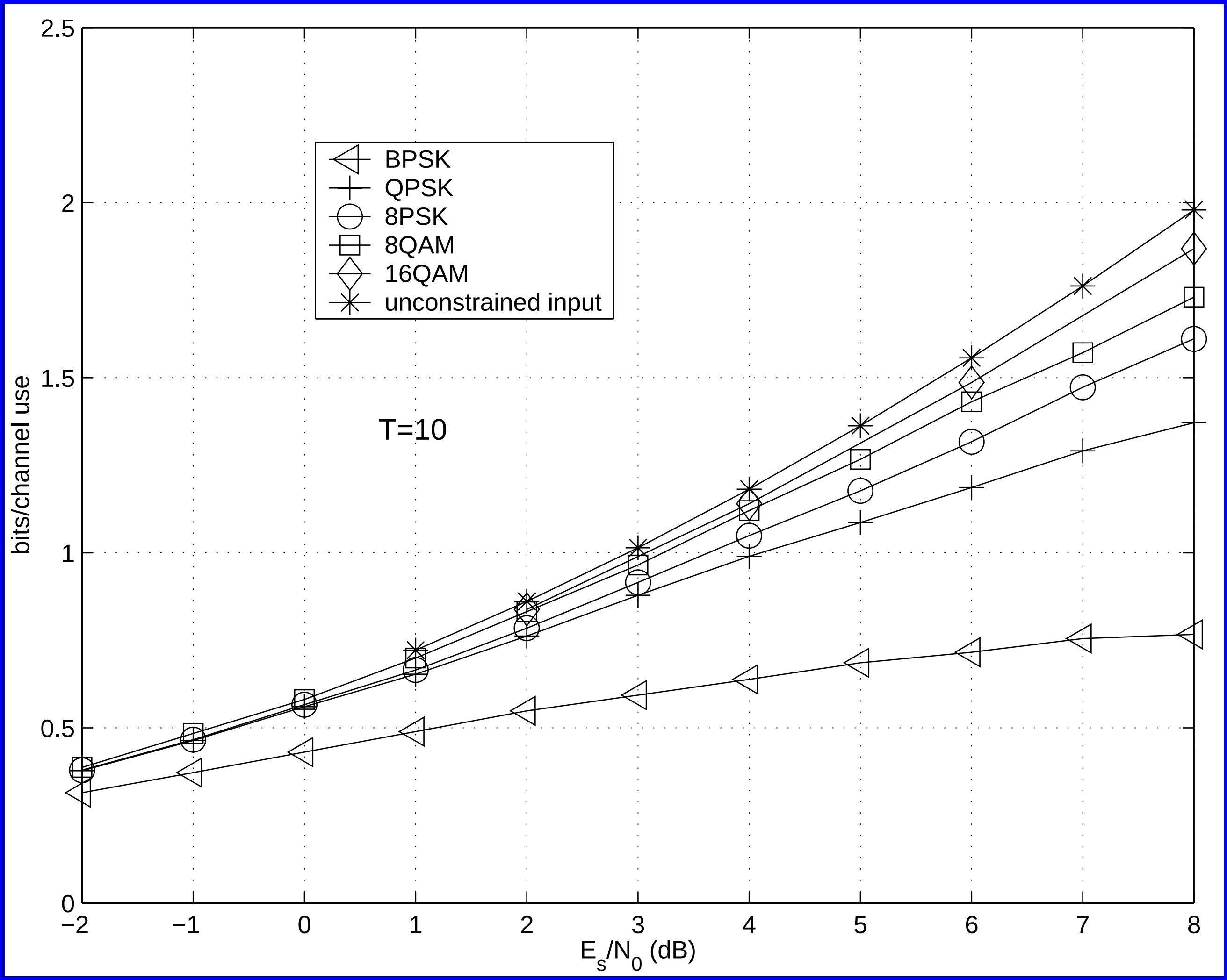
- Coherent detection done symbol by symbol
- Parallel coherent demodulators with quantized h
- Choose best match using noncoh metric
- Near-optimal
  - Eliminates 3 dB penalty
  - Number of quantizer levels small for DPSK

Shannon theory for block fading

- Capacity: Marzetta and Hochwald
  - Optimal input approx. uniform over sphere
- Our interpretation: can approximate by iid Gaussian symbols

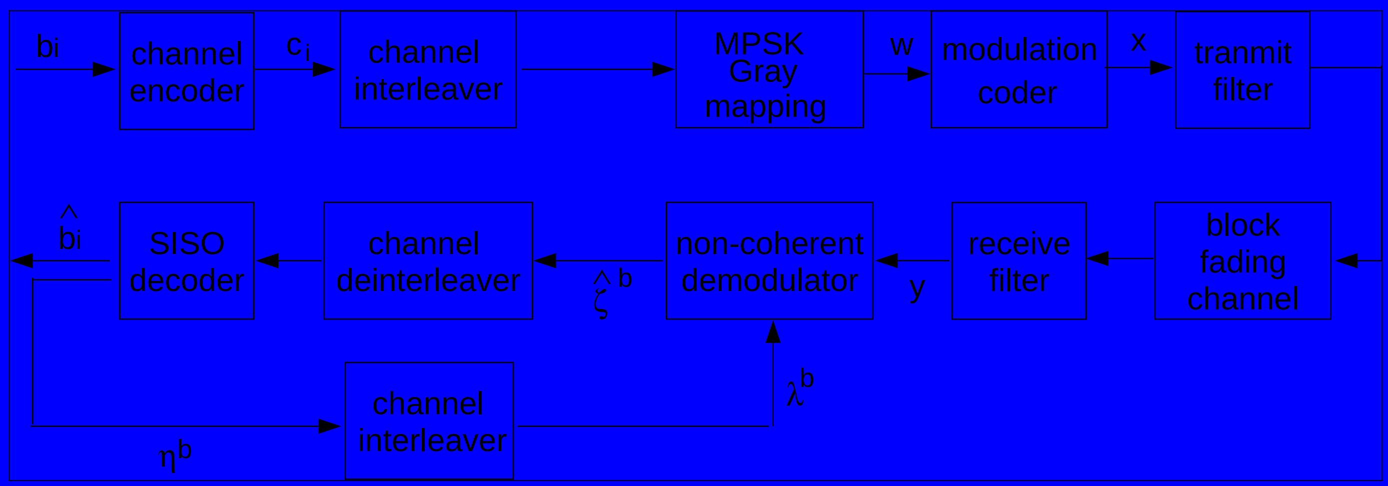
→ Standard PSK or QAM should also work

Noncoherent channel capacity for finite constellations



QPSK is appropriate for transmission rate 1/2 bits/channel use

Turbo noncoherent comm.

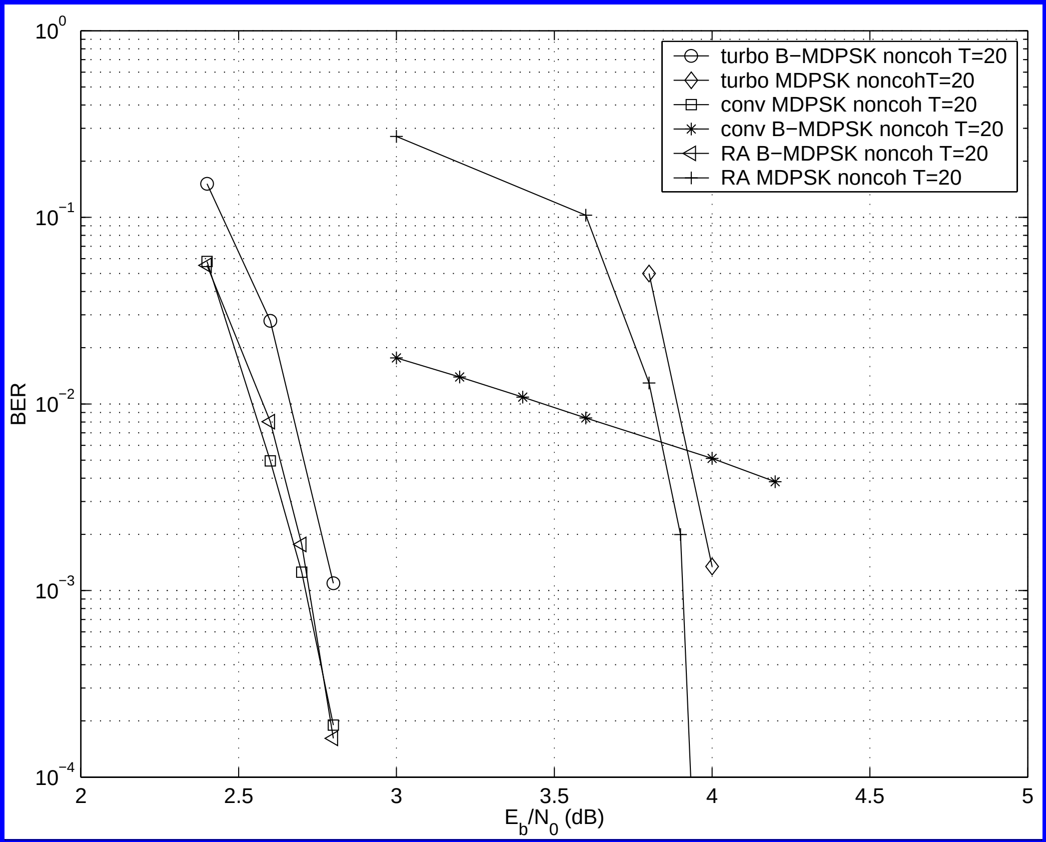


- Soft information exchange between demodulator and decoder
- SISO noncoherent demodulator for PSK avoids exponential complexity using parallel coherent demodulators

Design of modulation codes

- **The information-theoretical aspect**
  - Mutual information between input and output should approach unconstrained capacity
- **The complexity aspect**
  - Should allow for efficient decoding
- **The compatibility aspect**
  - Should match outer channel code

Matching modulation codes with outer channel codes



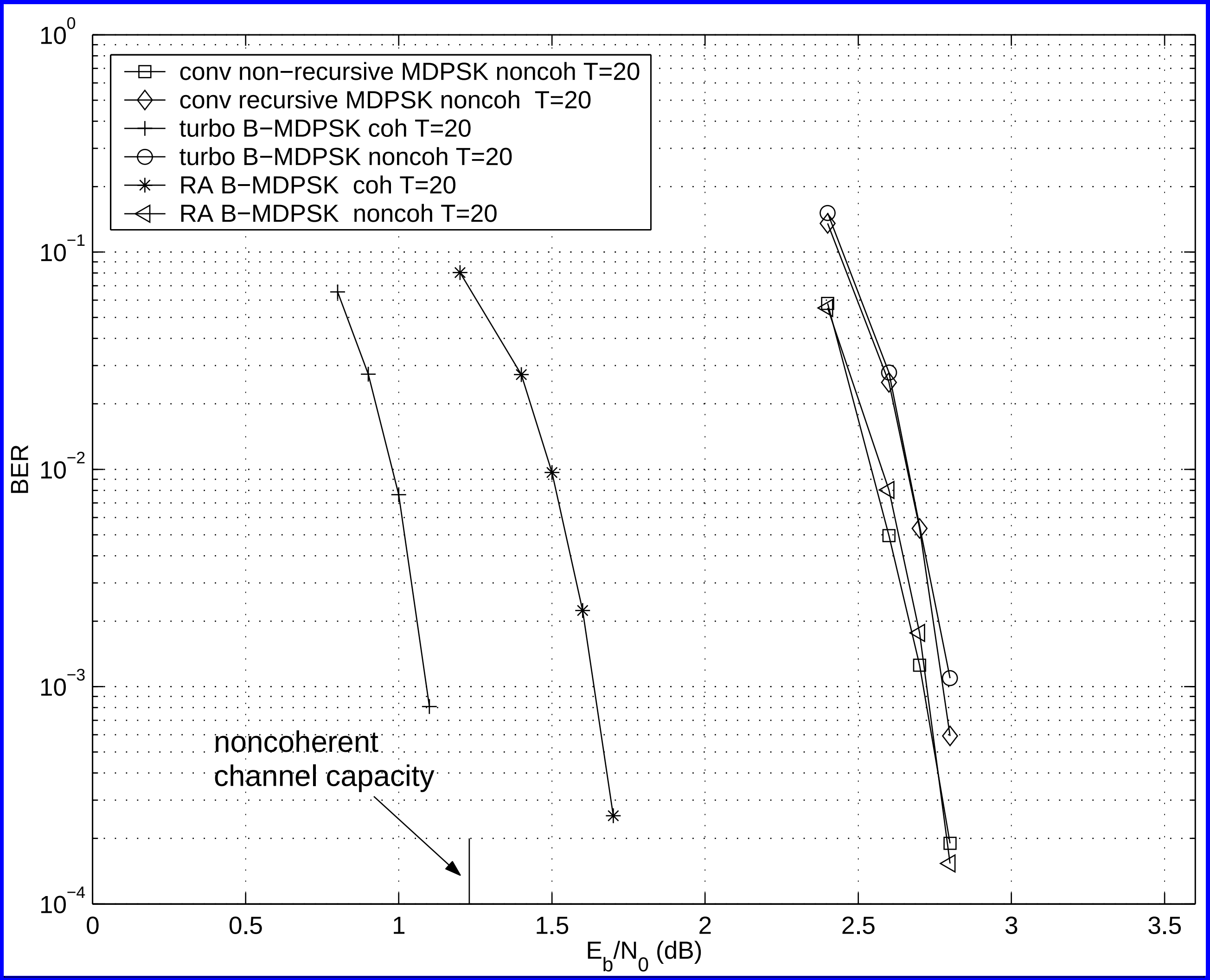
Good codes combinations:

•Turbo code + B-MDPSK;

• RA code + B-MDPSK;

•Convolutional code + MDPSK

Simulation results T=20



RA code + B-MDPSK within 1.6 dB of capacity at BER=10^{-4}

Convolutional code + MDPSK performs close to RA code + B-MDPSK

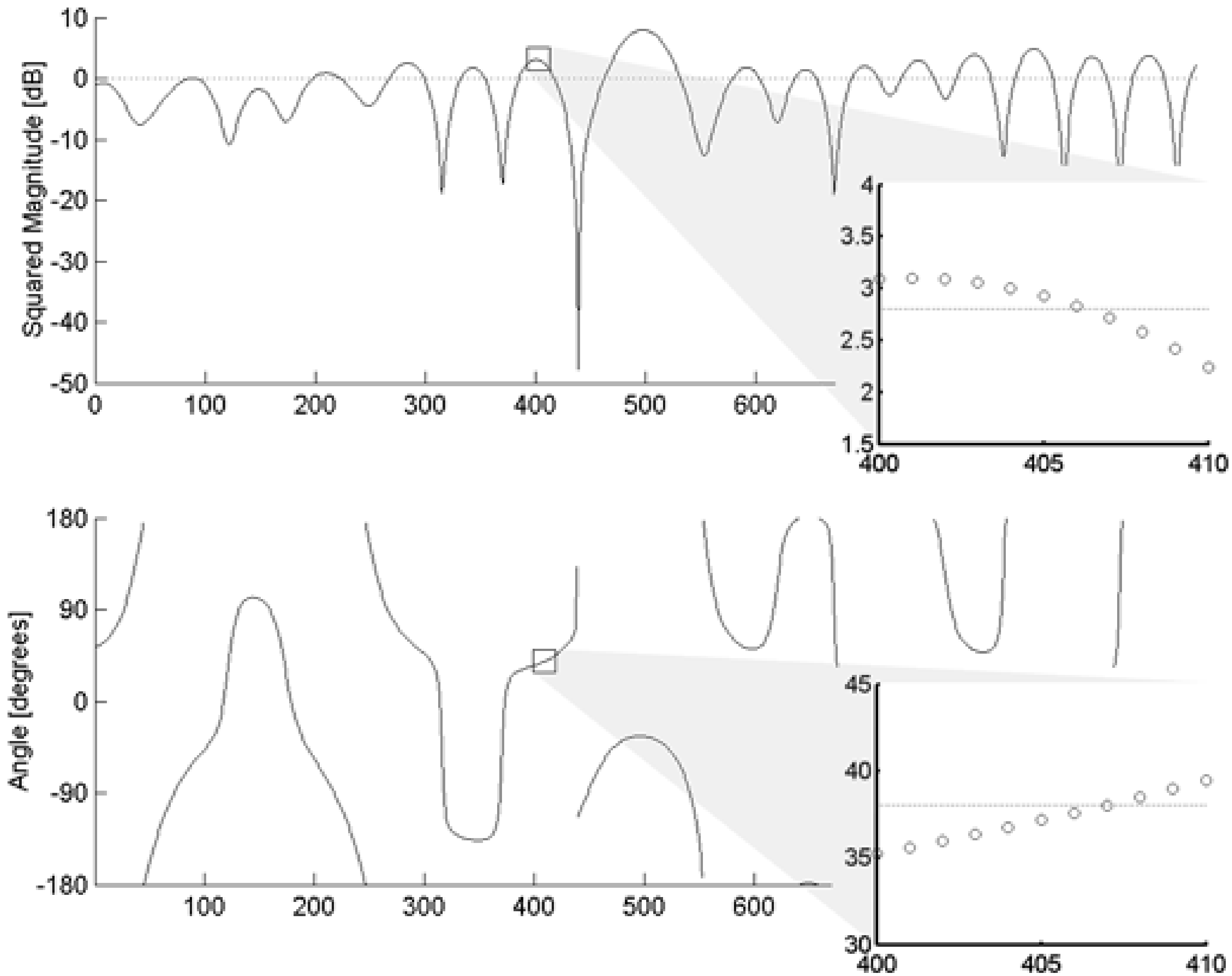
Turbo code + B-MDPSK performs best with coherent detection, inferior with noncoherent detection

I. So what?

- Turbo noncoherent comm works
  - Moderate SNR, moderate fading rates
  - Standard outer code
  - Standard constellations
  - Standard differential modulation
  - Soft information exchange
- What about high SNR, fast fading?

II. Continuous fading and high SNR

Errors in blk fading approx matter at high SNR



A bad operating regime

- $h[n] = \alpha h[n-1] + U[n]$ ***Gauss-Markov model***
- Signal-dependent noise due to channel estimation error dominates → nasty results
  - Standard Gaussian input a bad choice
  - O(log(log(SNR)) growth even with opt. input
- ***Bottomline: avoid high SNR, high mobility regime if at all possible***

Avoid Gaussian input!

- $h[n] = \alpha h[n-1] + U[n]$ ***Gauss-Markov model***
- Mutual info $\simeq -\log(1-\alpha^2)$ for large SNR
- Contrast with O(log(SNR)) for block fading model with blk length > 1

Comparison of mutual information

Mutual information in bits/channel use

SNR (dB)	10 dB	20 dB	∞
α=0.9 (upper bound)	2.1674	2.9795	3.2287
AWGN (exact)	3.4594	6.6582	∞

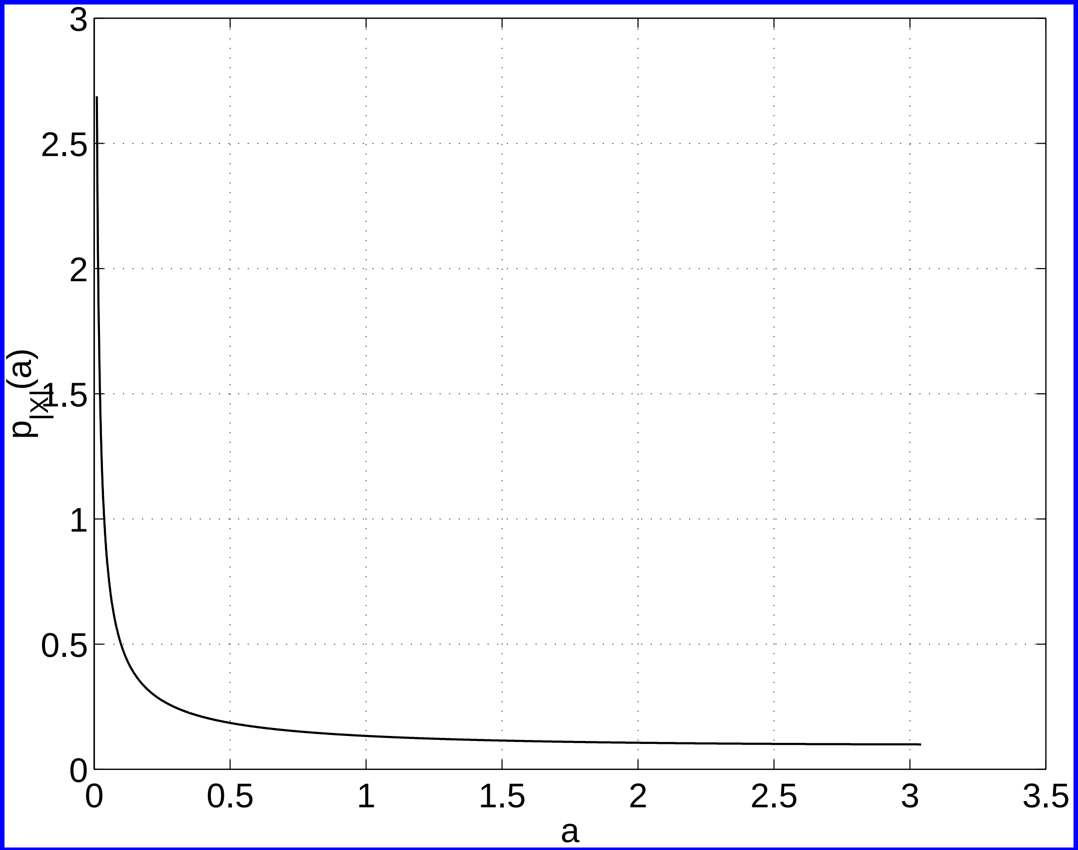
Infinite SNR with cont. fading comparable to AWGN channel with SNR = 9.23 dB!

Beyond Gaussian input

- Want fixed input distribution (scaled according to SNR)
  - Mutual information unbounded in SNR
  - Mutual information growth close to max possible
- Focus on worst-case memoryless fading
  - Information only carried in amplitude
  - High SNR limit → ignore additive noise

An example of a good continuous distribution

The density function of input amplitude:

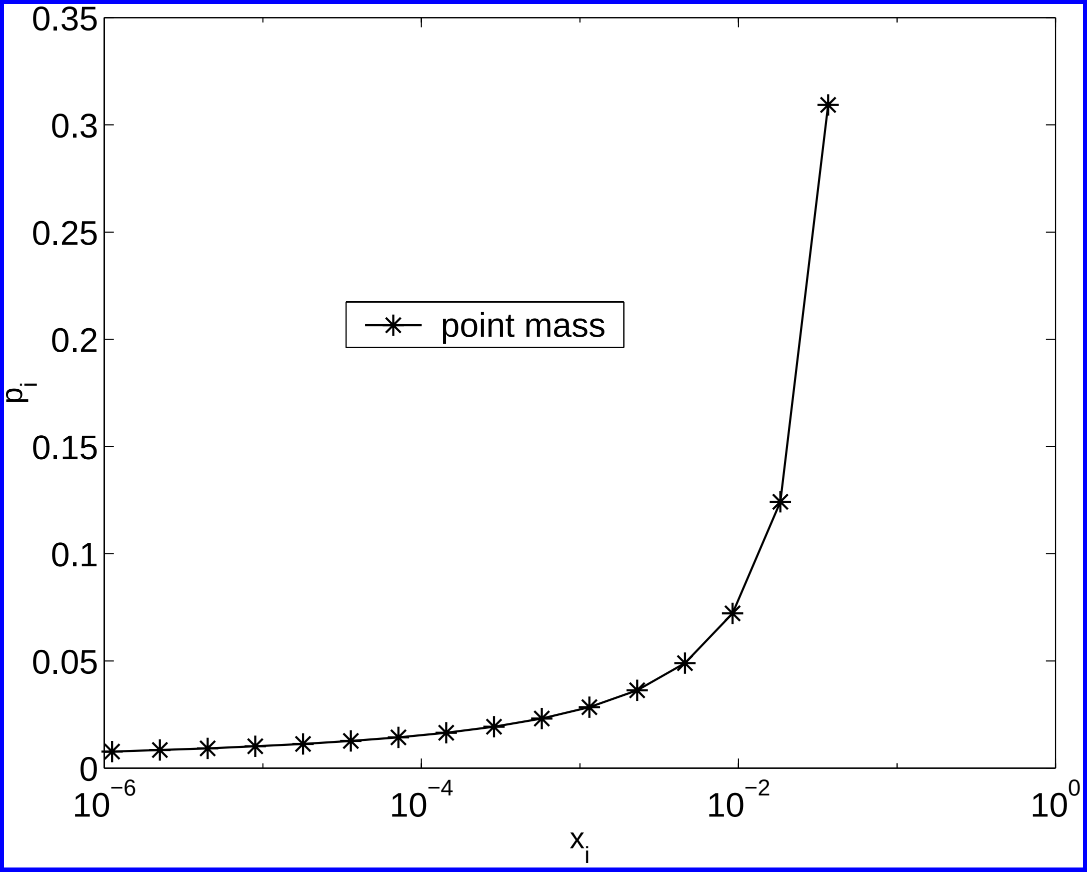


$$h(\log(|X|))=+\infty \Rightarrow I(X;Y)=+\infty.$$

Example of discrete distribution

- Fix L>1. Let X take discrete values at $x_i = L^{-i}$ with probability p_i.
- Infinite entropy: $H(X)=+\infty$.
- Let $p_i = t/[i(\log i)^{(1+u)}]$, for any 0<u<1.

  Mutual information growth rate > $O[(\log\log(SNR)]^{(1-u)}$.



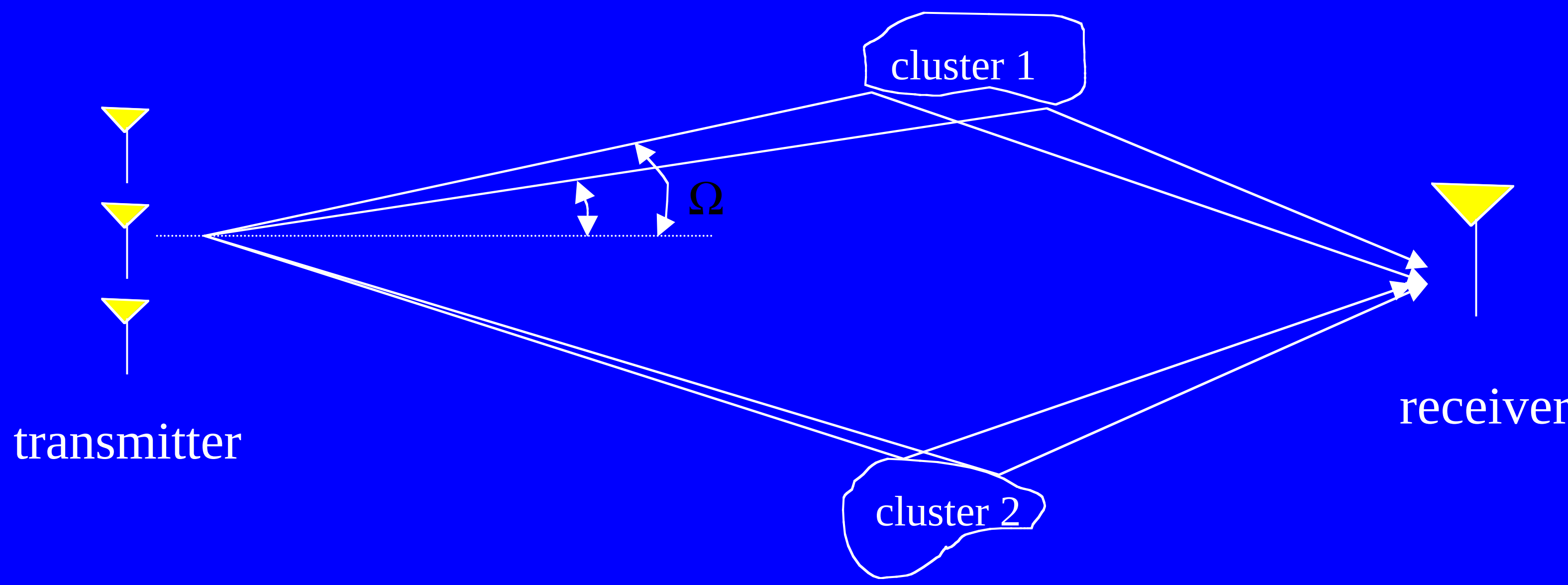
II. So what?

- High SNR, fast fading is a bad regime
  - Standard constellations do not work
  - Optimal constellations yield only log(log(SNR)) rate
- Can design insights be applied to improve moderate SNR regime?

III. Designing a wideband system

- Can I send 40 Mbps using a bandwidth of 20 MHz at SNR of 10 dB with 1% outage?
  - Desired spectral efficiency: 2 bits per second per Hz
  - Example: 16-QAM constellation with rate _ code
  - Want correct decoding 99% of the time

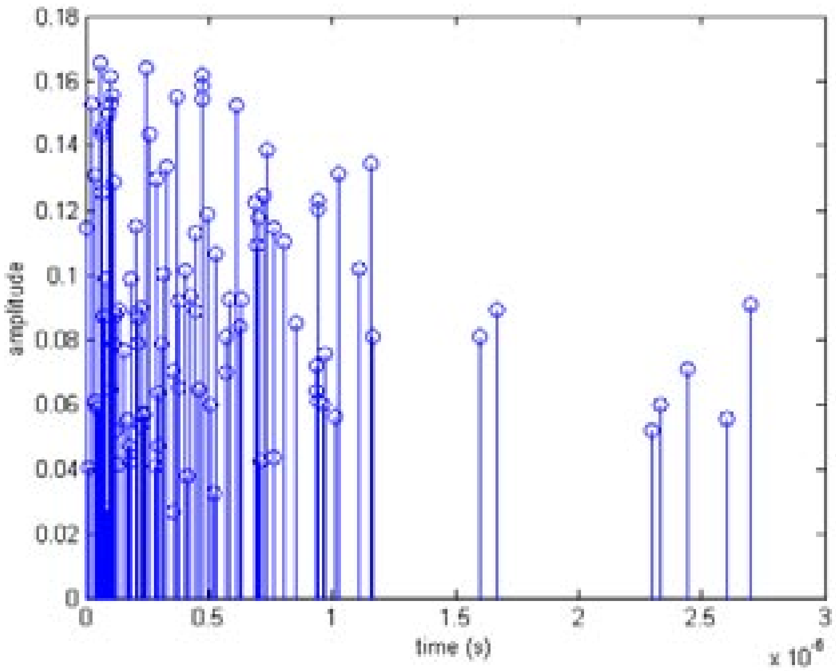
Outdoor Wideband Systems



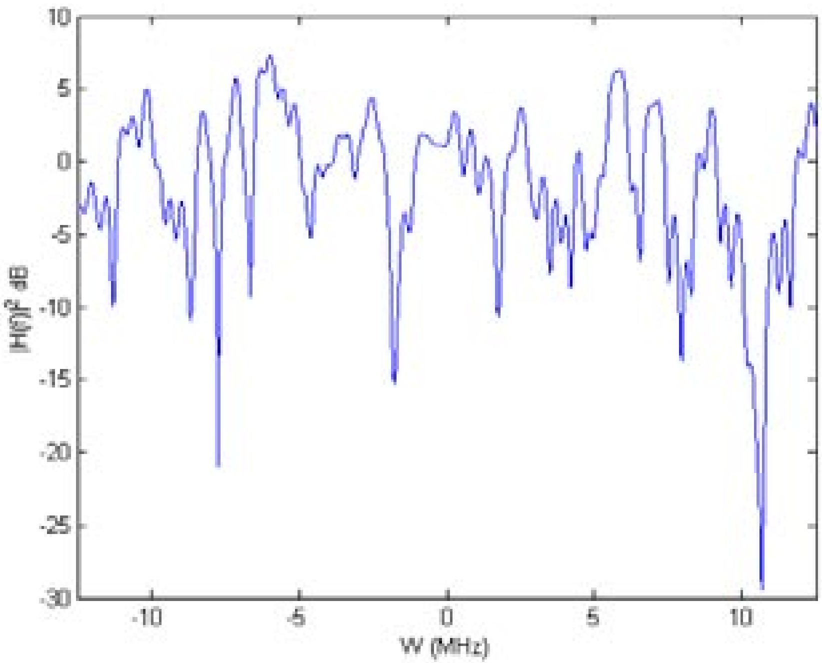
- few clusters
- small angular spreads

A wideband channel realization

Impulse response



Frequency response



The Problem

- Channel varies significantly over allocated band
- Channel feedback not available
- Ignore channel time variations over codeword
- Naturally matched to OFDM
  - No channel feedback ➔ no waterpouring in frequency
  - Use the same constellation on each subcarrier
  - Code across subcarriers
  - Channel realization random, then fixed over codeword
  - *Outage* occurs if code rate larger than channel capacity
- Goal: outage rate in terms of channel chcs.

Overview of Results

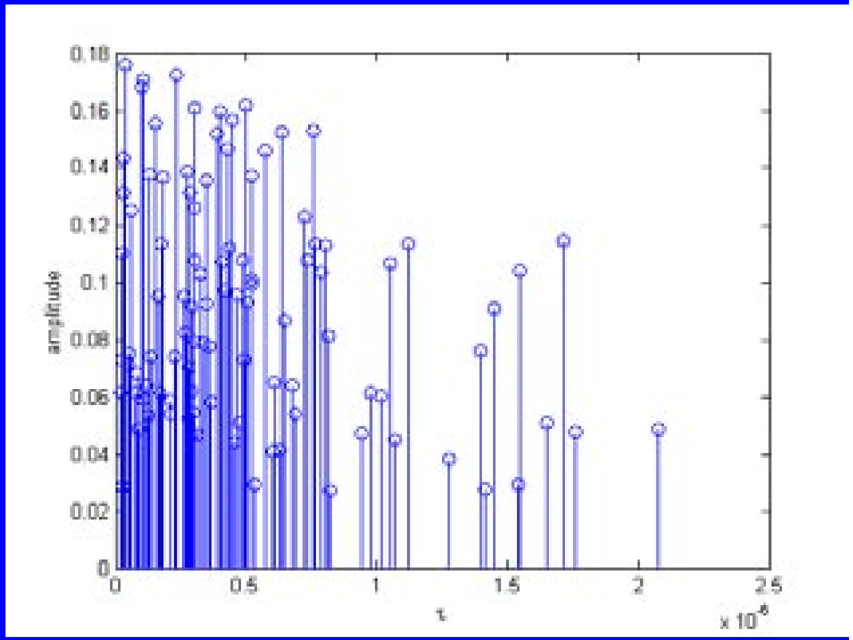
- Bandwidth-dependent TDL models
  - Provide analytical insight
  - Consistent with complex ray generation models
- Gaussian approximation for outage rates
  - SNR-independent defns of spatial and freq diversity

  ➔ Outage rates in terms of SNR, power delay profile, power angle profile, bandwidth, # transmit antennas

Ray-based Channel Model: Simulation

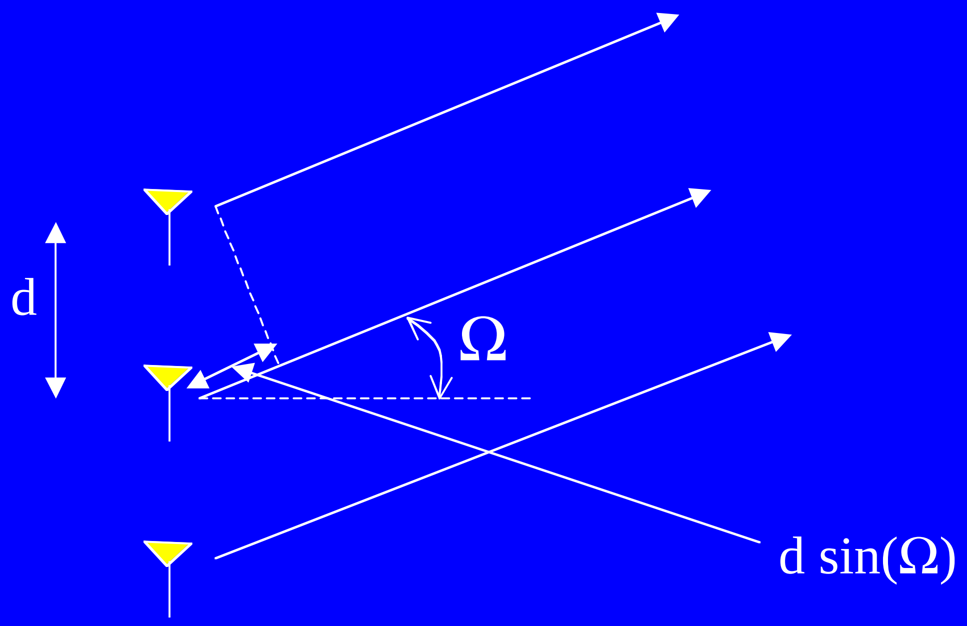
- Generate delays and angles of departure according to specified distributions
- Amplitudes from power profiles & distributions (consistency condn)
  - $\alpha_i^2 \sim P_\tau(\tau_i) P_\Omega(\Omega_i) / f_\tau(\tau_i) f_\Omega(\Omega_i)$
- Performance depends on power profiles, *not* distributions

➔ can replace by continuum model depending only on power profiles



sample channel realization
for 1 cluster (time domain)

Antenna Array Response



Running Example

- Uniform Linear Array
- $\mathbf{a}(\Omega)=[1\ a\ a^2\ \ldots\ldots a^{N_T-1}\ ]$

$a=\exp(j\ 2\pi\ d/\lambda\ \sin(\Omega))$

discrete ray based model $\xrightarrow{\text{paths} \to \infty}$ continuum model $\xrightarrow{\text{resolvability } 1/W}$ tap-delay line model

$$\mathbf{h}(t,\Omega) = \sum_{i=1}^{M} \alpha_i e^{j\Theta(\tau_i,\Omega_i')}\mathbf{a}(\Omega_i)\delta(t-\tau_i,\Omega-\Omega_i)$$

$$\Downarrow$$

$$h(t,\Omega) = \int_0^\infty \int_{-\pi}^{\pi} \sqrt{P_\tau(\tau)P_\Omega(\Omega)}e^{j\Theta(\tau,\Omega')}\mathbf{a}(\Omega)\delta(t-\tau,\Omega-\Omega')d\tau d\Omega'$$

$$\Downarrow$$

$$h_W(t) = \sum_{i=0}^{\infty} A(\frac{i}{W})\mathbf{v}_i\delta\left(t-\frac{i}{W}\right)$$

$$A(\tfrac{i}{W}) \propto \sqrt{P(\tfrac{i}{W})}$$

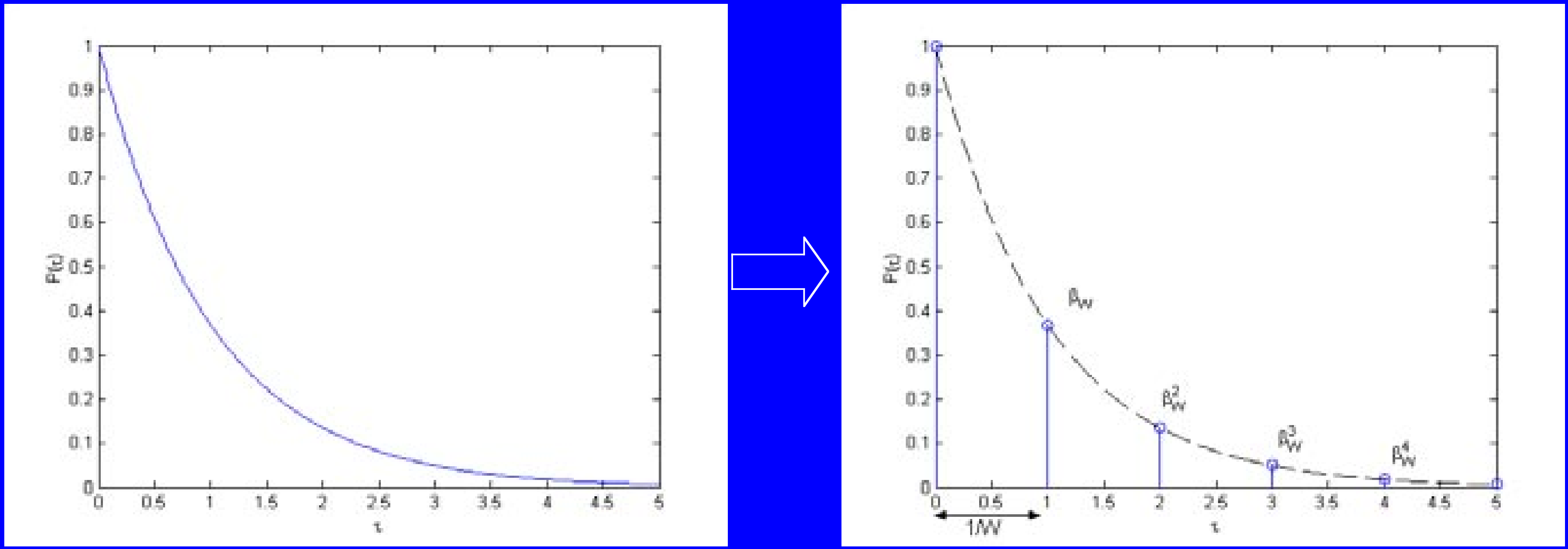
$$\mathbf{v}_i \sim CN(0,\mathbf{C})$$
$$\mathbf{C} = E[\mathbf{a}(\Omega)\mathbf{a}(\Omega)^H]$$

(central limit theorem)

TDL model: SISO System

$$h_W(t) = \sum_{i=0}^{\infty} A(\frac{i}{W}) v_i \delta\left(t - \frac{i}{W}\right)$$

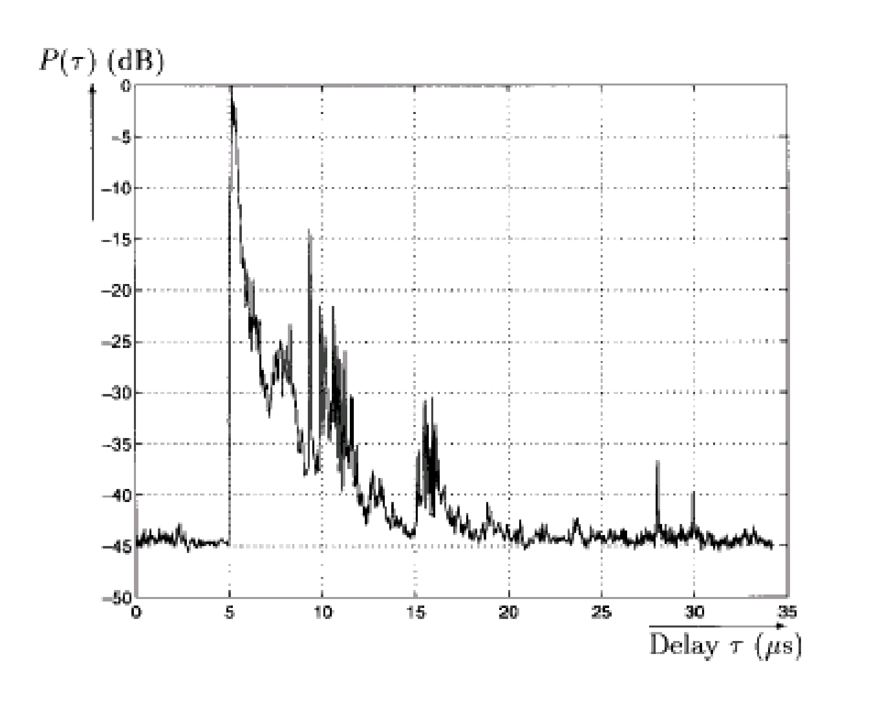
$$A(\tfrac{i}{W}) \propto \sqrt{P(\tfrac{i}{W})} \qquad v_i \sim CN(0,1)$$



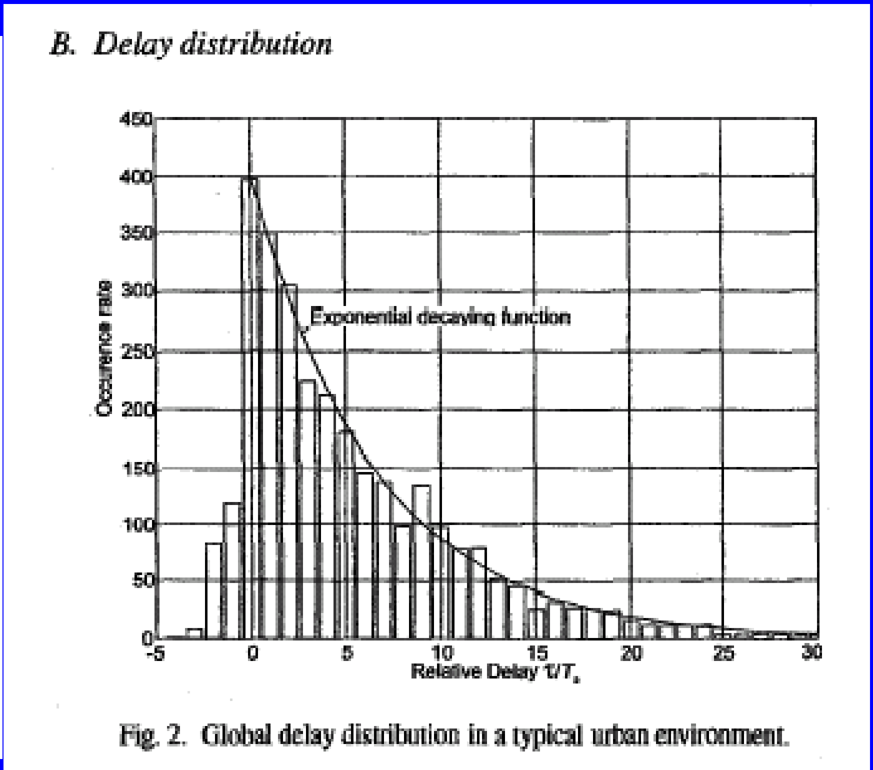
Running example: Exponential PDP

$$P(\tau) = \frac{1}{\tau_{rms}} e^{\frac{-\tau}{\tau_{rms}}}$$

Why Exponential PDP?



Fuhl, Rossi, Bonek: *Trans. on Antennas and propagation*, 1997



Pedersen, Mogensen,Fleury:*VTC* 95

Outage Spectral Efficiency

Spectral efficiency as a function of bandwidth

$$I_W = (1/W) \int_{-W/2}^{W/2} \log(1 + SNR \,|\, H(f) \,|^2) df$$

Outage occurs when transmitting at rate RW if $R > I_W$

Outage spectral efficiency:

$$R(\varepsilon) = \max \{ \; R : P[R > I_W] \leq \varepsilon \; \}$$

E.g., 1% outage rate corresponds to $\varepsilon = .01$

★ The Gaussian Approximation

Spectral efficiency is an average over frequency

$$I_W = (1/W)\int_{-W/2}^{W/2} \log(1 + SNR\,|H(f)|^2)\,df$$

Central limit theorem kicks in quickly

➔ I_W approximately Gaussian

$I_W \sim N(E[I_W], \mathrm{var}[I_W])$

Calculating Outage Rates is Now Easy

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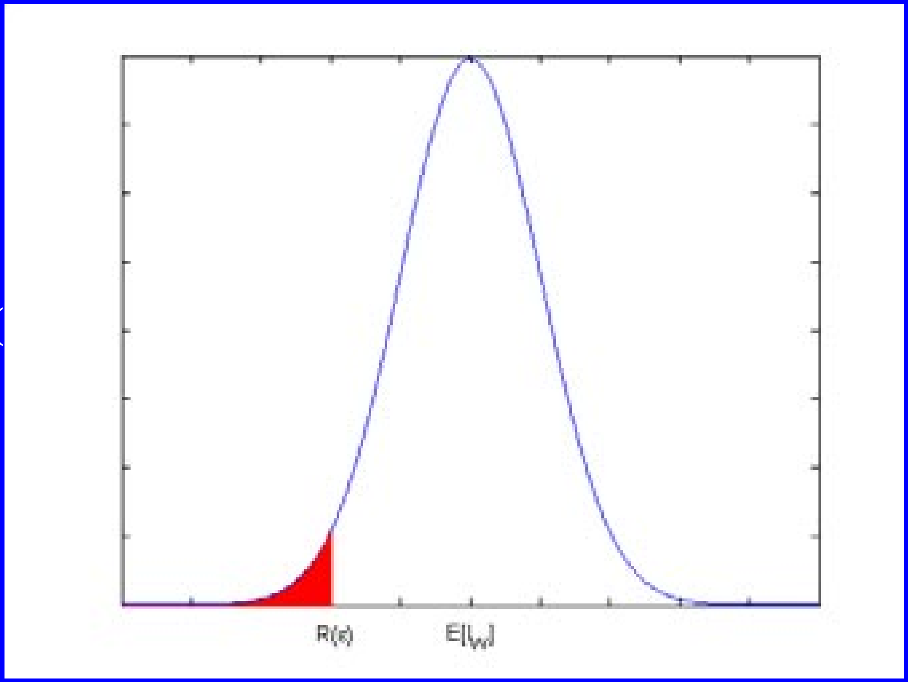
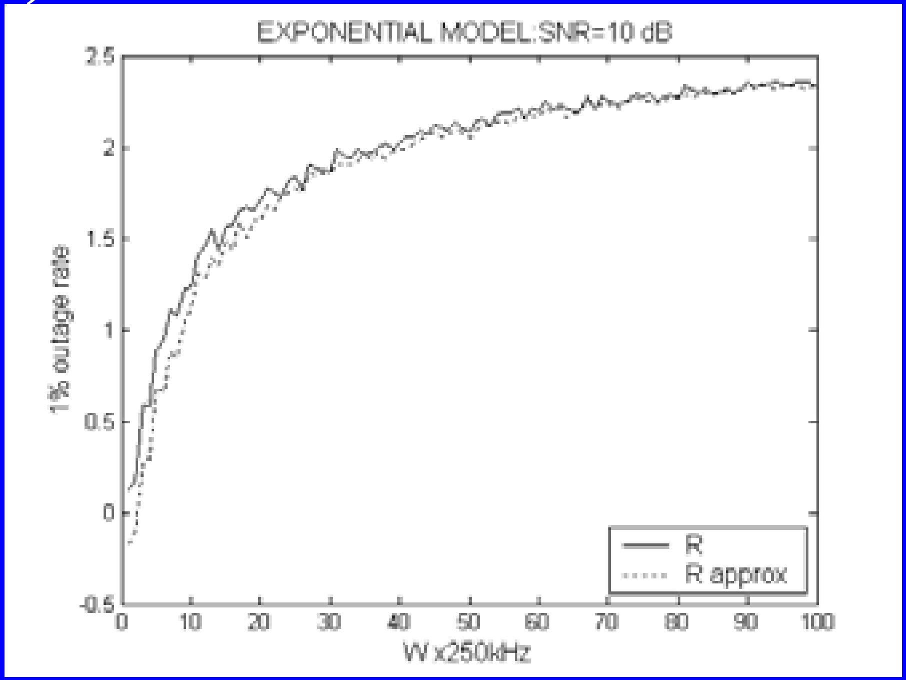


$$R(\varepsilon) \approx E[I_W] - \sqrt{Var(I_W)}\ Q^{-1}(\varepsilon)$$

Validating the Gaussian approx

- Compare $R(1\%),\ \hat{R}(1\%)$ for SISO system using the simulated values of $E[I_W]$ and $\mathrm{var}[I_W]$ for SNR=10 dB
- Gaussian approximation valid even for small W (and for a wide range of SNRs)



Mean and Variance (SISO)

- Mean = ergodic capacity: Rayleigh fading
- Variance ~ Variance(TDL channel energy)

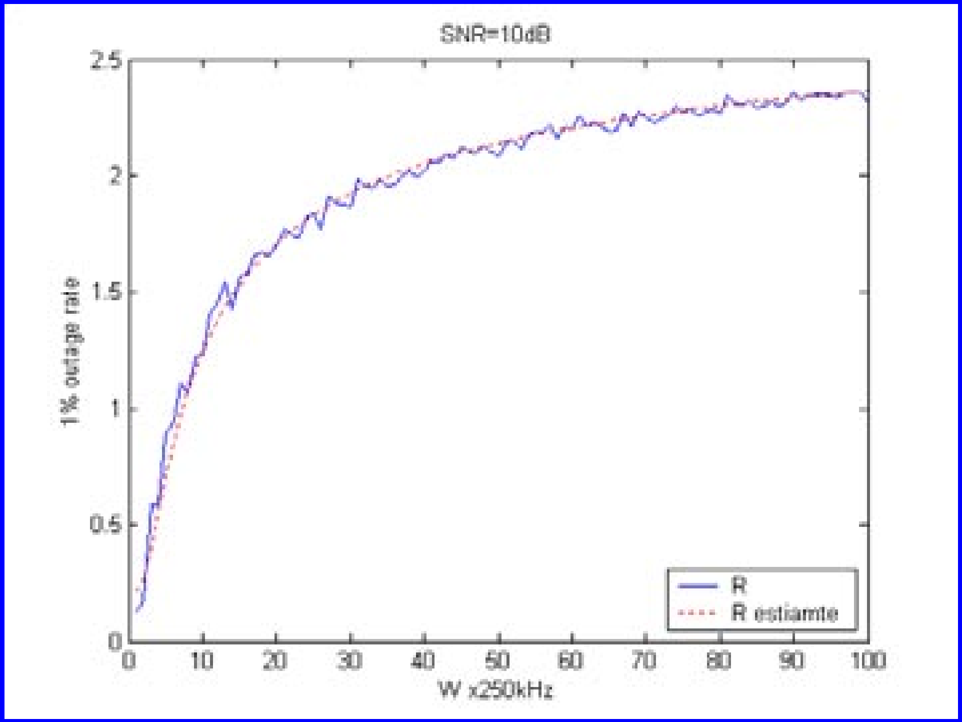
Proposition 1

$E[I_W]= E[\log(1+SNR\ X)]$ $X\sim Exp(1)$

$var[I_W]=\gamma^2\ var[E_C]$

- γ approx SNR/(SNR+1)
- E_C=total energy in the channel

Validating Proposition 1

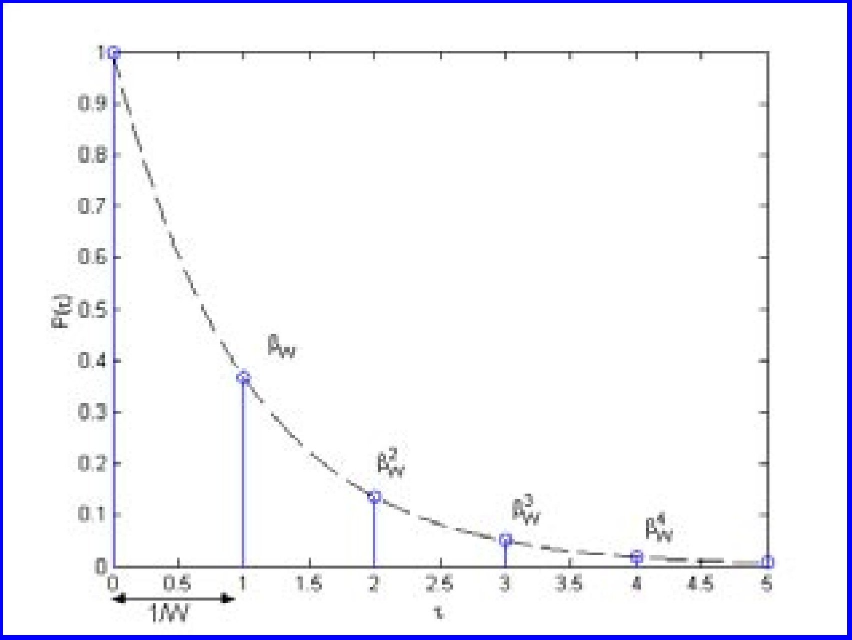


exponential PDP: 0.5 microsec rms delay

Effective Frequency Diversity

- D_f - effective number of iid fading paths in the time domain
- For D iid paths,

$$var[E_c] = \frac{1}{D}$$



- Define D_f as

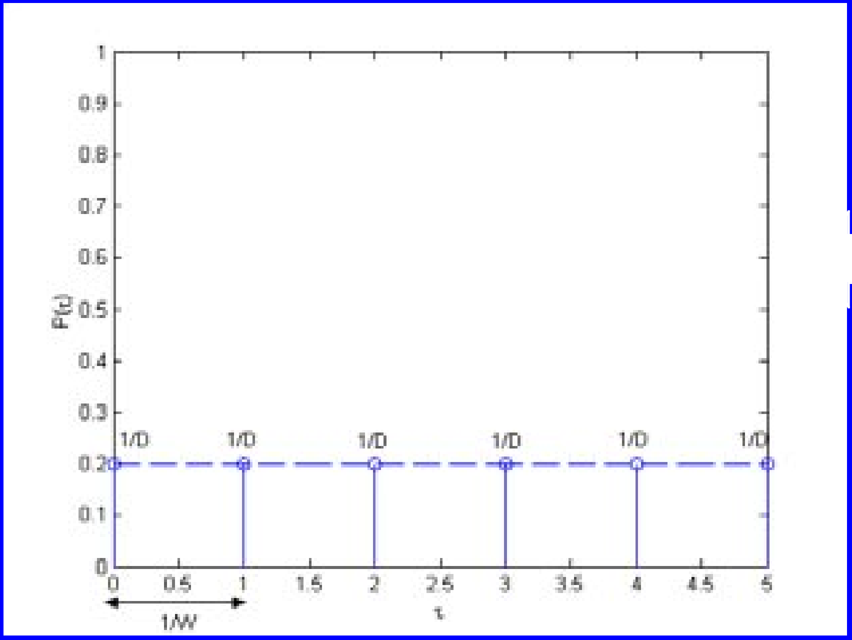
$$D_f = \frac{1}{var[E_c]}$$

➔ $$D_f = \frac{1+\beta_W}{1-\beta_W}$$

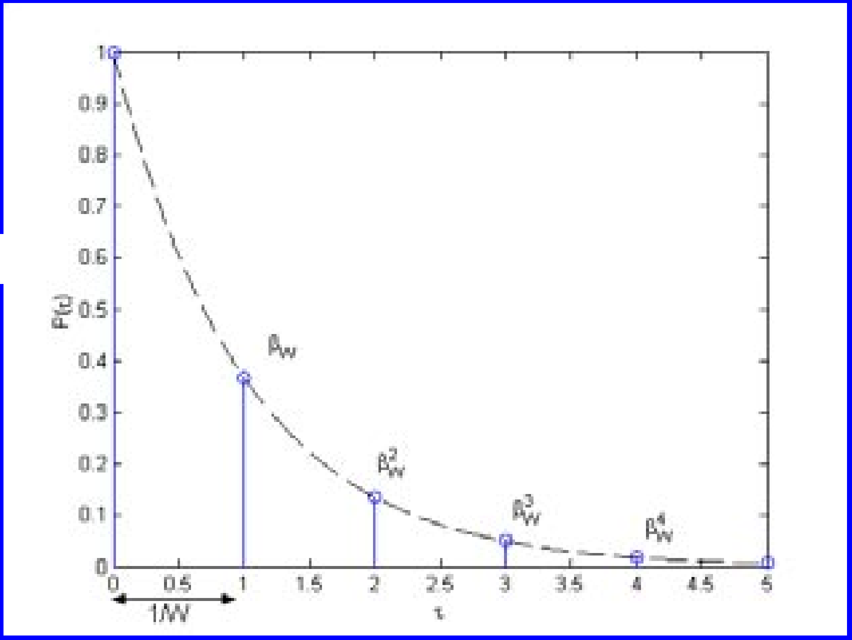
– $$\beta_W = e^{-\frac{1}{W\tau_{rms}}}$$

Physical Interpretation of D_f

D iid paths

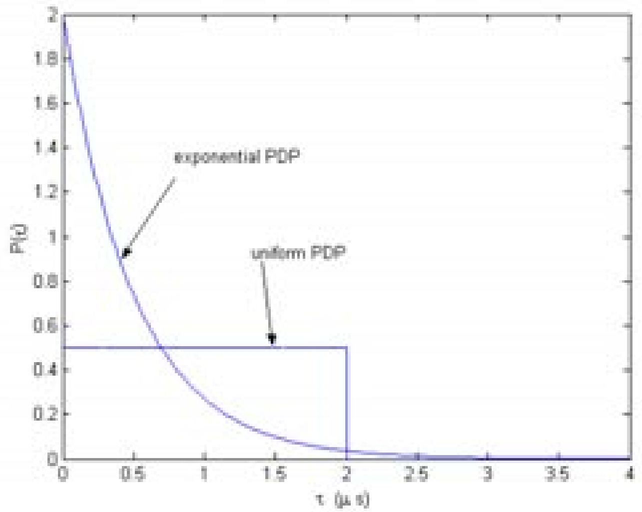


D_f effective iid paths

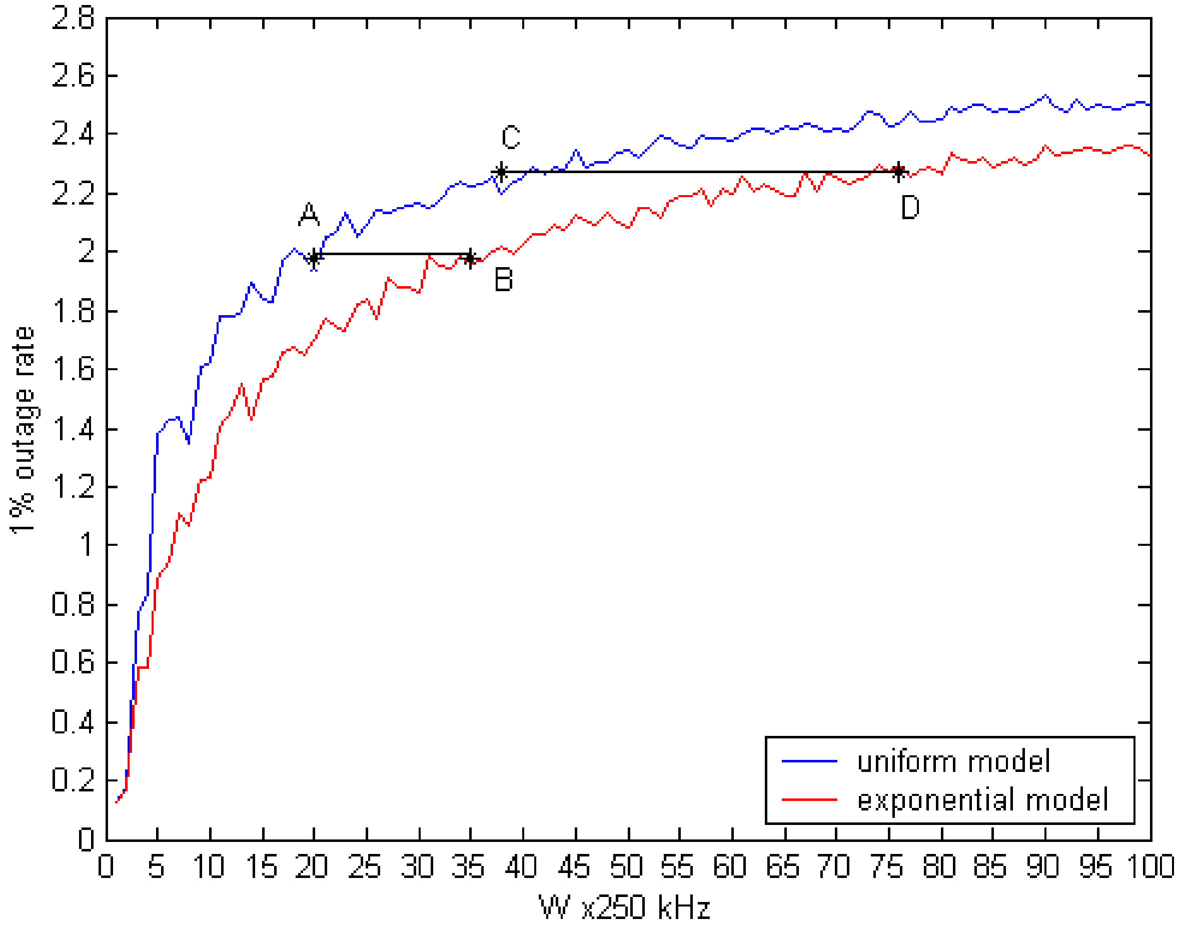


- outage rates should be equivalent when $D=D_f$

Outage rates *ARE* equivalent for D=D_f



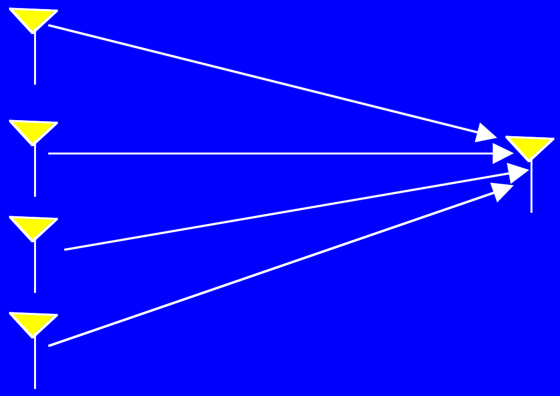
- SNR=10dB
- τ_{rms}=.5μ s
- Point A:D=10
- Point B:D_f=10
- Point C:D=19
- Point D:D_f=19



IIIa. So what? (SISO)

- Gaussian approximation works!
  - Mean = ergodic capacity (depends on SNR)
  - Variance depends on frequency diversity (strongly) and SNR (weakly)
- TDL model works!
- Concept of effective frequency diversity
  - Depends on PDP and bandwidth
  - Independent of SNR

MISO systems



- Full-blown space-time/frequency code
  - iid Gaussian input from all antennas at all frequencies
  - Complex to decode
- Suboptimal strategies
  - Alternate use of antennas

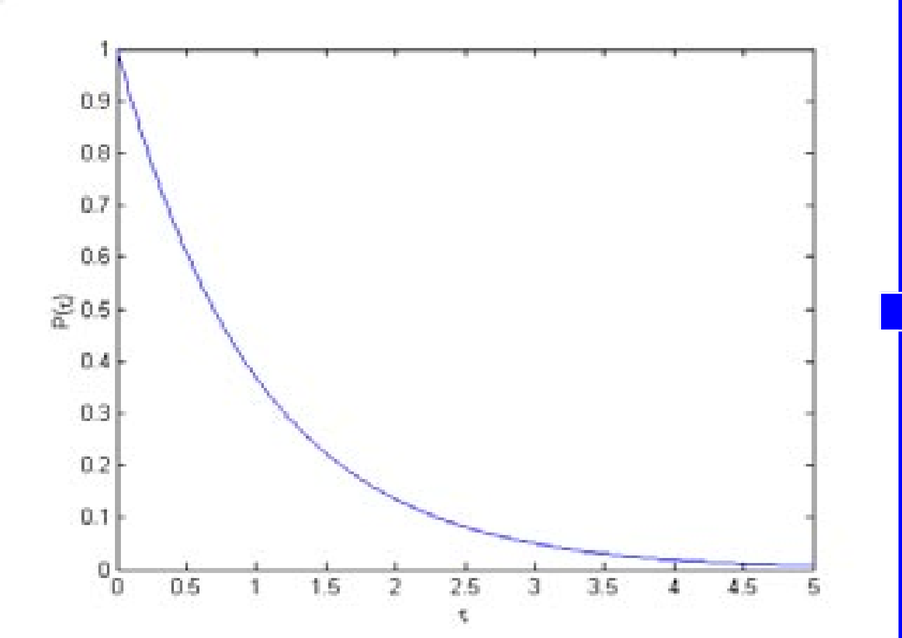
TDL Model: MISO

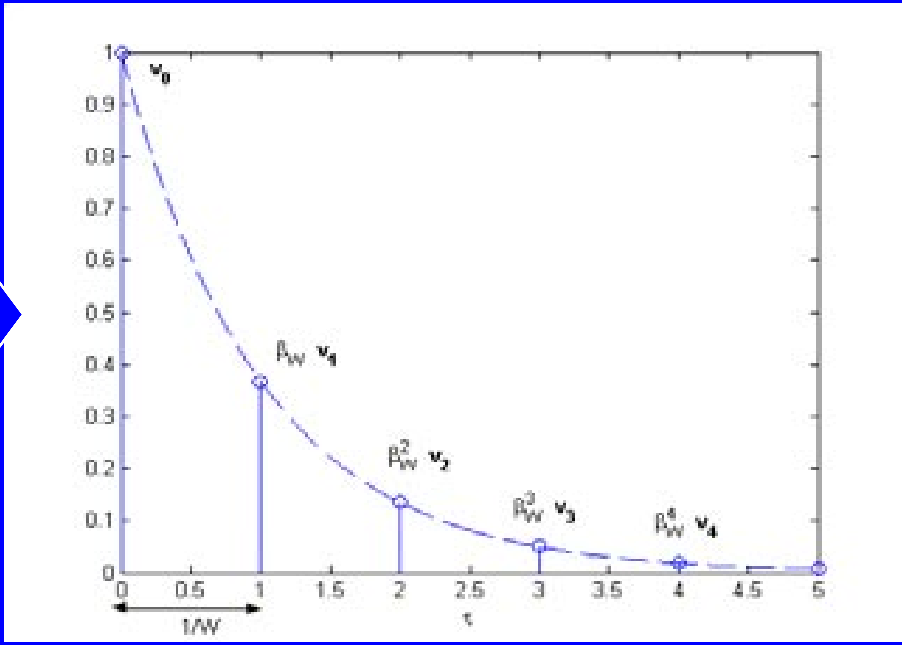
$$h_W(t) = \sum_{i=0}^{\infty} A(\frac{i}{W})\mathbf{v}_i \delta\left(t - \frac{i}{W}\right)$$

$A(\frac{i}{W}) \propto \sqrt{P(\frac{i}{W})}$

$\mathbf{v}_i \sim CN(0, \mathbf{C})$

$\mathbf{C} = E[\mathbf{a}(\Omega)\mathbf{a}(\Omega)^H]$





Narrowband capacity

- For a single frequency bin

$$\mathbf{H}(f) \sim CN(0, \mathbf{C})$$
$$\mathbf{C} = E[\mathbf{a}(\Omega)\mathbf{a}(\Omega)^H]$$

$$I(f) = \log(1 + \tfrac{SNR}{N_T} \| H(f) \|^2) = \log(1 + SNR \sum_{i=1}^{N_T} X_i)$$

X_i energy in ith eigen-direction: exponential with mean λ_i

$\lambda_1, \ldots, \lambda_{N_T}$ eigenvalues of **C**/N_T (same for all f)

Wideband spectral efficiency

$$I_W = (1/W)\int_{-W/2}^{W/2} I(f)\,df$$

Gaussian approximation holds again:

- Mean = E[I(f)] for any f (depends on eigenvalues strongly through sum of squares)
- Variance approx proportional to sum of squares of eigenvalues

Effective Spatial Diversity

Define as

$$D_s = \frac{1}{\sum_{i=1}^{N_T} \lambda_i^2} = \frac{1}{\mathrm{var}(\| H(f) \|^2)}$$

(equals N_T for iid spatial paths: equal e-values)

Proposition 2

$$E[I_W] \approx E\left[log(1 + SNR * X)\right]$$
$$X \sim \Gamma(D_s, 1/D_s)$$

$$var[I_W] \approx \gamma^2 \left(\frac{1}{D_f}\right)\left(\frac{1}{D_s}\right) \quad \gamma \approx \frac{SNR}{SNR+1}$$

Maximum spatial diversity

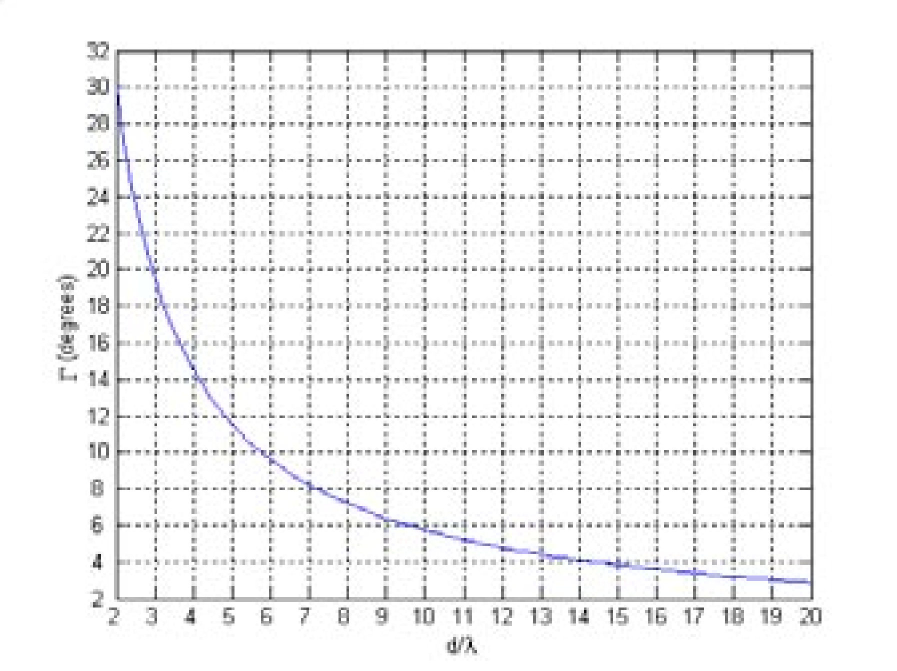
- $D_s = N_T$ if all e-values equal
  - mean maximized, variance minimized
  - ➔ outage rate maximized
- D_s depends on Power Angle Profile and antenna spacing

  For Laplacian PAP

$$\frac{1}{2\alpha}\exp\left(-\frac{|\Omega|}{\alpha}\right)$$

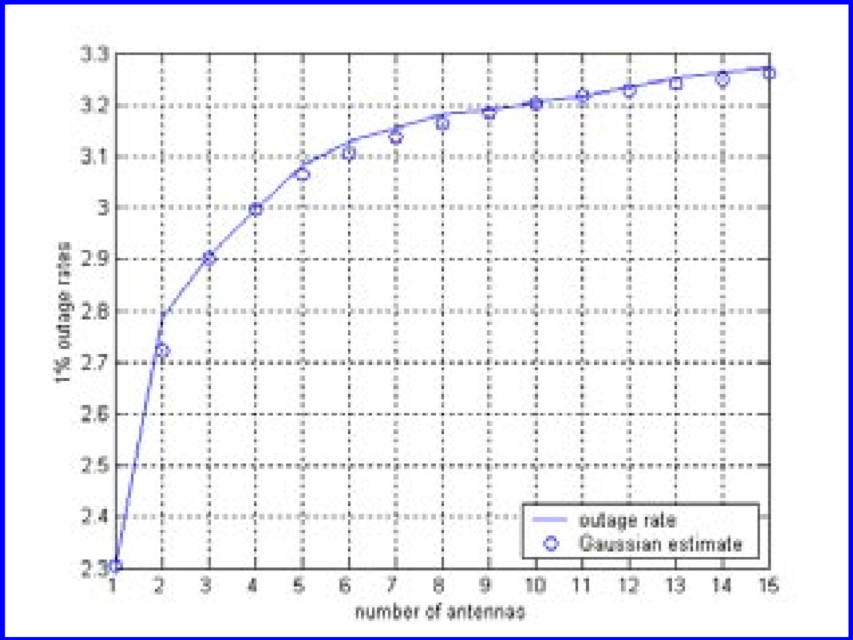
Max diversity requires

$$d > \frac{\lambda}{\sin(2\alpha)}$$

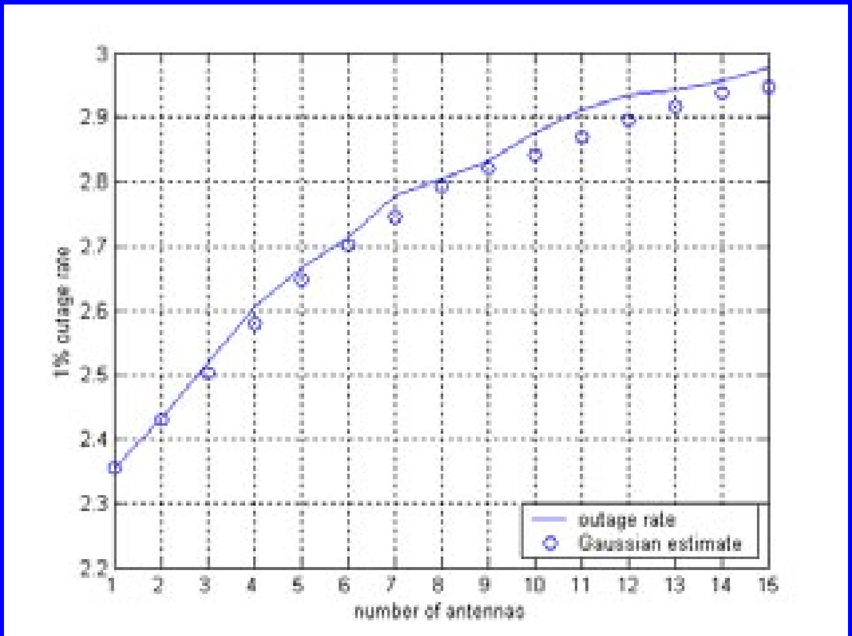


Validating Proposition 2

$D_s = N_T$ $d/\lambda=3$, $\Omega \sim L(0,10^o)$



$D_s \neq N_T$ $d/\lambda=.5$, $\Omega \sim L(0,5^o)$

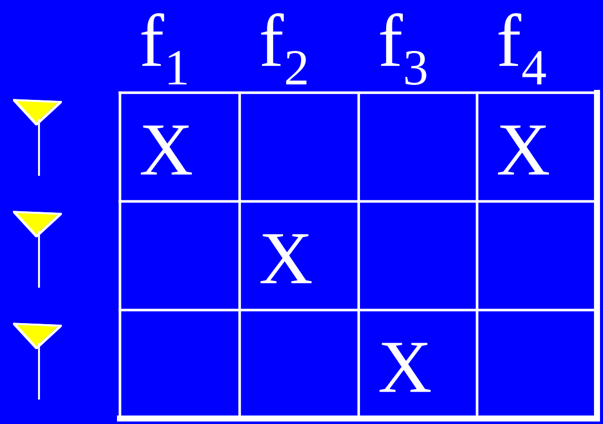


BW=25MHz

SNR=10 dB

τ_{rms}=.5μ s

Alternating Scheme



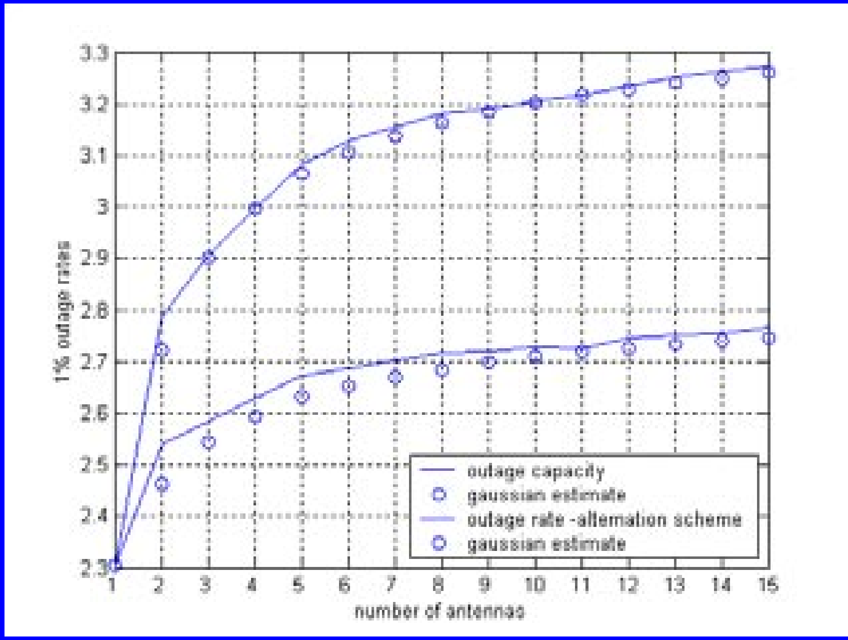
$$I_{alt} \approx N(E[I_{alt}, var[I_{alt}])$$

$$E[I_{alt}] = E[log(1 + SNR|H(f)|^2)]$$ $H(f) \sim CN(0,1)$, SISO case

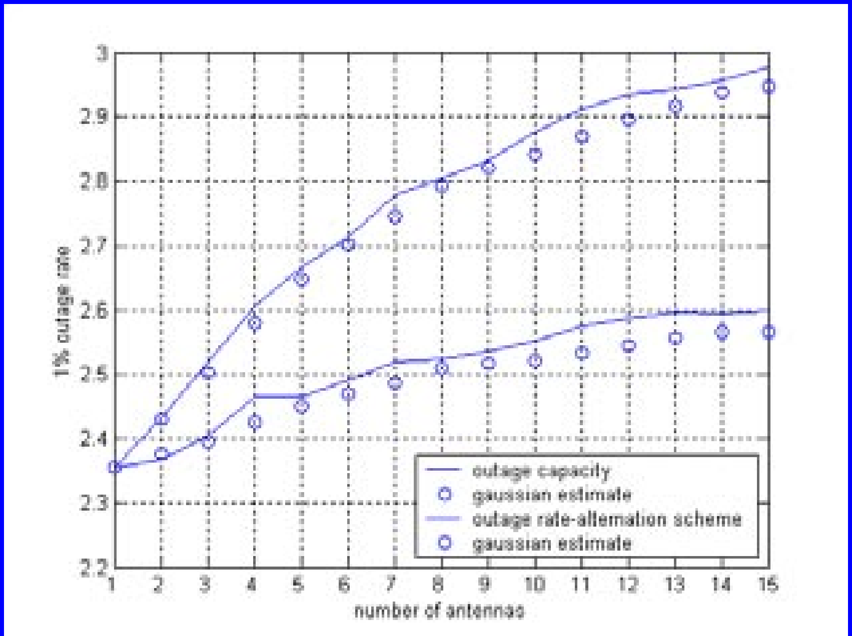
$$var[I_{alt}(N_T)] \approx var[I_W(N_T)]$$

Performance of Alternation

$D_S = N_T$



$D_S \neq N_T$



- Reduces variance as much as full-scale ST code

  ➔ achieves same predictability in performance
- But same mean as SISO

  ➔ obtains less than half of gains available relative to SISO

IIIb. So what? (MISO)

- Gaussian approx, TDL model still work!
- Characterization of spatial diversity
  - Variance reduction due to freq and spatial diversity is multiplicative
  - Predictability achievable simply in MISO
  - Nontrivial space-time/freq code reqd for full MISO gains

For more detail, see…

D. Warrier and U. Madhow, ``Spectrally efficient noncoherent communication," IEEE Trans. Information Theory, vol. 48, no. 3, pp. 651-668, March 2002.

R.-R. Chen, R. Koetter, U. Madhow, D. Agrawal, ``Joint demodulation and decoding for the noncoherent block fading channel: a practical framework for approaching channel capacity," submitted.

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G. Barriac, U. Madhow, ``Characterizing outage rates for space-time communication over wideband wireless channels,'' to appear, Proc. 2002 Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 2002.

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Open Issues

- Noncoherent amplitude/phase modulation
  - Low-complexity demodulation
  - Constellation choice
- Wideband, time-varying systems: theory and practice
- Role of channel feedback