332:541 Stochastic Signals and Systems Fall 1999

Quizlette 1

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1. Once upon a time there was a television game show called "Let's Make a Deal". At the end of every show, the host would give the winner a choice between three doors behind which were prizes. There was a grand prize, an OK prize and a booby prize (like a year's supply of lint).

   Assume that these three prizes are placed with equal probability behind each door (three prizes, three doors) so each door *a priori* was equally likely to have the grand prize, the OK prize and the booby prize behind it.

   You pick a door, say door number 1. Monty Hall (the actual name of the game show host) then tells you which of the remaining doors is NOT the booby prize. If you happen to have initially picked the booby prize, Monty always points to the Grand prize door and tells you that the booby prize is not behind it.

   Should you stick with the door your chose initially, or should you choose a new door? If the latter, which one? (Assume you're trying to maximize the probability you get the grand prize).

   Use probability trees to prove your answer.

   HINT: Make up three trees. One where you always stick with your first door, one where you always change to the door Monty said was NOT the booby prize and one where you change to the other door.

   Following is a description from which you should be able to construct the associated probability trees. Sorry, but drawing them in latex is a pain!

   You should change to the door Monty pointed to. If you stick you have probability 1/3 of getting the grand prize. If you always change to Monty's door you'll never get the booby prize and if you happen to have initially picked the booby prize or the OK prize, you'll always get the grand prize. Thus, always changing gives you a probability 2/3 of getting the grand prize.

   However, if you change to the other door, then if you've initially picked the grand or OK prize door, you'll always get the booby prize. And if you picked the booby prize door, you'll always get the OK prize. So you'll get the grand prize with probability 0 and the booby prize with probability 2/3.

   Monty was basically a nice guy, just trying to help :)