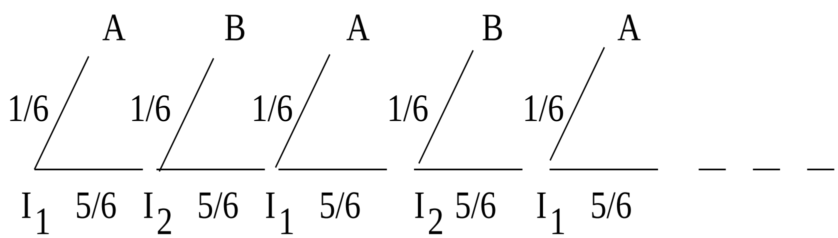
Homework 1

Problem7- Dueling Idiots: Two idiots decide to duel. Their weapon of choice is a six shooter with 5 empty chambers and one bullet. The first idiot takes the pistol, spins the chamber, puts the gun to some vital body part and pulls the trigger. He has 1/6 probability of killing himself, at which point the duel ends. If he does not kill himself, he passes the pistol to the other idiot duelist who re-spins the cartridge and thus also has a 1/6 probability of killing himself. This duel continues until one of the duelists is dead.

a) Draw a probability tree for this experiment b) What is the probability that the first duelist wins the duel? c) Suppose the probability of killing oneself is p (as opposed to 1/6). Is there some value of p which makes the duel fair (both duelists have equal probability of killing themselves)?

Solution: a)



A= the event that the first duelist dies

B= the event that the second duelist dies

b)

$$P[A] = \frac{1}{6} + \frac{5}{6}\frac{5}{6}\frac{1}{6} + \frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{1}{6} + \ldots = \frac{1}{6}\sum_{i=0}^{\infty}\left(\frac{5}{6}\right)^{2i} = \frac{1}{6}\frac{1}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

$$P[B] = \frac{5}{6}\frac{1}{6} + \frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{1}{6} + \frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{1}{6} + \ldots = \frac{1}{6}\sum_{i=0}^{\infty}\left(\frac{5}{6}\right)^{2i+1} = \frac{1}{6}\frac{\frac{5}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{5}{11}$$

The probability that the first duelist wins the duel = the probability that the second duelist dies =P[B] = 5/11.

Since $P[A] > P[B]$ the game is not fair.

c) In this case

$$P[A] = p\sum_{i=0}^{\infty}(1-p)^{2i} = p\frac{1}{1-(1-p)^2} = \frac{1}{2-p}$$

$$P[B] = p\sum_{i=0}^{\infty}(1-p)^{2i+1} = p\frac{1-p}{1-(1-p)^2} = \frac{1-p}{2-p}$$

To make this game fair (P[A]=P[B]) the only choice is p=0 (no more fun!).

Problem8 - Light Bulb Replacement: A given lightbulb has probability p of going out during a 1 minute interval. There are N lightbulbs.

a) What is the distribution on the number of lighbulbs which will go dark assuming the light bulbs are independent of each other? What is the expected number of lightbulbs which will go out?

b) If a repairman has to replace all lightbulbs which go out before the minute is up and can replace K lightbulbs per minute. What is the probability that he cannot replace all the lightbulbs which blow out and thereby loses his job?

Solution

a) X = the number of bulbs that go out during 1 minute interval.

$$P_X(x) = \begin{cases} \left(\begin{array}{c} N \\ x \end{array} \right) p^x (1-p)^{N-x} & x = 0, 1, \ldots, N \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = Np$$

b)

Denote A= the event that the repairman can change all the bulbs that blow out in a minute (no more then K bulbs blow out in a minute).

$$P[A] = P_X(0) + P_X(1) + \ldots + P_X(K) = \sum_{x=0}^{K} \left(\begin{array}{c} N \\ x \end{array} \right) p^x (1-p)^{N-x}$$

The probability of losing his job is 1-P[A].