Rutgers University
The State University Of New Jersey
College of Engineering
Department of Electrical and Computer Engineering

332:501 | Systems Analysis | Fall 2002

FINAL EXAMINATION

You have three hours to answer the following questions with point values as shown (total 150). You are allowed two sides of handwritten notes on an 8.5×11 sheet of paper. Think through each problem BEFORE you begin to write and don't get stuck on one problem. Move on if you are stumped. YOU MUST SHOW ALL WORK. ANSWERS GIVEN WITHOUT WORK RECEIVE NO CREDIT.

GOOD LUCK!

1. *(100 points)* **A Smorgasbord of Systems Analysis Facts:**

   Here are a set of questions which test your general knowledge of the course material. In many ways, they constitute the basic knowledge you MUST have. Keep your answers short and to the point.

   (a) *(10 points)* Suppose a set of N linearly independent vectors $\{\mathbf{x}_i\}$ span some space Ω. Let $\mathbf{v} = \sum_i a_i \mathbf{x}_i$. Is there another different set of coefficients b_i such that $\mathbf{v} = \sum_i b_i \mathbf{x}_i$? Why/why not?

   (b) *(10 points)* A system $T[]$ operates on elements X of some metric space to produce $Y = T[X]$. If T is linear, what properties must it satisfy? Let $T[x(t)] = ax(t) + b$ where $b \neq 0$ is a constant. Is this system linear?

   (c) *(10 points)* Let $\dot{x} + 2tx = 0$. Is this equation linear? Is it time invariant? What is $x(t)$, $t \geq 0$ with $x(0) = 1$?

   (d) *(10 points)* Describe the fundamental idea behind linearization of a nonlinear system. Describe the method by which we perform linearization. Be general and precisely mathematical starting from the differential equation $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, t)$.

   (e) *(10 points)* Carefully define the concept of "system state" for a system which evolves with time.

   (f) *(10 points)* The characteristic equation of an LTI system is $s^3 + s^2 + s - 1$. Is this system stable or unstable? How about $-s^3 - s^2 - s - 1$?

   (g) *(10 points)* What is a Lyapunov function for a system? Find a Lyapunov function for the system $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$.

   (h) *(10 points)* What is BIBO stability? If a general system is globally asymptotically stable, must it be BIBO stable as well? NOTE: not just LTI.

   (i) *(10 points)* For a continuous time system, what is the definition of complete controllability? How do we determine in general whether a linear system is completely controllable? What is the controllability condition for a linear time invariant system.

(j) *(10 points)* Repeat the previous question for observability.

2. *(40 points)* **Swaying Bridges:**

   Once when driving into Rutgers from New York City, I got stuck on the George Washington bridge in a traffic jam. Nothing moved for about an hour. During that hour I noticed just how much a bridge deck jostles up and down in the wind. This problem is inspired by that experience.

   The abstraction of my situation is depicted in FIGURE 1

   (a) *(10 points)* Derive a differential equation which describes the motion along the vertical axis (y axis) of the mass at the center of the bridge. Assume the springs themselves are massless. Assume the rest length of the springs is $L/2$ and that when the mass in in position $y = 0$, there is no tension in the springs.

   (b) *(20 points)* Is this system linear or nonlinear?

   (c) *(10 points)* If we want a rest displacement of $-1cm$ with a mass of 1000kg, what should the spring constant k be? $L = 1km$.

   (d) *(10 points)* What is the resonant frequency of the system?

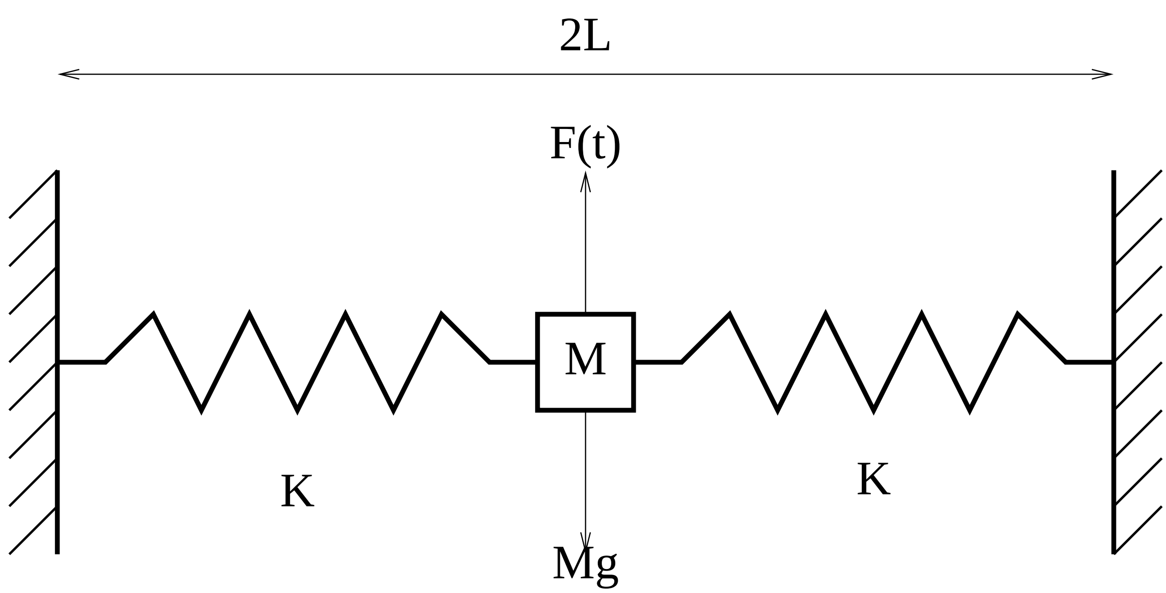

Figure 1: Abstraction of a professor stuck in the middle of a bridge.